MARK BOX		
PROBLEM	POINTS	
0	10	
1	14	
2-15	56=14x4	
16	10	
17	10	
%	100	

HAND IN PART

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

- This exam comes in two parts.
  1. HAND IN PART. Hand in only this part.
  2. STATEMENT OF MULTIPLE CHOICE PROBLEMS. Do not hand in this part.
     You can take this part home to learn from and to check your answers once the solutions are posted.
- **On Problem 0**, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, personal notes, electronic devices, and any device with which you can connect to the internet. Unauthorized materials (including cell phones, as well as your watch) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, give it to Prof. Girardi to hold for you during the exam (and it will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- Cheating is grounds for a F in the course.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13^{th} ed., ET): §10.1–10.6 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ______________________________

0. Fill-in the boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$, unless otherwise indicated.

0.1. For a formal series $\sum_{n=1}^{\infty} a_n$, where each $a_n \in \mathbb{R}$, consider the corresponding sequence $\{s_n\}_{n=1}^{\infty}$ of partial sums, so $s_n = \sum_{k=1}^{n} a_k$. By definition, the formal series $\sum a_n$ converges if and only if

[ ]

0.2. ***p*-series**. Fill in the boxes with the proper range of $p \in \mathbb{R}$.

- The series $\sum \frac{1}{n^p}$ converges if and only if [ ] .

0.3. **Geometric Series**. Fill in the boxes with the proper range of $r \in \mathbb{R}$.

- The series $\sum r^n$ converges if and only if r satisfies [ ] .

0.4. State the **Direct Comparison Test** for a positive-termed series $\sum a_n$.

- If [ ] when $n \geq 17$ and $\sum c_n$ converges, then $\sum a_n$ converges.
- If [ ] when $n \geq 17$ and $\sum d_n$ diverges, then $\sum a_n$ diverges.

Hint: sing the song to yourself.

0.5. State the **Limit Comparison Test** for a positive-termed series $\sum a_n$.

Let $b_n > 0$ and $L = \lim_{n\to\infty} \frac{a_n}{b_n}$.

- If [ ] , then [$\sum b_n$ converges $\iff$ $\sum a_n$ converges].
- If $L = 0$, then [ ] .
- If $L = \infty$, then [ ] .

Goal: cleverly pick positive b_n's so that you know what $\sum b_n$ does (converges or diverges) and the sequence $\left\{\frac{a_n}{b_n}\right\}_n$ converges.

0.6. State the **Ratio and Root Tests** for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$. Let

$$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \qquad \text{or} \qquad \rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} .$$

- If $\rho > 1$, then $\sum a_n$ [ ] .
- If $\rho < 1$, then $\sum a_n$ [ ] .

1. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.

The symbol $\sum$ is understood to mean $\sum_{n=1}^{\infty}$.

Scoring this problem: A problem with precisely one answer marked and the answer is correct, 1 point. All other cases, 0 points.

1.1	T	F	If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
1.2	T	F	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
1.3	T	F	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.
1.4	T	F	A series $\sum a_n$ is precisely one of the following: absolutely convergent, conditionally convergent, divergent.
1.5	T	F	If $a_n \geq 0$ for all $n \in \mathbb{N}$, then $\sum a_n$ is either absolutely convergent or divergent.
1.6	T	F	If $\sum \|a_n\|$ converges, then $\sum a_n$ converges.
1.7	T	F	If $\sum\ a_n$ diverges, then $\sum \|a_n\|$ diverges.
1.8	T	F	If $\sum \|a_n\|$ diverges, then $\sum a_n$ diverges.
1.9	T	F	If a function $f\colon [0,\infty) \to \mathbb{R}$ satisfies that $\lim_{x\to\infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n, then $\lim_{n\to\infty} a_n = L$.
1.10	T	F	If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n\to\infty} a_n = L$ and $f\colon [0,\infty) \to \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n, then $\lim_{x\to\infty} f(x) = L$.
1.11	T	F	The $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=17}^{\infty} a_n$ converges.
1.12	T	F	If $\sum a_n$ converges, then $\sum (a_n + 0.01)$ converges.
1.13	T	F	If $\sum_{n=1}^{\infty} p^n$ diverges, then $\sum_{n=1}^{\infty} (p+0.01)^n$ diverges, for a fixed real number p.
1.14	T	F	If $\sum_{n=1}^{\infty} n^{-p}$ converges, then $\sum_{n=1}^{\infty} n^{-p+0.01}$ converges, for a fixed real number p.

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to the multiple choice problems.
- You may choice up to **2** answers for each multiple choice problem. The scoring is as follows.
  - ∗ For a problem with precisely one answer marked and the answer is correct, 4 points.
  - ∗ For a problem with precisely two answers marked, one of which is correct, 1 points.
  - ∗ All other cases, 0 points.
- Fill in the "number of solutions circled" column.

Table for Your Muliple Choice Solutions							Do Not Write Below			
PROBLEM						number of solutions circled	1	2	B	x
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
11	11a	11b	11c	11d	11e					
12	12a	12b	12c	12d	12e					
13	13a	13b	13c	13d	13e					
14	14a	14b	14c	14d	14e					
15	15a	15b	15c	15d	15e					
							4	1	0	0
							Total:			

16. Below the choice-boxes (AC/CC/Divg), carefully justify the behavior of the series. Be sure to clearly explain your logic and specify which test(s) you are using. Then check the correct choice-box.

$$\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

17. Below the choice-boxes (AC/CC/Divg), carefully justify the behavior of the series. Be sure to clearly explain your logic and specify which test(s) you are using. Then check the correct choice-box. You may use Part 17.1 in Part 17.2.

17.1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(3n-2)^n}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

17.2. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(3n-2)^{n+\frac{1}{2}}}$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

To clarify, the quantity $(3n-2)$ in the denominator is raised to the power $(n+\frac{1}{2})$.

STATEMENT OF MULTIPLE CHOICE PROBLEMS
These sheets of paper are not collected.

Abbreviations used:

- DCT is Direct Comparison Test.
- LCT is Limit Comparison Test.
- AST is Alternating Series Test.

2. Limit of a sequence. Evaluate

$$\lim_{n\to\infty} \sqrt{\frac{2n}{n+1}}\ .$$

a. 0

b. 2

c. $\sqrt{2}$

d. ∞

e. None of the others.

3. Limit of a sequence. Evaluate

$$\lim_{n\to\infty} \frac{\sin n}{n}.$$

a. 0

b. 1

c. ∞

d. diverges but not to $\pm\infty$

e. None of the others.

4. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{1}{n+3^n}.$$

a. This series diverges since $\dfrac{1}{n+3^n} = \dfrac{1}{n} + \dfrac{1}{3^n}$ and $\sum \dfrac{1}{n}$ diverges.

b. This series diverges by the DCT, using for comparison $\sum \dfrac{1}{n}$.

c. This series diverges by the DCT, using for comparison $\sum \dfrac{1}{3^n}$.

d. This series converges by the DCT, using for comparison $\sum \dfrac{1}{3^n}$.

e. None of the others.

5. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}}\ .$$

a. This series is absolutely convergent by the LCT, using for comparison $\sum \frac{1}{n^2}$.

b. This series is conditionally convergent, as can be shown by using only the AST and no other tests.

c. This series is conditionally convergent by LCT (using for comparison $\sum \frac{1}{n}$) as well as AST.

d. This series diverges.

e. None of the others.

6. Consider the formal series

$$\sum_{k=1}^{\infty} \frac{k^2}{e^k}.$$

a. This series converges because the limit used in the Ratio Test is $\frac{1}{e}$.

b. This series diverges because the limit used in the Ratio Test is $\frac{1}{e}$.

c. This series converges because the limit used in the Ratio Test is e.

d. This series diverges because the limit used in the Ratio Test is e.

e. None of the others.

7. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}\ .$$

(Remember your Algebra Priorities: $e^{n^2} = e^{\left(n^2\right)}$.)

a. This series converges because the limit used in the Ratio Test is 0.

b. This series converges because the limit used in the Ratio Test is $\frac{1}{e}$.

c. This series converges because the limit used in the Ratio Test is $\frac{1}{e^2}$.

d. This series diverges because the limit used in the Ratio Test is ∞.

e. None of the others.

8. Consider the formal series

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right).$$

a. This series converges because the limit used in the Root Test is ∞.

b. This series converges because the limit used in the Root Test is 0 .

c. This series converges by the Integral Test.

d. This series diverges by the n^{th} term test.

e. This series diverges by the LCT, using for comparison $\sum \frac{1}{n}$.

f. None of the others.

9. What is the smallest integer N such that the Alternating Series Estimate/Remainder Theorem guarentees that

$$\left|\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} - \sum_{n=1}^{N} \frac{(-1)^n}{n^2}\right| \leq 0.05?$$

Note that $0.05 = \frac{0.05}{1.0000} = \frac{5}{100} = \frac{1}{20}$.

a. 3

b. 4

c. 5

d. 6

e. None of the others.

10. Find the vaule for b for which

$$e^b + e^{2b} + e^{3b} + e^{4b} + \ldots = 9.$$

a. $\ln \frac{8}{9}$

b. $\frac{8}{9}$

c. $\ln \frac{9}{10}$

d. $\frac{9}{10}$

e. None of the others.

11. Determine the behavior of the series

$$\sum_{n=1}^{\infty} \frac{2^n\, n!\, n!}{(2n)!}.$$

a. The series converges because the limit used in the Ratio Test is 0.

b. The series converges because the limit used in the Ratio Test is $\frac{1}{2}$.

c. The series diverges because the limit used in the Ratio Test is 2.

d. The series diverges because the limit used in the Ratio Test is ∞.

e. None of the others.

12. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1},$$

by the <u>Direct</u> Comparison Test,

a. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$.

b. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$.

c. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$.

d. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$.

e. None of the others.

13. The series

$$\sum_{n=1}^{\infty} \frac{1}{1+\ln n},$$

by the <u>Limit</u> Comparison Test,

a. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n}\right)} = 0$.

b. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n}\right)} = \infty$.

c. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n^2}\right)} = 0$.

d. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n^2}\right)} = \infty$.

e. None of the others.

14. The series

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n},$$

is

a. absolutely convergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

b. conditionally convergent, as shown by using only the Alternating Series Test (and no other tests).

c. conditionally convergent, as shown by using both the Alternating Series Test and the Integral Test.

d. divergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

e. None of the others.

15. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(2n+1)^n}{n^{2n}}.$$

a. This series diverges by the n^{th}-term test.

b. This series diverges because the limit used in the Root Test is ∞.

c. This series diverges because the limit used in the Root Test is 2.

d. This series is absolutely convergent because the limit used in the Root Test is 0.

e. None of the others.