MARK BOX		
PROBLEM	POINTS	
1-25	4x25	
%	100	

HAND IN PART

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND-IN PART. Hand-in only this part.
  (2) NOT TO HAND-IN PART. This part will not be collected.
  Take this part home to learn from and to check your answers when the solutions are posted.
- **For the Multiple Choice** problems, circle your answer(s) on the provided chart. No need to show work.
- The MARK BOX above indicates the problems (check that you have them all) along with their points.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, electronic devices, any device with which you can connect to the internet, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13^{th} ed., ET): §8.1–8.5, 8.7, 8.8, 10.1–10.10, 11.1–11.5 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ______________________________

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to the multiple choice problems.
- You may choice up to **2** answers for each multiple choice problem. The scoring is as follows.
  - ∗ For a problem with precisely one answer marked and the answer is correct, 4 points.
  - ∗ For a problem with precisely two answers marked, one of which is correct, 1 points.
  - ∗ All other cases, 0 points.
- Fill in the "number of solutions circled" column.

Table for Your Muliple Choice Solutions							Do Not Write Below			
PROBLEM						number of solutions circled	1	2	B	x
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
11	11a	11b	11c	11d	11e					
12	12a	12b	12c	12d	12e					
13	13a	13b	13c	13d	13e					
14	14a	14b	14c	14d	14e					
15	15a	15b	15c	15d	15e					
16	16a	16b	16c	16d	16e					
17	17a	17b	17c	17d	17e					
18	18a	18b	18c	18d	18e					
19	19a	19b	19c	19d	19e					
20	20a	20b	20c	20d	20e					
21	21a	21b	21c	21d	21e					
22	22a	22b	22c	22d	22e					
23	23a	23b	23c	23d	23e					
24	24a	24b	24c	24d	24e					
25	25a	25b	25c	25d	25e					
							4	1	0	0

<u>NOT</u> TO HAND-IN PART
STATEMENT OF MULTIPLE CHOICE PROBLEMS

- Hint. For a definite integral problems $\int_a^b f(x)\,dx$.
  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.
  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.

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1. Evaluate the integral

$$\int_0^{\pi/2} \cos^3 x\,dx.$$

a. $\frac{2}{3}$

b. $\frac{4}{3}$

c. $\frac{1}{4}$

d. $-\frac{1}{4}$

e. None of the others.

2. Evaluate the integral

$$\int_1^2 \frac{8}{x^2 - 2x + 2}\,dx.$$

Hint: complete the square in the denominator.

a. $8\left(\tan^{-1} 2 - \frac{\pi}{4}\right)$

b. 2π

c. $8 \ln 2$

d. $\frac{\pi}{4}$

e. None of the others.

3. Evaluate the integral

$$\int_1^2 x \ln x \, dx.$$

a. $\ln 2$

b. $\frac{1}{2}$

c. $\ln 4 + \frac{3}{4}$

d. $\ln 4 - \frac{3}{4}$

e. None of the others.

4. Evaluate the integral

$$\int_0^{\pi/8} e^{3x} \cos(4x) \; dx.$$

a. $e^{3\pi/8} + 1$

b. $\dfrac{4e^{3\pi/8} - 3}{25}$

c. $\dfrac{4e^{3\pi/8} - 3}{7}$

d. $\dfrac{e^{3\pi/8} - 1}{7}$

e. None of the others.

5. Evaluate the integral

$$\int_0^{\sqrt{3}/2} \frac{4x^2 \, dx}{(1 - x^2)^{3/2}}$$

a. $4\sqrt{3} - 2\ln\left|2 + \sqrt{3}\right|$

b. $4\sqrt{3} + 2\ln\left|2 + \sqrt{3}\right|$

c. $4\sqrt{3} - \dfrac{4\pi}{3}$

d. $4\sqrt{3}$

e. None of the others.

6. Evaluate the integral

$$\int_0^{\sqrt{3}} \frac{3t^2+t+4}{t^3+t}\,dt$$

a. $\ln\left(\frac{9}{\sqrt{2}}\right) + \frac{\pi}{12}$

b. $\ln\left(\frac{9}{\sqrt{2}}\right) - \frac{\pi}{12}$

c. $\ln\left(\frac{9}{2}\right) + \frac{\pi}{12}$

d. $\ln\left(\frac{9}{2}\right) - \frac{\pi}{12}$

e. None of the others.

7. Fill in the two blanks. By comparing the improper integral

$$\int_0^{\pi} \frac{dt}{\sqrt{t}\ +\ \sin t}$$

to the improper integral ____________________ ,
the Direct Comparison Test (for improper integrals) gives that $\displaystyle\int_0^{\pi} \frac{dt}{\sqrt{t}\ +\ \sin t}$ is __________.

a. $\displaystyle\int_0^{\pi} \frac{dt}{\sqrt{t}}$ and divergent

b. $\displaystyle\int_0^{\pi} \frac{dt}{\sqrt{t}}$ and convergent

c. $\displaystyle\int_0^{\pi} \frac{dt}{t}$ and divergent

d. $\displaystyle\int_0^{\pi} \frac{dt}{t}$ and convergent

e. None of the others.

8. Limit of a sequence. Evaluate

$$\lim_{n\to\infty} \sqrt{\frac{2n}{n+1}}$$

a. 0

b. 2

c. $\sqrt{2}$

d. ∞

e. None of the others.

9. Limit of a sequence.

$$\lim_{n\to\infty} \frac{\sin n}{n}.$$

a. 0

b. 1

c. ∞

d. diverges but not to $\pm\infty$

e. None of the others.

10. Find the vaule for b for which

$$e^b + e^{2b} + e^{3b} + e^{4b} + \ldots = 9.$$

a. $\ln \frac{8}{9}$

b. $\frac{8}{9}$

c. $\ln \frac{9}{10}$

d. $\frac{9}{10}$

e. None of the others.

11. Determine the behavior of the series

$$\sum_{n=1}^{\infty} \frac{2^n\, n!\, n!}{(2n)!}.$$

a. The series converges by the Ratio Test since the limit resulting from the test 0.

b. The series converges by the Ratio Test since the limit resulting from the test $\dfrac{1}{2}$

c. The series diverges by the Ratio Test since the limit resulting from the test 2

d. The series diverges by the Ratio Test since the limit resulting from the test ∞

e. None of the others.

12. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1},$$

by the <u>Direct</u> Comparison Test,

a. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$.

b. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$.

c. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$.

d. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1.5}}$.

e. None of the others.

13. The series

$$\sum_{n=1}^{\infty} \frac{1}{1+\ln n},$$

by the <u>Limit</u> Comparison Test,

a. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n}\right)} = 0$.

b. diverges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n}\right)} = \infty$.

c. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n^2}\right)} = 0$.

d. converges, using for comparison the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, since $\lim_{n\to\infty} \frac{\left(\frac{1}{1+\ln n}\right)}{\left(\frac{1}{n^2}\right)} = \infty$.

e. None of the others.

14. The series

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n},$$

is

a. absolutely convergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

b. conditionally convergent, as shown by using only the Alternating Series Test (and no other tests).

c. conditionally convergent, as shown by using both the Alternating Series Test and the Integral Test.

d. divergent, as shown by the Direct Comparison Test, using for comparison the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

e. None of the others.

15. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^{2n}\, (x-2)^n}{3n}$$

Recall that the interval of convergence is the set of x for which the power series converges, either absolutely or conditionally.

a. $\left[\frac{5}{3}, \frac{7}{3}\right)$

b. $\left(\frac{5}{3}, \frac{7}{3}\right]$

c. $\left[\frac{17}{9}, \frac{19}{9}\right)$

d. $\left(\frac{17}{9}, \frac{19}{9}\right]$

e. None of the others.

16. Using the geometric series, find a power series representation about (i.e., centered at) $x = 5$ for the function

$$g(x) = \frac{3}{x-2}$$

and indicate when the representation is valid.

a. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n (x-5)^n$, valid on $(2, 8)$.

b. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n (x-5)^n$, valid on $(2, 8)$.

c. $\displaystyle\sum_{n=0}^{\infty} (-1)^n (x-5)^n$, valid on $(4, 6)$.

d. $\displaystyle\sum_{n=0}^{\infty} (x-5)^n$, valid on $(4, 6)$.

e. None of the others.

17. Find the 3$^{\text{rd}}$ order Taylor polynomial, about the center $x_0 = 1$, for the function $f(x) = x^5 - x^2 + 5$.

a. $p_3(x) = 5 + 3(x-1) + 9(x-1)^2 + 10(x-1)^3$

b. $p_3(x) = 5 + 3(x-1) + 18(x-1)^2 + 60(x-1)^3$

c. $p_3(x) = 5 + 3x + 9x^2 + 10x^3$

d. $p_3(x) = 5 + 3x + 18x^2 + 60x^3$

e. None of the others.

18. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) \;=\; \frac{1}{(1-x)^4}$$

about the center $x_0 = 0$ which is valid for $|x| < 1$. Hint. Start with the Taylor series expansion

$$\frac{1}{1-x} \;=\; \sum_{k=0}^{\infty} x^k \qquad \text{valid for } |x| < 1$$

and differentiate (as many times as needed). Be careful and don't forget the chain rule:

$$D_x(1-x)^{-1} \;=\; (-1)(1-x)^{-2}\, D_x(1-x) \;=\; (-1)(1-x)^{-2}\,(-1) \;=\; (1-x)^{-2}\,.$$

a. $\sum_{n=0}^{\infty} \frac{(n)(n-1)(n-2)}{6}\, x^{n-3}$

b. $\sum_{n=0}^{\infty} (n)(n-1)(n-2)\, x^{n}$

c. $\sum_{n=0}^{\infty} \frac{(n+3)(n+2)(n+1)}{6}\, x^{n}$

d. $\sum_{n=0}^{\infty} (-1)^n\, \frac{(n+3)(n+2)(n+1)}{6}\, x^{n}$

e. None of the others.

19. Consider the function

$$f(x) = e^{-x}\,.$$

The 5^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_5(x) \;=\; \sum_{n=0}^{5} \frac{(-x)^n}{n!} \;=\; 1 \;-\; x \;+\; \frac{x^2}{2!} \;-\; \frac{x^3}{3!} \;+\; \frac{x^4}{4!} \;-\; \frac{x^5}{5!}\,.$$

The 5^{th} order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $e^{-x} \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\frac{(e)(3^5)}{5!}$

b. $\frac{(e^{-3})(3^5)}{5!}$

c. $\frac{(e)(3^6)}{6!}$

d. $\frac{(e^{-3})(3^6)}{6!}$

e. None of the others.

20. Find an equation for the line tangent to the curve parameterized by

$$x = 2t^2 + 3$$
$$y = t^4$$

at the point defined by the value $t = -1$.

a. $y = x - 6$

b. $y = x - 4$

c. $y = -x - 6$

d. $y = -x - 4$

e. None of the others.

21. Find the Cartesian coordinates of the point with polar coordinates

$$\left(-3, \frac{5\pi}{6}\right)$$

a. $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

b. $\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

c. $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

d. $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

e. None of the others.

22. A parametrization of a circle with center at $(0,0)$ and radius 1, which is traced out twice in the clockwise direction is

a. $x(t) = \cos t$ and $y(t) = \sin t$ for $0 \le t \le 2\pi$

b. $x(t) = \cos t$ and $y(t) = \sin t$ for $0 \le t \le 4\pi$

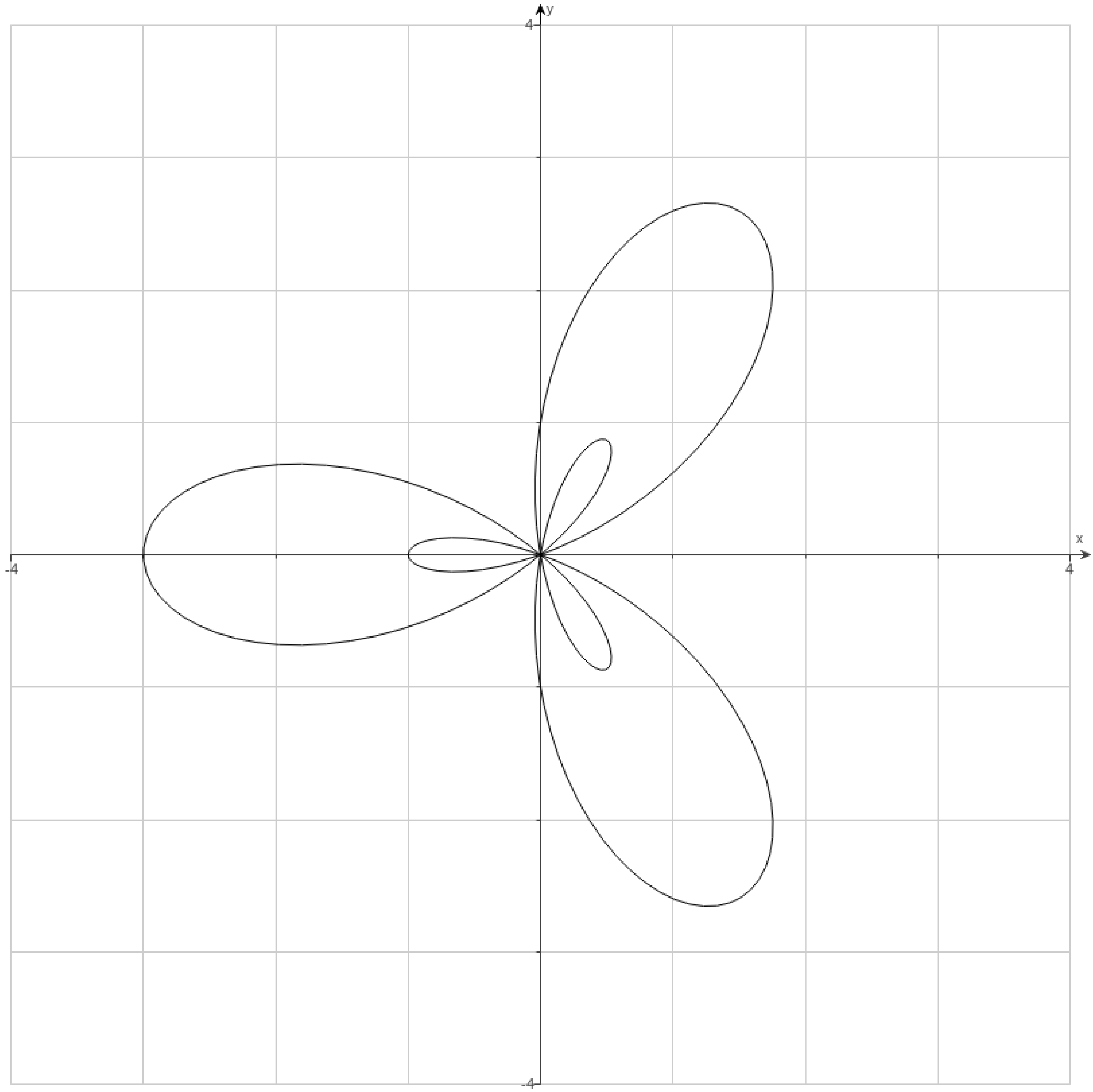
c. $x(t) = \cos t$ and $y(t) = -\sin t$ for $0 \le t \le 2\pi$

d. $x(t) = \cos t$ and $y(t) = -\sin t$ for $0 \le t \le 4\pi$

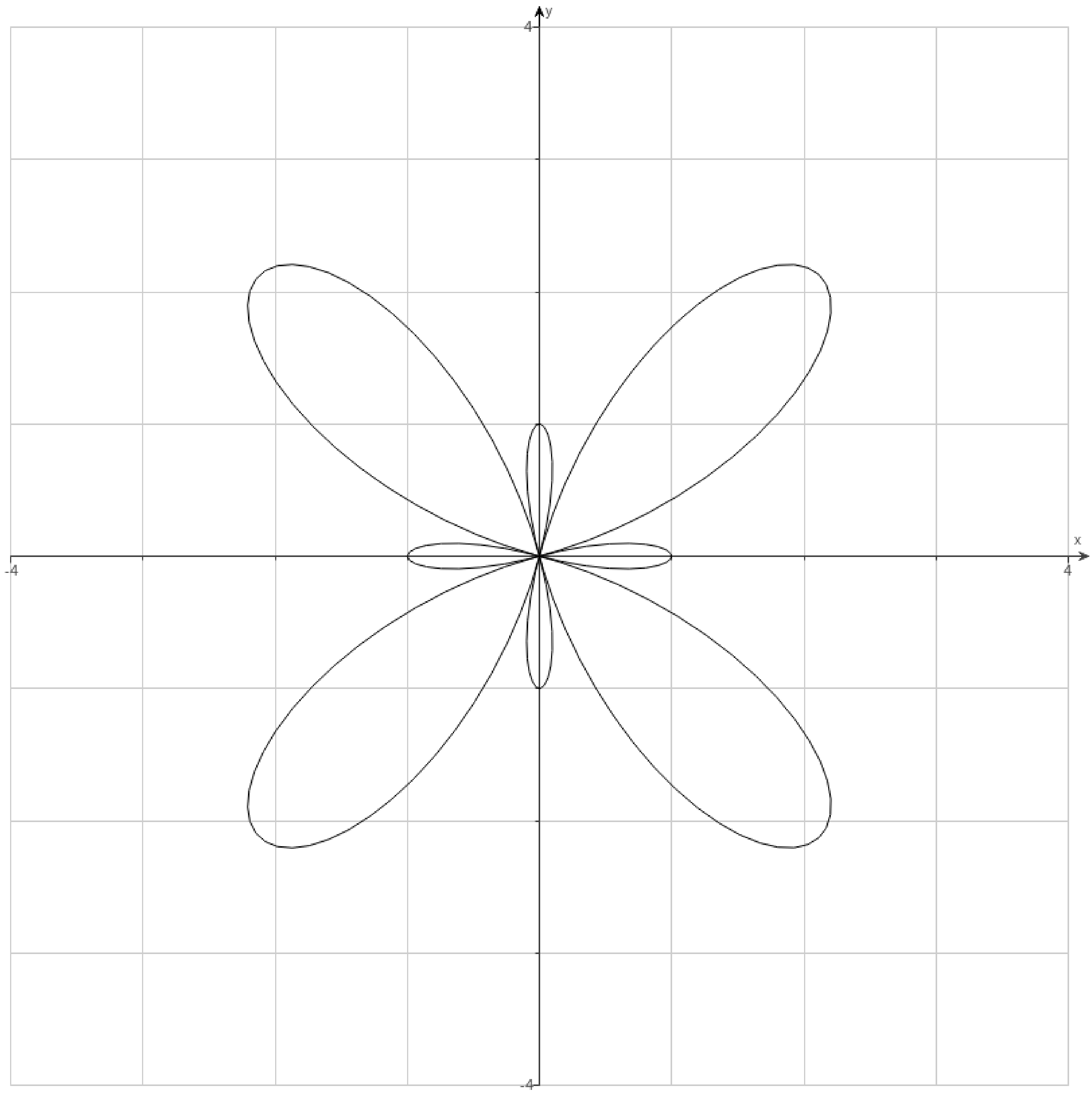
e. None of the others.

23. Choose the correct graph of the polar equation $r = 1 - 2\sin(3\theta)$.

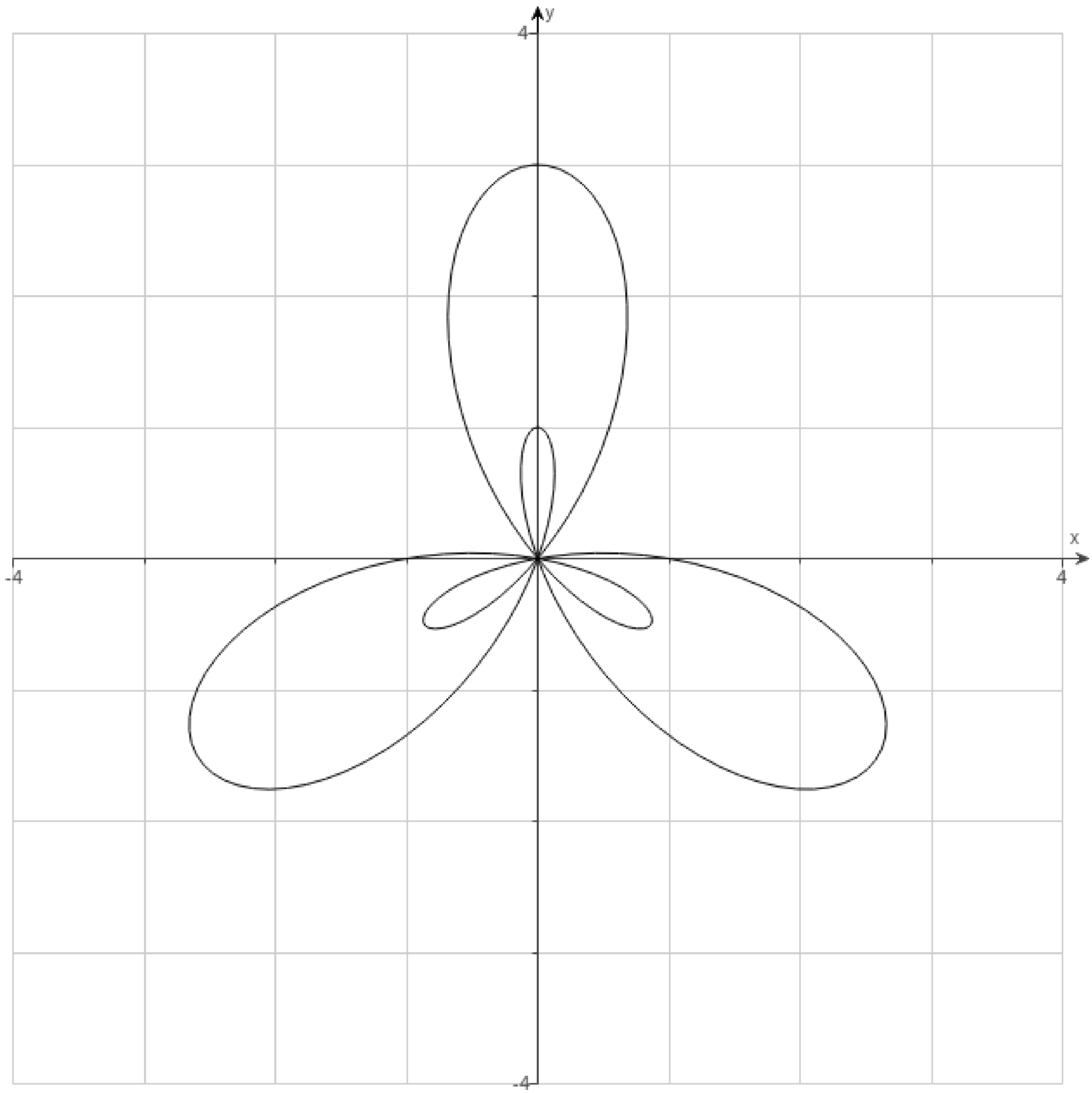
a. graph a



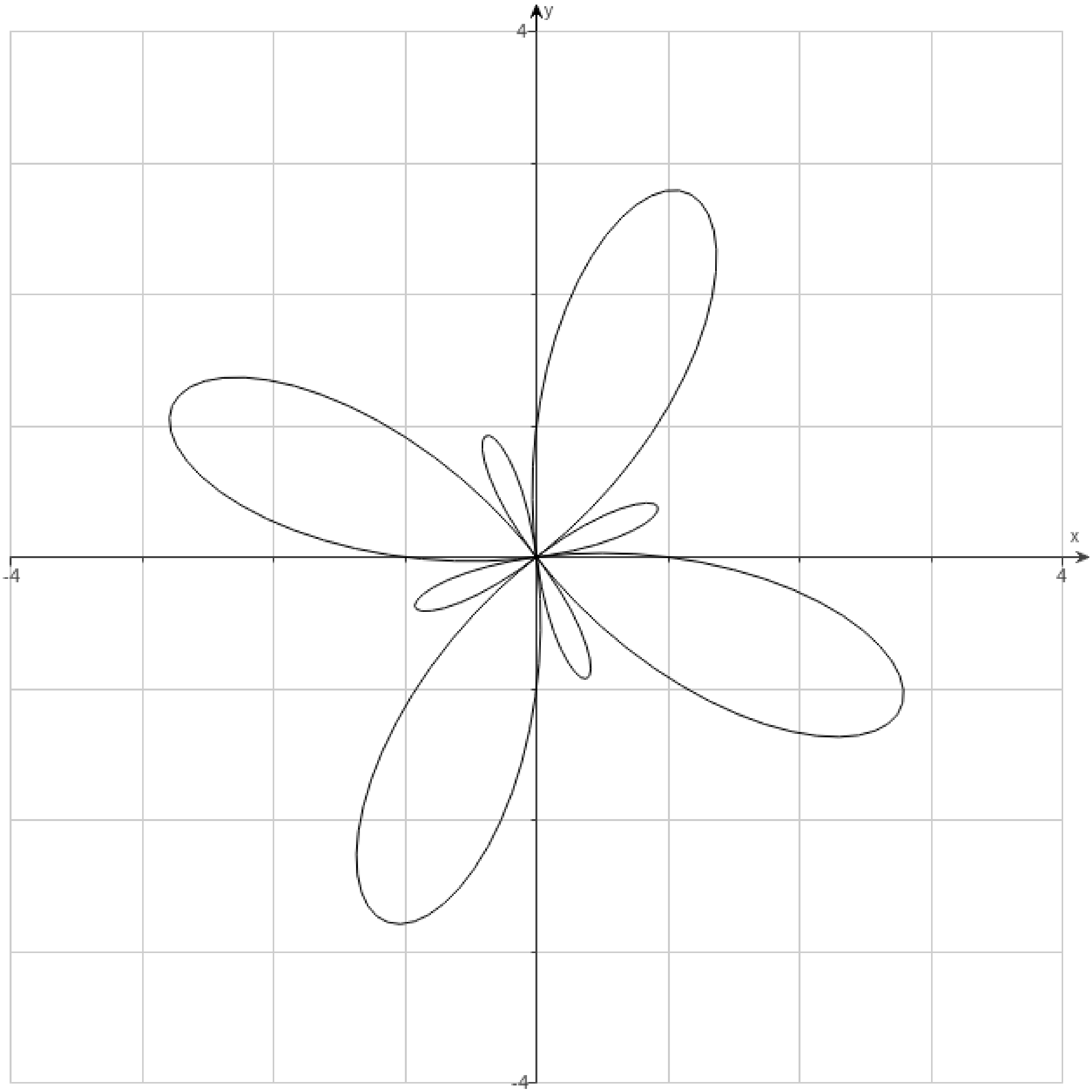
b. graph b



c. graph c



d. graph d



e. None of the shown graphs.

24. Express the area, as an integral (or as integrals) with respect to θ, of the region that (described using polar equations) lies inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$.

a. $\displaystyle \frac{1}{2}\int_{\pi/3}^{\pi/2} (3\sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/3}^{\pi/2} (1+\sin\theta)^2 \, d\theta$

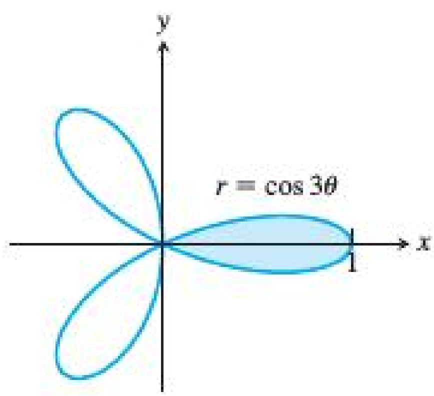
b. $\displaystyle \frac{1}{2}\int_{\pi/6}^{\pi/2} (3\sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{\pi/2} (1+\sin\theta)^2 \, d\theta$

c. $\displaystyle \int_{\pi/3}^{\pi/2} (3\sin\theta)^2 \, d\theta - \int_{\pi/3}^{\pi/2} (1+\sin\theta)^2 \, d\theta$

d. $\displaystyle \int_{\pi/6}^{\pi/2} (3\sin\theta)^2 \, d\theta - \int_{\pi/6}^{\pi/2} (1+\sin\theta)^2 \, d\theta$

e. None of the others.

25. Below is a graph of the (polar) equation $r = \cos(3\theta)$, with one leaf shaded in of this three-leaved rose. Express the arc length of this ONE leaf as an integral with respect to θ.



a. $\displaystyle \int_0^{2\pi} \sqrt{\cos^2(3\theta) + 3\sin^2(3\theta)} \; d\theta$

b. $\displaystyle \int_0^{2\pi} \sqrt{\cos^2(3\theta) + 9\sin^2(3\theta)} \; d\theta$

c. $\displaystyle 2\int_0^{\pi/6} \sqrt{\cos^2(3\theta) + 3\sin^2(3\theta)} \; d\theta$

d. $\displaystyle 2\int_0^{\pi/6} \sqrt{\cos^2(3\theta) + 9\sin^2(3\theta)} \; d\theta$

e. None of the others.