MARK BOX		
PROBLEM	POINTS	
0	10	
1-13	65=13x5	
14	5	
15	10	
16	10	
%	100	

$\boxed{\textbf{HAND IN PART}}$

NAME: _____

PIN: _____

INSTRUCTIONS

- This exam comes in two parts.
  - (1) HAND IN PART. Hand in <u>only</u> this part.
  - (2) STATEMENT OF MULTIPLE CHOICE PROBLEMS. Do <u>not</u> hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- This exam has 3 types of problems: Problem 0, Multiple Choice, and Show All Your Work.
  - **On Problem 0**, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
  - **For Multiple Choice problems**, circle your answer(s) on the provided chart. No need to show work. The STATEMENT OF MULTIPLE CHOICE PROBLEMS will not be collected.
  - **For the Show All Your Work problems**, to receive credit you **MUST**:
    - (1) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
    - (2) if a line/box is provided, then:
      - — show you work BELOW the line/box
      - — put your answer on/in the line/box
    - (3) if no such line/box is provided, then box your answer.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, electronic devices, any device with which you can connect to the internet, and personal notes. Unauthorized materials (including cell phones, as well as your watch) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- Cheating is grounds for a F in the course.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13th ed., ET): §8.1-8.5, 8.7, 8.8 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : _____

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0. Fill-in the blanks.

0.1. $\arcsin(\,-\frac{1}{2}\,) = $ _____ (Your answers should be an angle in **RADIANS**.)

0.2. Double-angle Formula. Your answer should involve trig functions of θ, and not of 2θ.

$\sin(2\theta) = $ _____

0.3. $\int \dfrac{du}{u} \overset{u \neq 0}{=}$ _____ $+ \ C$

0.4. $\int u^n \, du \overset{n \neq -1}{=}$ _____ $+C$

0.5. $\int e^u \, du = $ _____ $+C$

0.6. $\int \sec^2 u \, du \ = $ _____ $+ \ C$

0.7. $\int \sec u \, du \ = $ _____ $+ \ C$

0.8. $\int \dfrac{1}{a^2 + u^2} \, du \overset{a \geq 0}{=}$ _____ $+ \ C$

0.9. Trig sub.: if the integrand involves $u^2 - a^2$, then one makes the substitution $u \ = $ _____

0.10. Integration by parts formula: $\int u \, dv \ = $ _____

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to the multiple choice problems.
- You may choice up to **2** answers for each multiple choice problem. The scoring is as follows.
    * For a problem with precisely one answer marked and the answer is correct, 5 points.
    * For a problem with precisely two answers marked, one of which is correct, 2 points.
    * All other cases, 0 points.
- Fill in the "number of solutions circled" column. (Worth a total of 1 point of extra credit.)

Table for Your Muliple Choice Solutions						number of solutions circled	Do Not Write Below			
PROBLEM							1	2	B	x
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
11	11a	11b	11c	11d	11e					
12	12a	12b	12c	12d	12e					
13	13a	13b	13c	13d	13e					
							5	2	0	0
							Extra Credit:			

14. Evaluate $\int \left(1 + \sqrt{x}\right)^{123} dx.$ Show your work <u>below</u> the box then put your answer <u>in</u> the box.

$$\int \left(1 + \sqrt{x}\right)^{123} \; dx \; = \hspace{8cm} + \; C$$

15. Show your work below the box then put your answer in the box. You may use part (a) in part (b).

15a. Evaluate $\int \sin^2 x \, dx$.

$$\int \sin^2 x \, dx \;=\; \hspace{10cm} + \text{C}$$

15b. Evaluate $\int x \sin^2 x \, dx$.

$$\int x \sin^2 x \, dx \;=\; \hspace{10cm} + \text{C}$$

16. Part 16a should help with part 16b. Show your work below the box then put your answer in the box.

16a. Evaluate $\int e^{\left(x^2\right)} (2x) \, dx$.

$$\int e^{\left(x^2\right)} (2x) \; dx \;\; = \hspace{9cm} + \; C$$

16b. We cannot integrate the functions $y = e^{x^2}$ and $y = x^2 \, e^{x^2}$ with techniques we have learned thus far (in fact, they do not have elementary antiderivatives). But we can integrate $y = (2x^2 + 1) \, e^{x^2}$ with techniques we know thus far. Evaluate $\int (2x^2 + 1) \, e^{x^2} \, dx$.

$$\int (2x^2 + 1) \, e^{x^2} \; dx \;\; = \hspace{8cm} + \; C$$

<div style="border:1px solid black; display:inline-block; padding:10px;">

STATEMENT OF MULTIPLE CHOICE PROBLEMS
These sheets of paper are <u>not</u> collected.

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- <u>Hint</u>. For a definite integral problems $\int_a^b f(x)\,dx$.

  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.

  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).

  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.

- <u>Hint</u>. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r\ln a$.

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1. Evaluate the integral

$$\int_0^1 \frac{x}{x^2 + 9}\,dx.$$

    a. $\frac{1}{2}\left(\ln 10 - \ln 9\right)$

    b. $\frac{1}{2}\left(\ln 1 - \ln 0\right)$

    c. $\left(\ln 10 - \ln 9\right)$

    d. $\left(\ln 1 - \ln 0\right)$

    e.   None of the others.

2. Evaluate the integral

$$\int_0^4 \frac{x}{x + 9}\,dx\ .$$

    a. $4 - 9\ln(13) + 9\ln(9)$

    b. $13 - 9\ln(4) + \ln(3)$

    c. $(1/(9\ln(13))) - \ln(3)$

    d. $4 - 13\ln(9) + 3\ln(18)$

    e.   None of the others.

3. Evaluate

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \, dx \ .$$

    a. $\dfrac{3}{10}$

    b. $\dfrac{7}{10}$

    c. $\dfrac{8}{15}$

    d. $\dfrac{2}{15}$

    e. None of the others.

4. Evaluate

$$\int_{x=5}^{x=10} \frac{\sqrt{x^2 - 25}}{x} \, dx$$

AND specify the initial substitution.

    a. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$.

    b. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$

    c. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$.

    d. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$

    e. None of the others.

5. Evaluate the integral

$$\int_0^{\pi/2} \cos^3 x \, dx.$$

    a. $\dfrac{2}{3}$

    b. $\dfrac{4}{3}$

    c. $\dfrac{1}{4}$

    d. $-\dfrac{1}{4}$

    e. None of the others.

6. Evaluate the integral

$$\int_1^2 \frac{8}{x^2 - 2x + 2}\, dx.$$

Hint: complete the square in the denominator.

    a. $8\left(\tan^{-1} 2 - \frac{\pi}{4}\right)$

    b. 2π

    c. $8\ln 2$

    d. $\frac{\pi}{4}$

    e. None of the others.

7. Evaluate the integral

$$\int_1^2 x \ln x \, dx.$$

    a. $\ln 2$

    b. $\frac{1}{2}$

    c. $\ln 4 + \frac{3}{4}$

    d. $\ln 4 - \frac{3}{4}$

    e. None of the others.

8. Evaluate the integral

$$\int_0^{\pi/8} e^{3x} \cos(4x)\, dx.$$

    a. $e^{3\pi/8} + 1$

    b. $\dfrac{4e^{3\pi/8} - 3}{25}$

    c. $\dfrac{4e^{3\pi/8} - 3}{7}$

    d. $\dfrac{e^{3\pi/8} - 1}{7}$

    e. None of the others.

9. Evaluate the integral

$$\int_0^{\sqrt{3}/2} \frac{4x^2\,dx}{(1-x^2)^{3/2}}$$

    a. $4\sqrt{3} - 2\ln\left|2+\sqrt{3}\right|$

    b. $4\sqrt{3} + 2\ln\left|2+\sqrt{3}\right|$

    c. $4\sqrt{3} - \dfrac{4\pi}{3}$

    d. $4\sqrt{3}$

    e. None of the others.

10. Evaluate the integral

$$\int_0^{\sqrt{3}} \frac{3t^2 + t + 4}{t^3 + t}\,dt$$

    a. $\ln\left(\dfrac{9}{\sqrt{2}}\right) + \dfrac{\pi}{12}$

    b. $\ln\left(\dfrac{9}{\sqrt{2}}\right) - \dfrac{\pi}{12}$

    c. $\ln\left(\dfrac{9}{2}\right) + \dfrac{\pi}{12}$

    d. $\ln\left(\dfrac{9}{2}\right) - \dfrac{\pi}{12}$

    e. None of the others.

11. Let $y = p(x)$ be a polynomial of degree 5.
What is the form of the partial fraction decomposition of

$$\frac{p(x)}{(x^2 - 1)(x^2 + 1)^2} \quad ?$$

Here A, B, C, D, E and F are constants.

    a. $\dfrac{A}{x^2 - 1} + \dfrac{B}{(x^2 + 1)^2}$

    b. $\dfrac{Ax + B}{x^2 - 1} + \dfrac{Cx + D}{(x^2 + 1)^2}$

    c. $\dfrac{Ax + B}{x^2 - 1} + \dfrac{Cx + D}{x^2 + 1} + \dfrac{Ex + F}{(x^2 + 1)^2}$

    d. $\dfrac{A}{x - 1} + \dfrac{B}{x + 1} + \dfrac{Cx + D}{x^2 + 1} + \dfrac{Ex + F}{(x^2 + 1)^2}$

    e. None of the others.

12. Evaluate

$$\int_{-1}^{1} \frac{dx}{x^2} \ .$$

   a. -2

   b. 2

   c. diverges to infinity

   d. does not exist but also does not diverge to infinity

   e. None of the others.

13. Evaluate

$$\int_{-1}^{1} \frac{dx}{x^3} \ .$$

   a. 0

   b. $\frac{1}{4}$

   c. diverges to infinity

   d. does not exist but also does not diverge to infinity

   e. None of the others.