1. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.

T	F	B	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
T	F	B	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.
T	F	B	If $a_n \geq 0$ for all $n \in \mathbb{N}$, then $\sum a_n$ is either absolutely convergent or divergent.
T	F	B	If $\sum \lvert a_n \rvert$ converges, then $\sum a_n$ converges.
T	F	B	If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

5. Evaluate

$$\lim_{n\to\infty} \frac{\sqrt{25n^3 + 4n^2 + n - 5}}{7n^{\frac{3}{2}} + 6n - 1} .$$

a. 0
b. ∞
c. $\frac{25}{7}$
d. $\frac{5}{7}$
e. None of the others.

6. Find **all** real numbers r satisfying that

$$\sum_{n=2}^{\infty} r^n = \frac{1}{12} .$$

a. $\dfrac{1}{12}$
b. $\dfrac{1}{12}$ and $\dfrac{-1}{12}$
c. $\dfrac{1}{4}$ and $\dfrac{-1}{3}$
d. $\dfrac{1}{3}$ and $\dfrac{-1}{4}$
e. None of the others.

7. Consider the following two series.

Series A is $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

Series B is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

a. Both series converge absolutely.
b. Both series diverge.
c. Series A converges conditionally and Series B diverges.
d. Series A diverges and Series B converges conditionally.
e. None of the others.

8. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}} .$$

a. This series is absolutely convergent, as can be shown by the limit comparison test (LCT) with $b_n = \frac{1}{n^2}$.
b. This series is conditionally convergent, as can be shown by using only the AST and not other tests.
c. This series converges conditionally, as can be shown by

using the LCT with $b_n = \frac{1}{n}$ as well as the AST .

d. This series diverges.

e. None of the others.

9. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \, \frac{n!}{(3n)!} \, .$$

Let

$$a_n \; = \; (-1)^n \, \frac{n!}{(3n)!} \qquad \text{and} \qquad \rho \; = \; \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \, .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = \frac{1}{3}$.

b. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = 0$.

c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$.

d. $\rho > 1$ so by the Ratio Test $\sum_{n=1}^{\infty} a_n$ diverges.

e. None of the others.

11. By using the Limit Comparison Test, one can show that the formal series

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{(n+1)\,(n+2)\,(n+3)\,(n+4)\,(n+5)}} \, . \tag{11}$$

is:

a. convergent by comparing the series in (11) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
b. convergent by comparing the series in (11) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
c. divergent by comparing the series in (11) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
d. divergent by comparing the series in (11) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
e. none of the others

12. The formal series

$$\sum_{n=17}^{\infty} \frac{1}{n \, \ln n}$$

is:

a. convergent by the integral test
b. convergent by the ratio test
c. divergent by the integral test
d. divergent by the ratio test
e. none of the others

13. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{1}{n \; (n+1)} \tag{13}$$

and let

$$s_N \; = \; \sum_{n=1}^{N} \frac{1}{n \; (n+1)} \, .$$

Note that the partial fractions decompostion of $\frac{1}{n \; (n+1)}$ is $\frac{1}{n} - \frac{1}{n+1}$.

a. $s_N = 1 - \frac{1}{N+1}$ and the series in (13) converges to 1.
b. $s_N = 1 + \frac{1}{N+1}$ and the series in (13) converges to 1.
c. $s_N = 1 + \frac{1}{N}$ and the series in (13) converges to 1.
d. $s_N = 1 - \frac{1}{N}$ and the series in (13) converges to 1.
e. none of the others

14. Geometric Series. (On this page, you should do basic algebra but you do NOT have to do any grade-school arithmetic (eg, you can leave $(\frac{17}{18})^{171}$ as just that.) Let, for $N \geq 51$,

$$s_N \;=\; \sum_{n=51}^{N} 2\frac{3^{n+1}}{5^n} \;.$$

14a. Do some algebra to write s_N as $\sum_{n=51}^{N} \; c\, r^n$ for an appropriate constant c and ratio r.

$$s_N \;=\; \sum_{n=51}^{N}$$

14b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation $\sum$ sign nor some dots $\ldots$.).

$s_N \;=$

14c. Does $\sum_{n=51}^{\infty} 2\frac{3^{n+1}}{5^n}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=51}^{\infty} 2\frac{3^{n+1}}{5^n}$$

15. Check the correct box and then indicate your reasoning below. **SHOW ALL YOUR WORK**. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.23}}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent