

MARK BOX		
PROBLEM	POINTS	
0	30	
1-10	50=10x5	
11	10	
12	10	
%	100	

HAND IN PART

NAME: _____

PIN: _____

INSTRUCTIONS

- This exam comes in two parts.
 - (1) **HAND IN PART.** Hand in only this part.
 - (2) **STATEMENT OF MULTIPLE CHOICE PROBLEMS.** Do not hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- **On Problem 0,** fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- **MultipleChoice problems 1–10,** circle your answer(s) on the provided chart. No need to show work. The STATEMENT OF MULTIPLE CHOICE PROBLEMS will not be collected.
- **For problems > 10,** to receive credit you **MUST:**
 - (1) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
 - (2) if a line/box is provided, then:
 - show your work **BELOW** the line/box
 - put your answer on/in the line/box
 - (3) if no such line/box is provided, then box your answer.
- The MARK BOX above indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, electronic devices, any device with which you can connect to the internet, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student’s request, I will project my watch upon the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13th ed., ET): §8.1-8.5, 8.7, 8.8 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina’s Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University’s Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : _____

0. Fill-in the blanks.

0.1. $\arccos(-\frac{1}{2}) =$ _____ (Your answers should be an angle in **RADIANS**.)

0.2. $\arcsin(-\frac{1}{2}) =$ _____ (Your answers should be an angle in **RADIANS**.)

0.3/0.4. Double-angle Formulas. Your answer should involve trig functions of θ , and not of 2θ .

$\cos(2\theta) =$ _____ and $\sin(2\theta) =$ _____

0.5/0.6. Half-Angle Formula. Your answer should involve $\cos(2\theta)$.

$\cos^2(\theta) =$ _____ and $\sin^2(\theta) =$ _____.

0.7. $\int \frac{du}{u} \stackrel{u \neq 0}{=} \text{_____} + C$

0.8. $\int u^n du \stackrel{n \neq -1}{=} \text{_____} + C$

0.9. $\int e^u du = \text{_____} + C$

0.10. $\int \cos u du = \text{_____} + C$

0.11. $\int \sec^2 u du = \text{_____} + C$

0.12. $\int \sec u \tan u du = \text{_____} + C$

0.13. $\int \sin u du = \text{_____} + C$

0.14. $\int \csc^2 u du = \text{_____} + C$

0.15. $\int \csc u \cot u du = \text{_____} + C$

0.16. $\int \tan u du = \text{_____} + C$

0.17. $\int \cot u du = \text{_____} + C$

0.18. $\int \sec u du = \text{_____} + C$

0.19. $\int \csc u du = \text{_____} + C$

0.20. $\int \frac{1}{\sqrt{a^2-u^2}} du \stackrel{a>0}{=} \underline{\hspace{10em}} + C$

0.21. $\int \frac{1}{a^2+u^2} du \stackrel{a>0}{=} \underline{\hspace{10em}} + C$

0.22. $\int \frac{1}{u\sqrt{u^2-a^2}} du \stackrel{a>0}{=} \underline{\hspace{10em}} + C$

0.23. Trig sub.: if the integrand involves $a^2 - u^2$, then one makes the substitution $u = \underline{\hspace{10em}}$

0.24. Trig sub.: if the integrand involves $u^2 - a^2$, then one makes the substitution $u = \underline{\hspace{10em}}$

0.25. Integration by parts formula: $\int u dv = \underline{\hspace{10em}}$

Improper Integrals

Below, $a, b, c \in \mathbb{R}$ with $a < c < b$.

0.26. If $f: [0, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_0^\infty f(x) dx$ by

$$\int_0^\infty f(x) dx = \boxed{\hspace{15em}} .$$

0.27. If $f: (-\infty, \infty) \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_{-\infty}^\infty f(x) dx$ by

$$\int_{-\infty}^\infty f(x) dx = \boxed{\hspace{15em}} .$$

0.28. If $f: [a, c) \cup (c, b] \rightarrow \mathbb{R}$ is continuous, then we define the improper integral $\int_a^b f(x) dx$ by

$$\int_a^b f(x) dx = \boxed{\hspace{15em}} .$$

0.29. An improper integral as above *converges* precisely when

0.30. An improper integral as above *diverges* precisely when

MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to the multiple choice problems.
- You may choose up to **2** answers for each multiple choice problem. The scoring is as follows.
 - * For a problem with precisely one answer marked and the answer is correct, 5 points.
 - * For a problem with precisely two answers marked, one of which is correct, 2 points.
 - * For a problem with nothing marked (i.e., left blank) 1 point.
 - * All other cases, 0 points.
- Fill in the “number of solutions circled” column. (Worth a total of 1 point of extra credit.)

Table for Your Multiple Choice Solutions							Do Not Write Below			
PROBLEM						number of solutions circled	1	2	B	x
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
							5	2	1	0
							Extra Credit:			

11. Show your work below the box then put your answer in the box. Your final answer should not have a trig function in it.

11a. Complete the square. $x^2 - 4x - 5 =$

11b.

$$\int \sqrt{5 + 4x - x^2} dx =$$

+ C

12. Derive a reduction formula for $\int x^n e^x dx$ where $n \in \mathbb{N} = \{1, 2, 3, 4, \dots\}$. Show your work below the box then put your answer in the box.

$$\int x^n e^x dx =$$

STATEMENT OF MULTIPLE CHOICE PROBLEMSThese sheets of paper are not collected.

- Hint. For a definite integral problems $\int_a^b f(x) dx$.
 - (1) First do the indefinite integral, say you get $\int f(x) dx = F(x) + C$.
 - (2) Next check if you did the indefinite integral correctly by using the Fundamental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
 - (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
 - Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.
-

1. Evaluate the integral

$$\int_0^1 \frac{x}{x^2 + 9} dx.$$

- a. $\frac{1}{2} (\ln 10 - \ln 9)$
- b. $\frac{1}{2} (\ln 1 - \ln 0)$
- c. $(\ln 10 - \ln 9)$
- d. $(\ln 1 - \ln 0)$
- e. None of the others.

2. Evaluate the integral

$$\int_0^4 \frac{x}{x+9} dx .$$

- a. $4 - 9 \ln(13) + 9 \ln(9)$
- b. $13 - 9 \ln(4) + \ln(3)$
- c. $(1/(9 \ln(13))) - \ln(3)$
- d. $4 - 13 \ln(9) + 3 \ln(18)$
- e. None of the others.

3. Evaluate

$$\int_0^{\ln(2\pi)} e^x \cos(e^x) dx$$

- a. $e^{2\pi}$
- b. $e^{2\pi} - 1$
- c. $-\sin(1)$
- d. $\sin(1)$
- e. None of the others.

vspbtw

4. Evaluate

$$\int_{x=0}^{x=\frac{3\pi}{2}} e^x \cos x dx .$$

- a. $\frac{1 + e^{3\pi/2}}{2}$
- b. $\frac{1 - e^{3\pi/2}}{2}$
- c. $\frac{-1 + e^{3\pi/2}}{2}$
- d. $\frac{-1 - e^{3\pi/2}}{2}$
- e. None of the others.

5. Investigate the convergence of

$$\int_{x=1}^{x=\infty} \frac{1 - e^{-x}}{x} dx .$$

- a. The integral converges by the Limit Comparison Test, comparing the integrand with $g(x) = \frac{1}{x}$.
- b. The integral diverges by the Limit Comparison Test, comparing the integrand with $g(x) = \frac{1}{x}$.
- c. The integral converges by the Direct Comparison Test, comparing the integrand with $g(x) = \frac{1}{x}$.
- d. The integral diverges by the Direct Comparison Test, comparing the integrand with $g(x) = \frac{1}{x}$.
- e. None of the others.

6. Evaluate

$$\int_{x=0}^{x=1} \sin^4 x \, dx .$$

- a. 1
- b. π
- c. $1 + \sin 2 + \sin 4$
- d. $\frac{3}{8} - \frac{1}{4} \sin 2 + \frac{1}{32} \sin 4$
- e. None of the others.

7. Evaluate

$$\int_{x=5}^{x=10} \frac{\sqrt{x^2 - 25}}{x} \, dx$$

AND specify the initial substitution.

- a. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sec \theta$.
- b. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sec \theta$
- c. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sin \theta$.
- d. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sin \theta$
- e. None of the others.

8. Evaluate

$$\int_{x=1}^{x=3} \frac{5x^2 + 3x - 2}{x^3 + 2x^2} \, dx .$$

- a. $3 \ln 5 - \ln 3 - \frac{2}{3}$
- b. $3 \ln 5 - \ln 3 - \frac{8}{3}$
- c. $\ln 5 - \frac{2}{3}$
- d. $\frac{2}{3} - \ln 5$
- e. None of the others.

9. For which value of p does

$$\int_0^1 \frac{1}{x^p} dx = 1.25 ?$$

- a. $p = 0.2$
- b. $p = 0.5$
- c. $p = 2.0$
- d. $p = 2.5$
- e. None of the others.

10. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^3} dx .$$

- a. 0
- b. $\frac{1}{4}$
- c. diverges to infinity
- d. does not exist but also does not diverge to infinity
- e. None of the others.