MARK BOX		
PROBLEM	POINTS	
1-30	90=3x30	
Ch 11 on MML	10	
%	100	

HAND IN PART

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND-IN PART. Hand-in only this part.
  (2) NOT TO HAND-IN PART. This part will not be collected.
  Take this part home to learn from and to check your answers when the solutions are posted.
- **For the Multiple Choice** problems, circle your answer(s) on the provided chart. No need to show work.
- The MARK BOX above indicates the problems (check that you have them all) along with their points.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, personal notes, electronic devices, and any device with which you can connect to the internet. Unauthorized materials (including cell phones, as well as your watch) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, give it to Prof. Girardi to hold for you during the exam (and it will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- Cheating is grounds for a F in the course.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13^{th} ed., ET): §8.1–8.5, 8.7, 8.8, 10.1–10.10, 11.1–11.5 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

I verify that I did NOT receive help from other people or devices on the MML portion of this exam.

Signature : ______________________________

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to the multiple choice problems.
- You may choose up to **1** answers for each multiple choice problem.
  The scoring is as follows.
  - ∗ For a problem with precisely one answer marked and the answer is correct, 3 points.
  - ∗ All other cases, 0 points.

At most ONE choice per problem. Table for Your Muliple Choice Solutions

PROBLEM						leave this column blank
1	1a	1b	1c	1d	1e	
2	2a	2b	2c	2d	2e	
3	3a	3b	3c	3d	3e	
4	4a	4b	4c	4d	4e	
5	5a	5b	5c	5d	5e	
6	6a	6b	6c	6d	6e	
7	7a	7b	7c	7d	7e	
8	8a	8b	8c	8d	8e	
9	9a	9b	9c	9d	9e	
10	10a	10b	10c	10d	10e	
11	11a	11b	11c	11d	11e	
12	12a	12b	12c	12d	12e	
13	13a	13b	13c	13d	13e	
14	14a	14b	14c	14d	14e	
15	15a	15b	15c	15d	15e	
16	16a	16b	16c	16d	16e	
17	17a	17b	17c	17d	17e	
18	18a	18b	18c	18d	18e	
19	19a	19b	19c	19d	19e	
20	20a	20b	20c	20d	20e	
21	21a	21b	21c	21d	21e	
22	22a	22b	22c	22d	22e	
23	23a	23b	23c	23d	23e	
24	24a	24b	24c	24d	24e	
25	25a	25b	25c	25d	25e	
26	26a	26b	26c	26d	26e	
27	27a	27b	27c	27d	27e	
28	28a	28b	28c	28d	28e	
29	29a	29b	29c	29d	29e	
30	30a	30b	30c	30d	30e	

NOT TO HAND-IN PART
STATEMENT OF MULTIPLE CHOICE PROBLEMS

- Hint. For a definite integral problems $\int_a^b f(x)\,dx$.
  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.
  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint. Laws of Logs. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.
- Abbreviations used with Series:
  - DCT is Direct Comparison Test.
  - LCT is Limit Comparison Test.
  - AST is Alternating Series Test.

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1. Evaluate

$$\int_3^{27} \frac{1}{2x}\,dx.$$

You can use the Laws of Logs (see above Hint).

a. $\ln 3$

b. $\ln 5$

c. $\ln 9$

d. $\ln 25$

e. $\ln 27$

2. Find the polynomial $y = p(x)$ so that

$$\int (p(x))\, e^{x^2}\,dx = x e^{x^2} + C\ .$$

Recall that $e^{x^2} = e^{\left(x^2\right)}$. Also note that we cannot integrate the function $y = e^{x^2}$ with techniques we have learned thus far (in fact, $y = e^{x^2}$ does not have elementary antiderivative). Have you yet read the Hints at the top of page?

a. $p(x) = x^3$

b. $p(x) = 2x$

c. $p(x) = x + 1$

d. $p(x) = 2x^2 + 1$

e. None of the others.

►. Problems **1** and **2** were meant to reiterate the importance of the above two Hints.
Kept them in mind while doing the rest of the integration problems.

3. Evaluate

$$\int_0^1 \frac{36\,dx}{(2x+1)^3}\ .$$

Answer:

a. 8

b. $\frac{4}{9}$

c. $\frac{40}{9}$

d. $\frac{20}{81}$

e. None of the others.

4. Evaluate

$$\int_0^{\pi} \sin^2 5r\,dr\ .$$

Answer:

a. $\frac{5\pi}{2}$

b. $\frac{5\pi}{2} - \frac{1}{4}$

c. $\frac{\pi}{2}$

d. $\frac{\pi}{2} - \frac{1}{20}$

e. None of the others.

5. Evaluate

$$\int_0^{\pi/2} \sin^3 x \cos^4 x\,dx\ .$$

Answer:

a. 0

b. $\frac{2}{35}$

c. $\frac{4}{35}$

d. $\frac{12}{35}$

e. None of the others.

6. Evaluate

$$\int_0^1 x^2 e^x \, dx \ .$$

Answer:

a. $e-2$

b. $e+1$

c. $2e+1$

d. $2e+2$

e. None of the others.

7. Evaluate

$$\int_1^e x^2 \ln x \, dx \ .$$

Answer:

a. e

b. $e+1$

c. $\dfrac{e^3+1}{9}$

d. $\dfrac{2e^3+1}{9}$

e. None of the others.

8. Evaluate

$$\int\limits_{x=0}^{x=\pi} e^{3x} \, \cos 2x \ dx \ .$$

Answer:

a. 0

b. $\dfrac{3}{13}\left(e^{3\pi}-1\right)$

c. $\dfrac{3}{5}\left(e^{3\pi}\right)$

d. $\dfrac{3}{5}\left(e^{3\pi}-1\right)$

e. None of the others.

9. Evaluate

$$\int_3^7 \frac{dx}{x^2 - 6x + 25} \ .$$

Hint. Complete the square: $x^2 - 6x + 25 = (x \pm \ ? \ )^2 \pm \ ?$.

a. $\ln \frac{4}{3}$

b. $\ln \frac{32}{25}$

c. $\frac{\pi}{16}$

d. $\frac{\pi}{4}$

e. None of the others.

10. Evaluate

$$\int_3^5 \frac{\sqrt{25 - x^2}}{x} \, dx \ .$$

Answer:

a. $5 \ln 3 - 4$

b. $\ln \frac{5}{4} \ 8$

c. $\ln \frac{5}{3}$

d. $\frac{1}{5} \ln \frac{5}{3}$

e. None of the others.

11. Evaluate

$$\int_1^2 \frac{dx}{x\,(x+1)^2} \ .$$

Answer:

a. $\ln \frac{4}{3}$

b. $\ln \frac{4}{3} - \frac{1}{6}$

c. $\ln 2$

d. $\ln 2 - \frac{1}{6}$

e. None of the others.

12. Evaluate

$$\int_0^1 \frac{dx}{(x+1)\,(x^2+1)} \ .$$

Answer:

a. $\dfrac{\ln 2}{4} + \dfrac{\pi}{8}$

b. $\dfrac{\ln 2}{2} + \dfrac{\pi}{8}$

c. $\dfrac{\ln 2}{4} + \dfrac{5\pi}{8}$

d. $\dfrac{\ln 2}{2} + \dfrac{5\pi}{8}$

e. None of the others.

13. Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} \ .$$

Answer:

a. $\dfrac{\pi}{2}$

b. π

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

14. Evaluate

$$\int_{x=-1}^{x=1} \frac{1}{x^6}\, dx \ .$$

Answer:

a. $\frac{2}{5}$

b. $\frac{-2}{5}$

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

15. Evaluate

$$\int_{x=-1}^{x=1} \frac{1}{x^5}\, dx \ .$$

Answer:

a. 0

b. $\frac{1}{2}$

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

16. Investigate the convergence of

$$\int_{x=1}^{x=\infty} \frac{1-e^{-x}}{x}\, dx \ .$$

a. The integral converges by the Limit Comparison Test for Improper Integrals, comparing the integrand with $g(x) = \dfrac{1}{x}$.

b. The integral diverges by the Limit Comparison Test for Improper Integrals, comparing the integrand with $g(x) = \dfrac{1}{x}$.

c. The integral converges by the Direct Comparison Test for Improper Integrals, comparing the integrand with $g(x) = \dfrac{1}{x}$.

d. The integral diverges by the Direct Comparison Test for Improper Integrals, comparing the integrand with $g(x) = \dfrac{1}{x}$.

e. None of the others.

17. Evaluate

$$\lim_{n\to\infty} \frac{\sqrt{25n^3+4n^2+n-5}}{7n^{\frac{3}{2}}+6n-1} \ .$$

Answer:

a. 0

b. $\frac{5}{7}$

c. $\frac{25}{7}$

d. ∞

e. None of the others.

18. Find **all** real numbers r satisfying that

$$\sum_{n=2}^{\infty} r^n \;=\; \frac{1}{2}\,.$$

a. $\frac{1}{2}$

b. $\frac{-1}{2}$ and $\frac{1}{3}$

c. $\frac{-1}{3}$ and $\frac{1}{4}$

d. $\frac{-1}{4}$ and $\frac{1}{5}$

e. None of the others.

19. Consider the following two series.

Series A is $\sum_{n=1}^{\infty} \frac{1}{n}$

Series B is $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

a. both series converge absolutely

b. both series diverge

c. series A converges conditionally and series B diverges

d. series A diverges and series B converges conditionally

e. None of the others.

20. The formal series (note: in the demoninator is the cube root $\sqrt[3]{\ }$, not the square root $\sqrt[2]{\ }$)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt[3]{(n+1)(n+2)(n+3)}}\,.$$

is

a. absolutely convergent, as seen by the LCT (using for comparison $\sum \frac{1}{n^{3/2}}$).

b. conditionally convergent, as seen by using only the AST and not other tests.

c. conditionally convergent, as seen by using the LCT (using for comparison $\sum \frac{1}{n}$) as well as the AST.

d. divergent.

e. None of the others.

21. The formal series

$$\sum_{n=17}^{\infty} \frac{1}{n \ln n}$$

is:

a. convergent by the integral test.

b. divergent by the integral test

c. convergent by the ratio test.

d. divergent by the ratio test

e. None of the others.

22. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \, n!}{(3n)!} \; .$$

Let

$$a_n = \frac{(-1)^n \, n!}{(3n)!} \qquad \text{and} \qquad \rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \; .$$

Then

a. $\sum_{n=1}^{\infty} \frac{(-1)^n \, n!}{(3n)!}$ converges absolutely by the Ratio Test because $\rho = 0$.

b. $\sum_{n=1}^{\infty} \frac{(-1)^n \, n!}{(3n)!}$ converges absolutely by the Ratio Test because $\rho = \frac{1}{3}$.

c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} \frac{(-1)^n \, n!}{(3n)!}$.

d. $\rho > 1$ so by the Ratio Test $\sum_{n=1}^{\infty} \frac{(-1)^n \, n!}{(3n)!}$ diverges

e. None of the others.

23. The formal series

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.23}}$$

is

a. divergent by the n^{th}-term test

b. divergent by the Direct Comparison Test, using for comparison $\sum \frac{1}{n}$

c. absolutely convergent by the Direct Comparison Test, using for comparison $\sum \frac{1}{n^2}$

d. absolutely convergent by the Direct Comparison Test, using for comparison $\sum \frac{1}{n^{1.01}}$

e. None of the others.

24. Let c be a natural number (i.e., $c \in \{1, 2, 3, 4, \ldots\}$). The series

$$\sum_{n=1}^{\infty} \frac{(n!)^6}{(cn)!}$$

a. converges when $c < 6$ and diverges when $c \geq 6$

b. converges when $c \leq 6$ and diverges when $c > 6$

c. diverges when $c < 6$ and converges when $c \geq 6$

d. diverges when $c \leq 6$ and converges when $c > 6$

e. None of the others.

25. What is the LARGEST interval for which the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+15)^n}{4^n}$$

is absolutely convergent?

a. $\left(\frac{11}{5}, \frac{19}{5}\right)$

b. $\left[\frac{11}{5}, \frac{19}{5}\right]$

c. $\left(\frac{-19}{5}, \frac{-11}{5}\right)$

d. $\left[\frac{-19}{5}, \frac{-11}{5}\right]$

e. None of the others.

26. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{2}{3-x}$$

about the center $x_0 = 0$ and state when this Taylor series is valid. Hint:

$$f(x) = \frac{2}{3-x} = \left(\frac{2}{3}\right)\left(\frac{1}{1-\frac{x}{3}}\right).$$

by simple algebra.

a. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n x^n$, valid for $|x| < 1$

b. $\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n$, valid for $|x| < 3$

c. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 1$

d. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 3$

e. None of the others.

27. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{1}{(1-x)^4}$$

about the center $x_0 = 0$ which is valid for $|x| < 1$. Hint. Start with the Taylor series expansion

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \qquad \text{valid for } |x| < 1$$

and differentiate (as many times as needed). Be careful and don't forget the chain rule:

$$D_x(1-x)^{-1} = (-1)(1-x)^{-2}\, D_x(1-x) = (-1)(1-x)^{-2}\,(-1) = (1-x)^{-2}.$$

Answer:

a. $\sum_{n=0}^{\infty} \frac{(n)(n-1)(n-2)}{6} x^{n-3}$

b. $\sum_{n=0}^{\infty} (n)(n-1)(n-2)\, x^n$

c. $\sum_{n=0}^{\infty} \frac{(n+3)(n+2)(n+1)}{6} x^n$

d. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+3)(n+2)(n+1)}{6} x^n$

28. Let the function $y = f(x)$ have a power series power series representation $\sum_{n=0}^{\infty} c_n (x - x_0)^n$, which is valid in some interval $(x_0 - R, x_0 + R)$ with $R > 0$. What, if anything, can you say about the second derivative $f''(x_0)$ of f evaluated at x_0?

a. Then $f''(x_0)$ must be c_0.

b. Then $f''(x_0)$ must be c_2.

c. Then $f''(x_0)$ must be $2c_2$.

d. Then we know that $f''(x_0)$ exists but we do not know what the value of $f''(x_0)$ is.

e. None of the others.

29. Find the 2nd order Taylor polynomial for the function $f(x) = \sqrt[3]{x}$ about the center $x_0 = 8$.

a. $P_2(x) = 2 + \dfrac{x}{12} - \dfrac{x^2}{9(2^5)}$

b. $P_2(x) = 2 + \dfrac{(x-8)}{12} + \dfrac{(x-8)^2}{9(2^5)}$

c. $P_2(x) = 2 + \dfrac{(x-8)}{12} - \dfrac{(x-8)^2}{9(2^5)}$

d. $P_2(x) = 2 + \dfrac{(x-8)}{12} - \dfrac{(x-8)^2}{9(2^4)}$

e. None of the others.

30. Consider the function
$$f(x) = e^{-x} \ .$$
The 5^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is
$$P_5(x) \ = \ \sum_{n=0}^{5} \frac{(-x)^n}{n!} \ = \ 1 \ - \ x \ + \ \frac{x^2}{2!} \ - \ \frac{x^3}{3!} \ + \ \frac{x^4}{4!} \ - \ \frac{x^5}{5!} \ .$$
The 5^{th} order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $e^{-x} \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\dfrac{(e)(3^5)}{5!}$

b. $\dfrac{(e^{-3})(3^5)}{5!}$

c. $\dfrac{(e)(3^6)}{6!}$

d. $\dfrac{(e^{-3})(3^6)}{6!}$

e. None of the others.