MARK BOX		
PROBLEM	POINTS	
0	12	
1	10	
2	10	
3	10	
4	10	
5-16	48=12x4	
%	100	

HAND IN PART

NAME: _____

PIN: _____

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND IN PART. Hand in only this part.
  (2) STATEMENT OF MULTIPLE CHOICE PROBLEMS. Do not hand in this part.
  You can take this part home to learn from and to check your answers once the solutions are posted.
- On Problem 0, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, personal notes, electronic devices, and any device with which you can connect to the internet. Unauthorized materials (including cell phones, as well as your watch) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold during the exam (it will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets.
- Cheating is grounds for a F in the course.
- At a student's request, I will project my watch upon the projector screen.
  Upon request, you will be given as much (blank) scratch paper as you need.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- This exam covers (from *Calculus* by Thomas, 13th ed., ET): §10.1–10.6 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : _____

0. Fill-in the boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$, unless otherwise indicated.

0.1. State the $n^{\mathbf{th}}$**-term test** for an arbitrary series $\sum a_n$.

0.2. Fix $r \in \mathbb{R}$ with $r \neq 1$. For $N \geq 7$, let $s_N = \sum_{\mathbf{n=7}}^{N} r^n$. (Note the sum starts at 7.) For each $N \geq 7$, the partial sums s_N can be written as: (your answer should NOT contain a "…" nor a "$\sum$" sign)

$$s_N = \boxed{}.$$

0.3. **Geometric Sequence**. $\lim_{n\to\infty} r^n = 0$ if and only if r satisfies $\boxed{}$.

0.4. **Geometric Series**. The series $\sum r^n$ converges if and only if r satisfies $\boxed{}$.

0.5. p**-series**. The series $\sum \frac{1}{n^p}$ converges if and only if $\boxed{}$.

0.6. State the **Direct Comparison Test** for a positive-termed series $\sum a_n$.

- If $0 \leq a_n \leq c_n$ when $n \geq 17$, then $\boxed{}$.
- If $0 \leq d_n \leq a_n$ when $n \geq 17$, then $\boxed{}$.

Hint: sing the song to yourself.

0.7. State the **Limit Comparison Test** for a positive-termed series $\sum a_n$.
Let $b_n > 0$ and $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

- If $\boxed{}$, then $[\sum b_n$ converges $\iff \sum a_n$ converges $]$.

Goal: cleverly pick positive b_n's so that you know what $\sum b_n$ does (converges or diverges) and the sequence $\left\{\frac{a_n}{b_n}\right\}_n$ converges.

0.8. State the **Ratio and Root Tests** for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$. Let

$$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \qquad \text{or} \qquad \rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} .$$

- If $\boxed{}$, then $\sum a_n$ is absolutely convergent.
- If $\boxed{}$, then the test is inconclusive.

0.9. The **Integral Test with Remainder Estimate** says that if $\sum a_n$ satisfies the conditions of the Integral Test for a function $y = f(x)$ with $f(n) = a_n$, then

$$0 \leq \left(\sum_{k=1}^{\infty} a_k\right) - \left(\sum_{k=1}^{N} a_k\right) \leq \boxed{}.$$

0.10. The **Alternating Series Estimation Theorem** says that if $\{u_n\}_{n=1}^{\infty}$ satisfies the conditions of the Alternating Series Test, then for $N \in \mathbb{N}$,

$$\left|\left(\sum_{k=1}^{\infty} (-1)^k u_k\right) - \left(\sum_{k=1}^{N} (-1)^k u_k\right)\right| \leq \boxed{}.$$

1. Determine the behavior of the given series. Circle only one choice per series.

Series	absolutely convergent	conditionally convergent	divergent
$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{1}{n}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{1}{n^2}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{1}{e^n}$	AC	CC	DVG
$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$	AC	CC	DVG
$\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$	AC	CC	DV

▶
▶
▶.

On the by-hand problems: 2,3 and 4,
justify your answer **below** the box and then put your answer **in** the box.
Show all your work neatly and concisely. Explicitly specify which test(s) you use.
You will be graded on the quality and correctness of your justification.

2. For a natural number $n > 2$, let

$$a_n = \frac{(n-1)!}{(3n)!}$$

2.1. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} =$$

2.2. Determine the behavior of the given series. You may use the previous part of this problem.

$$\sum_{n=2}^{\infty} \frac{(n-1)!}{(3n)!}$$

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

3. Consider the series

$$\sum_{n=1}^{\infty}\left[\cos\left(\frac{1}{n+1}\right) - \cos\left(\frac{1}{n+2}\right)\right] . \tag{3.1}$$

3.1. Find an expression for the Nth partial sum s_N of the series in (3.1).
Your solution should NOT contain a summation $\sum$ sign or the ... sign.

$s_N =$

3.2. If the series in (3.1) converges, then in the box write the number to which it converges.
If the series in (3.1) diverges, then in the box write "diverges".

$$\sum_{n=1}^{\infty}\left[\cos\left(\frac{1}{n+1}\right) - \cos\left(\frac{1}{n+2}\right)\right]$$

4. Determine the behavior of the given series.

$$\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$$

- [ ] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

MULTIPLE CHOICE PROBLEMS

The statement of the multiple choice problems, along with the scoring, can be found in the Statement of Multiple Choice Problems portion of the exam.

Table for Multiple Choice Solutions								Do Not Write Below			
PROBLEM							# of sol'ns circled	1	2	B	x
5	5a	5b	5c	5d	5e	5f					
6	6a	6b	6c	6d	6e	6f					
7	7a	7b	7c	7d	7e	7f					
8	8a	8b	8c	8d	8e	8f					
9	9a	9b	9c	9d	9e	9f					
10	10a	10b	10c	10d	10e	10f					
11	11a	11b	11c	11d	11e	11f					
12	12a	12b	12c	12d	12e	12f					
13	13a	13b	13c	13d	13e	13f					
14	14a	14b	14c	14d	14e	14f					
15	15a	15b	15c	15d	15e	15f					
16	16a	16b	16c	16d	16e	16f					
Fill in the "number of solutions circled" column with: 0, 1, or 2.											
								4	1	0	0

STATEMENT OF MULTIPLE CHOICE PROBLEMS
This portion of the exam is **NOT** collected.
Thus you do not have to show your work.

Instructions:

- **When you initially work through the Multiple Choice Problems**, indicate your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS portion of the exam, which is not collected.
- **After you have finished working all of the Multiple Choice Problems**, circle your answers in the TABLE FOR MULTIPLE CHOICE SOLUTIONS, which is collected.
- You may choice up to **2** answers for each multiple choice problem. The scoring is as follows.
  - ∗ For a problem with precisely one answer marked and the answer is correct, 4 points.
  - ∗ For a problem with precisely two answers marked, one of which is correct, 1 points.
  - ∗ All other cases, 0 points.

Abbreviations used:

- DCT is Direct Comparison Test.
- LCT is Limit Comparison Test.
- AST is Alternating Series Test.

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5. The $\lim_{n\to\infty} \dfrac{\sqrt[2]{9n^2-8n+7}}{\sqrt[3]{8n^3+7n^2-6n-5}}$

a. converges to 0

b. diverges to ∞

c. converges to $\frac{9}{8}$

d. converges to $\frac{3}{2}$

e. diverges but does not diverges to ∞.

f. None of the others.

6. The $\lim_{n\to\infty} \dfrac{\sin n}{n}$

a. converges to 0

b. converges to 1

c. diverges to ∞

d. diverges to $-\infty$

e. diverges but does not diverge to $\pm\infty$

f. None of the others.

7. The sequence $\{a_n\}_{n=1}^{\infty}$, where $a_n = \dfrac{(-1)^n\ (n+1)}{5-2n}$,

a. converges to $\dfrac{1}{2}$

b. diverges

c. converges to $-\dfrac{1}{2}$

d. converges to 0

e. converges to 1

f. None of the others.

8. The sequence $\{a_n\}_{n=1}^{\infty}$, where $a_n = \dfrac{\ln n}{\ln(2n)}$,

a. converges to 1

b. converges to 2

c. converges to $\dfrac{1}{2}$

d. converges to 0

e. diverges

f. None of the others.

9. Which of the following statements is always true?

I. If $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$.

II. If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

III. $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n-1}}{7^{2n}}$ is a geometric series with ratio $r = \dfrac{4}{7}$.

a. I only

b. II and III only

c. I and II only

d. III only

e. I and III only

f. None of the others.

10. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, 2^n}{5^{n+1}} =$

a. $\dfrac{1}{3}$

b. $\dfrac{1}{7}$

c. $-\dfrac{2}{15}$

d. $\dfrac{2}{15}$

e. $-\dfrac{2}{35}$

f. None of the others.

11. Suppose $s_n = \dfrac{n+1}{3n-2}$ is the sequence of partial sums for the series $\displaystyle\sum_{n=1}^{\infty} a_n$. Find a_3 and $\displaystyle\sum_{n=1}^{\infty} a_n$.

a. $a_3 = \dfrac{4}{7}$ and $\displaystyle\sum_{n=1}^{\infty} a_n = \frac{1}{3}$

b. $a_3 = \dfrac{1}{3}$ and $\displaystyle\sum_{n=1}^{\infty} a_n = -\frac{5}{28}$

c. $a_3 = -\dfrac{1}{14}$ and $\displaystyle\sum_{n=1}^{\infty} a_n = \frac{1}{3}$

d. $a_3 = -\dfrac{5}{28}$ and $\displaystyle\sum_{n=1}^{\infty} a_n = \frac{1}{3}$

e. $a_3 = -\dfrac{5}{28}$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges

f. None of the others.

12. For what values of p does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge?

a. $p \geq 1$

b. $p > 1$

c. $p > 0$

d. $0 < p < 1$

e. for all p

f. None of the others.

13. The series $\sum_{n=1}^{\infty} ne^{-n^2}$

a. converges since $\int_1^{\infty} xe^{-x^2}\,dx$ converges to $\frac{1}{2}$.

b. converges since $\int_1^{\infty} xe^{-x^2}\,dx$ converges to $\frac{1}{2e}$.

c. converges since $\int_1^{\infty} xe^{-x^2}\,dx$ converges to $\frac{1}{e}$.

d. converges since $\int_1^{\infty} xe^{-x^2}\,dx$ converges to $\frac{2}{e}$.

e. diverges.

f. None of the others.

14. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(2n)+3}{\sqrt{n}}$

a. converges by the DCT, using for comparison $\displaystyle\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}}$

b. diverges by the DCT, using for comparison $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$

c. diverges by the DCT, using for comparison $\displaystyle\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}}$

d. converges by the DCT, using for comparison $\displaystyle\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$

e. diverges by the DCT, using for comparison $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}}$

f. None of the others.

15. The series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(n+1)!}$

a. diverges by the integral test

b. converges by the ratio test

c. converges by the integral test

d. diverges by the ratio test

e. diverges by the nth-terms test

f. None of the others.

16. What is your favorite series convergence test?

a. n^{th}-term test

b. limit comparison test

c. ratio test

d. I love each one equally.

e. I hate them ALL, big time!

f. None of the others.