MARK BOX		
PROBLEM		
1-30	30	
%	100	

HAND IN PART

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND IN PART. Hand in only this part.
  (2) NOT TO HAND-IN PART. This part will not be collected.
  Take this part home to learn from and to check your answers when the solutions are posted.
- **For the Multiple Choice** problems, circle your answer(s) on the provided chart.
  No need to show work.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, personal notes, electronic devices, and any device with which you can connect to the internet. Unauthorized materials (including cell phones, as well as your watch) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold during the exam (it will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets.
- Cheating is grounds for a F in the course.
- At a student's request, I will project my watch upon the projector screen.
  Upon request, you will be given as much (blank) scratch paper as you need.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- This exam covers (from *Calculus* by Thomas, 13^{th} ed., ET):
  §8.1–8.5, 8.7, 8.8, 10.1–10.10, 11.1–11.5 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ______________________________

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table your solution to the multiple choice problems.
- You may choose up to **1** answers for each multiple choice problem.
  The scoring is as follows.
  * For a problem with precisely one answer marked and the answer is correct, $\frac{10}{3} = 3.\overline{3}$ points.
  * All other cases, 0 points.

At most ONE choice per problem. Table for Your Muliple Choice Solutions						
PROBLEM						leave this column blank
1	1a	1b	1c	1d	1e	
2	2a	2b	2c	2d	2e	
3	3a	3b	3c	3d	3e	
4	4a	4b	4c	4d	4e	
5	5a	5b	5c	5d	5e	
6	6a	6b	6c	6d	6e	
7	7a	7b	7c	7d	7e	
8	8a	8b	8c	8d	8e	
9	9a	9b	9c	9d	9e	
10	10a	10b	10c	10d	10e	
11	11a	11b	11c	11d	11e	
12	12a	12b	12c	12d	12e	
13	13a	13b	13c	13d	13e	
14	14a	14b	14c	14d	14e	
15	15a	15b	15c	15d	15e	
16	16a	16b	16c	16d	16e	
17	17a	17b	17c	17d	17e	
18	18a	18b	18c	18d	18e	
19	19a	19b	19c	19d	19e	
20	20a	20b	20c	20d	20e	
21	21a	21b	21c	21d	21e	
22	22a	22b	22c	22d	22e	
23	23a	23b	23c	23d	23e	
24	24a	24b	24c	24d	24e	
25	25a	25b	25c	25d	25e	
26	26a	26b	26c	26d	26e	
27	27a	27b	27c	27d	27e	
28	28a	28b	28c	28d	28e	
29	29a	29b	29c	29d	29e	
30	30a	30b	30c	30d	30e	

NOT TO HAND-IN PART
STATEMENT OF MULTIPLE CHOICE PROBLEMS

- Hint. For a definite integral problems $\int_a^b f(x)\,dx$.
  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.
  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint. Laws of Logs. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.
- Abbreviations used with Series:
  - DCT is Direct Comparison Test.
  - LCT is Limit Comparison Test.
  - AST is Alternating Series Test.

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1. Evaluate the integral

$$\int_0^1 \frac{x}{x^2+9}\,dx.$$

a. $\frac{1}{2}(\ln 10 - \ln 9)$

b. $\frac{1}{2}(\ln 1 - \ln 0)$

c. $(\ln 10 - \ln 9)$

d. $(\ln 1 - \ln 0)$

e. None of the others.

2. Evaluate the integral

$$\int_0^4 \frac{x}{x+9}\,dx\ .$$

a. $4 - 9\ln(13) + 9\ln(9)$

b. $13 - 9\ln(4) + \ln(3)$

c. $(1/(9\ln(13)) - \ln(3)$

d. $4 - 13\ln(9) + 3\ln(18)$

e. None of the others.

3. Evaluate

$$\int_{0}^{\ln(2\pi)} e^{x} \cos\left(e^{x}\right) \, dx$$

a. $e^{2\pi}$

b. $e^{2\pi} - 1$

c. $-\sin(1)$

d. $\sin(1)$

e. None of the others.

4. Evaluate

$$\int_{x=0}^{x=\frac{3\pi}{2}} e^{x} \cos x \, dx \; .$$

a. $\dfrac{1 + e^{3\pi/2}}{2}$

b. $\dfrac{1 - e^{3\pi/2}}{2}$

c. $\dfrac{-1 + e^{3\pi/2}}{2}$

d. $\dfrac{-1 - e^{3\pi/2}}{2}$

e. None of the others.

5. Investigate the convergence of

$$\int_{x=1}^{x=\infty} \frac{1 - e^{-x}}{x} \, dx \; .$$

a. The integral converges by the Limit Comparison Test, comparing the integrand with $g(x) = \dfrac{1}{x}$.

b. The integral diverges by the Limit Comparison Test, comparing the integrand with $g(x) = \dfrac{1}{x}$.

c. The integral converges by the Direct Comparison Test, comparing the integrand with $g(x) = \dfrac{1}{x}$.

d. The integral diverges by the Direct Comparison Test, comparing the integrand with $g(x) = \dfrac{1}{x}$.

e. None of the others.

6. Evaluate

$$\int_{x=0}^{x=1} \sin^4 x \, dx \ .$$

a. 1

b. π

c. $1 + \sin 2 + \sin 4$

d. $\frac{3}{8} - \frac{1}{4}\sin 2 + \frac{1}{32}\sin 4$

e. None of the others.

7. Evaluate

$$\int_{x=5}^{x=10} \frac{\sqrt{x^2 - 25}}{x} \, dx$$

AND specify the initial substitution.

a. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$.

b. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$

c. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$.

d. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$

e. None of the others.

8. Evaluate

$$\int_{x=1}^{x=3} \frac{5x^2 + 3x - 2}{x^3 + 2x^2} \, dx \ .$$

a. $3\ln 5 - \ln 3 - \frac{2}{3}$

b. $3\ln 5 - \ln 3 - \frac{8}{3}$

c. $\ln 5 - \frac{2}{3}$

d. $\frac{2}{3} - \ln 5$

e. None of the others.

9. Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} \ .$$

Answer:

a. $\frac{\pi}{2}$

b. π

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

10. For which value of p does

$$\int_0^1 \frac{1}{x^p}\, dx = 1.25 \ ?$$

a. $p = 0.2$

b. $p = 0.5$

c. $p = 2.0$

d. $p = 2.5$

e. None of the others.

11. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^3}\ dx \ .$$

a. 0

b. $\frac{1}{4}$

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

12. Limit of a sequence. Evaluate

$$\lim_{n\to\infty} \frac{\sqrt{25n^3+4n^2+n-5}}{7n^{\frac{3}{2}}+6n-1} \ .$$

a. 0

b. ∞

c. $\frac{25}{7}$

d. $\frac{5}{7}$

e. None of the others.

13. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{1}{n+3^n}.$$

a. This series diverges since $\dfrac{1}{n+3^n} = \dfrac{1}{n} + \dfrac{1}{3^n}$ and $\sum \dfrac{1}{n}$ diverges.

b. This series diverges by the DCT, using for comparison $\sum \dfrac{1}{n}$.

c. This series diverges by the DCT, using for comparison $\sum \dfrac{1}{3^n}$.

d. This series converges by the DCT, using for comparison $\sum \dfrac{1}{3^n}$.

e. None of the others.

14. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n+2)(n+7)} \ .$$

a. This series is absolutely convergent by the LCT, using for comparison $\sum \frac{1}{n^2}$.

b. This series is conditionally convergent, as can be shown by using only the AST and no other tests.

c. This series is conditionally convergent by LCT (using for comparison $\sum \frac{1}{n}$) as well as AST.

d. This series is conditionally convergent by LCT (using for comparison $\sum \frac{1}{n^2}$) as well as AST.

e. None of the others.

15. By using the Limit Comparison Test, one can show that the formal series

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{(n+1)\,(n+2)\,(n+3)\,(n+4)}}\,. \tag{15}$$

is:

a. convergent by comparing the series in (15) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 1$.

b. convergent by comparing the series in (15) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = \dfrac{3}{2}$.

c. divergent by comparing the series in (15) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 1$.

d. divergent by comparing the series in (15) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = \dfrac{3}{2}$

e. None of the others.

16. Find **<u>all</u>** real numbers r satisfying that

$$\sum_{n=2}^{\infty} r^n = \frac{1}{6}\,.$$

a. $\dfrac{1}{6}$

b. $\dfrac{1}{4}$ and $\dfrac{-1}{3}$

c. $\dfrac{-1}{2}$ and $\dfrac{1}{3}$

d. $\dfrac{-1}{3}$ and $\dfrac{1}{3}$

e. None of the others.

17. The series

$$\sum_{n=2}^{\infty} (-1)^{n+1}\,\frac{1}{n\ln n},$$

is

a. absolutely convergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

b. conditionally convergent, as shown by using only the Alternating Series Test (and no other tests).

c. conditionally convergent, as shown by using both the Alternating Series Test and the Integral Test.

d. divergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

e. None of the others.

18. What is the smallest integer N such that the Alternating Series Estimate/Remainder Theorem guarentees that

$$\left|\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}-\sum_{n=1}^{N}\frac{(-1)^n}{n^2}\right|\leq 0.05?$$

Note that $0.05=\frac{0.05}{1.0000}=\frac{5}{100}=\frac{1}{20}$.

a. 3

b. 4

c. 5

d. 6

e. None of the others.

19. Find the 4^{th} order Maclaurin polynonial for

$$f(x)=\frac{1}{(x+1)^3}.$$

a. $-3x+6x^2-10x^3+15x^4$

b. $1-3x+6x^2-10x^3+15x^4$

c. $1-3(x+1)+6(x+1)^2-10(x+1)^3+15(x+1)^4$

d. $1-3x+12x^2-30x^3+360x^4$

e. None of the others.

20. Find the 10^{th}-order Taylor polynomial $y=P_{10}(x)$, centered about $x_0=17$, of the function

$$f(x)=5+6x^7.$$

a. $P_{10}(x)=5+6(x-17)^7$

b. $P_{10}(x)=5+6(x+17)^7$

c. $P_{10}(x)=5+6x^7$

d. It does not exist.

e. None of the others.

21. Find a power series representation for

$$f(x) = \ln(10 - x).$$

Hint: $\ln(ab) = \ln a + \ln b$.

a. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n10^n}$

b. $\displaystyle\sum_{n=1}^{\infty} \frac{10x^n}{n^n}$

c. $\displaystyle\ln 10 - \sum_{n=1}^{\infty} \frac{x^n}{10^n}$

d. $\displaystyle\ln 10 - \sum_{n=1}^{\infty} \frac{x^n}{n\,10^n}$

e. None of the others.

22. Consider the function $f(x) = e^x$ over the interval $(-1, 3)$. The 4^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{4} \frac{x^n}{n!}.$$

The 4^{th} order Remainder term $R_4(x)$ is defined by $R_4(x) = f(x) - P_4(x)$ and so $e^x \approx P_4(x)$ where the approximation is within an error of $|R_4(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_4(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\displaystyle\frac{(e^3)(3^4)}{4!}$

b. $\displaystyle\frac{(e^{-1})(3^4)}{4!}$

c. $\displaystyle\frac{(e^3)(3^5)}{5!}$

d. $\displaystyle\frac{(e^{-1})(3^5)}{5!}$

e. None of the others.

23. Suppose that the interval of convergence of the series $\sum_{n=1}^{\infty} c_n(x-x_0)^n$ is $(x_0-R, x_0+R]$. What can be said about the series at $x = x_0 + R$?

a. It must be absolutely convergent.

b. It must be conditionally convergent.

c. It must be divergent.

d. Nothing can be said.

e. None of the others.

24. Suppose that the power series $\sum_{n=1}^{\infty} c_n(x-10)^n$ has interval of convergence $(1, 19)$. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} c_n x^{2n}$?

a. $[9, 19]$

b. $[-3, 3]$

c. $(-3, 3)$

d. $(-81, 81)$

e. None of the others.

25. Describe the motion of a puffo whose position (x, y) is parameterized by

$$x = 6\sin t$$
$$y = 3\cos t$$

for $0 \leq t \leq 2\pi$.

a. Moves once counterclockwise along the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ starting and ending at $(6, 0)$.

b. Moves once counterclockwise along the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ starting and ending at $(-3, 0)$.

c. Moves once clockwise along the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ starting and ending at $(0, 3)$.

d. Moves once clockwise along the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ starting and ending at $(-3, 0)$.

e. None of the others.

26. Eliminate the parameter to find a Cartesian equation of the curve

$x = 5e^t$

$y = 21e^{-t}$.

a. $y = \dfrac{5}{21x}$

b. $y = \dfrac{105}{x}$

c. $y = 105x$

d. $y = \dfrac{105}{e^x}$

e. None of the others.

27. Find the arc length of the curve

$x = 3t^2$

$y = 2t^3$

for $0 \leq t \leq 3$

a. $20\sqrt{10} - 2$

b. $2\sqrt{10} - 1$

c. $2\sqrt{10} - 2$

d. $10\sqrt{2} - 2$

e. None of the others.

28. Find a parameterization for the line segment from $(-1, 2)$ to $(10, -6)$ for $0 \leq t \leq 1$.

a. $x = 10 - 8t$ and $y = -1 + t$

b. $x = -1 + 11t$ and $y = 2 - 8t$

c. $x = -1 + 11t$ and $y = -6 - 8t$

d. $x = -1 - 11t$ and $y = -8t$

e. None of the others.

29. Find an equation of the tangent line to the curve at the point corresponding to $t = 11\pi$.

$x = t \sin t$

$y = t \cos t.$

a. $y = \dfrac{x}{11\pi} + 12\pi$

b. $y = \dfrac{x}{11\pi} - 11\pi$

c. $y = \dfrac{x}{11\pi} + 11\pi$

d. $y = \dfrac{x}{11\pi} - 12\pi$

e. None of the others.

30. Prof. Girardi likes

a. moose

b. colored chalk

c. the number 17

d. mathematics

e. All of the above.

Good Luck in your math fun to come!