$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$

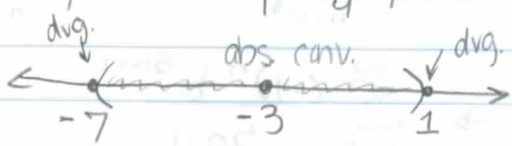
center @ -3

$|a_n|^{1/n} = \left|\frac{(-1)^n n}{4^n}(x+3)^n\right|^{1/n} = \left|\frac{n^{1/n}}{4}(x+3)\right|$

$\lim_{n\to\infty} \frac{n^{1/n}}{4}|x+3| \to |x+3| \lim_{n\to\infty} \frac{n^{1/n}}{4}$

$\frac{|x+3|}{4} = p$

$p<1 \quad \left|\frac{x+3}{4}\right| < 1 \quad |x+3| < 4$



Test endpoints: (1) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\cancel{4^n}} \cdot \cancel{4^n} = \sum_{n=1}^{\infty} (-1)^n n$ dvg.

(-7) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(-4^n) = \sum_{n=1}^{\infty} \frac{(-1)^n n}{\cancel{4^n}} \cdot (-1)^n \cancel{4^n}$

$= \sum_{n=1}^{\infty} (-1)^{2n} n = \sum_{n=1}^{\infty} n$ dvg. (nth term test)

$\boxed{2}$ $\sum_{n=1}^{\infty} \frac{2^n}{n}(4x-8)^n$

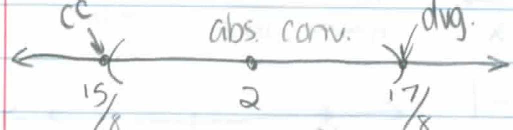
$|a_n|^{1/n} = \left|\frac{2^n}{n}(4x-8)^n\right|^{1/n} = \frac{2}{n^{1/n}}|4x-8|$

$|4x-8| \lim_{n\to\infty} \frac{2}{n^{1/n}} = |4x-8| \cdot 2 = p < 1 \quad \frac{2|4x-8|}{2} < \frac{1}{2}$

$|4x-8| < \frac{1}{2}$

$4|x-2| < 1/2$

$|x-2| < 1/8$



$\frac{15}{8}$: $\sum_{n=1}^{\infty} \frac{2^n}{n}\left(\frac{15}{2}-8\right)^n = \sum_{n=1}^{\infty} \frac{2^n}{n}\left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{2^n}{n} \cdot \frac{(-1)^n}{2^n}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ $|a_n| = \frac{1}{n}$ dvg. p-series $p \le 1$

a_n use AST $u_n = \frac{1}{n}$ pos ✓ cont ✓ dec ✓ therefore conv.

overall $\boxed{CC}$

$\frac{17}{8}$: $\sum_{n=1}^{\infty} \frac{2^n}{n}\left(\frac{17}{2}-8\right)^n = \sum_{n=1}^{\infty} \frac{2^n}{n}\left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{2^n}{n}\cdot\frac{1^n}{2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

dvg. p series
p = 1

$\boxed{4.}$ $f(x) = \frac{x}{4+x^2}$

$\frac{x}{4}\left(\frac{1}{1-\left(-\frac{x^2}{4}\right)}\right) = \frac{x}{4}\sum_{n=0}^{\infty}\left(-\frac{x^2}{4}\right)^n$ valid when $\left|\frac{-x^2}{4}\right| < 1 \to (-2,2)$

$\frac{x}{4}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{4^n} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{4^n}\cdot\frac{x}{4} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{4^{n+1}}$

$\boxed{5}$ $f(x) = \int_0^b \frac{1}{1+x^5}$ Valid when $|-x^5| < 1 \to (-1,1)$

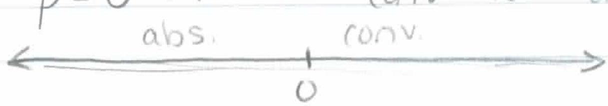
$\int_0^b \frac{1}{1-(-x^5)}$ $\sum_{n=0}^{\infty}\int_0^b (-x^5)^n = \sum_{n=0}^{\infty}\int_0^b (-1)^n x^{5n} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{5n+1}}{5n+1}\Big|_0^b$

$\sum_{n=0}^{\infty}\left[\frac{(-1)^n b^{5n+1}}{5n+1} - 0\right] \to \sum_{n=0}^{\infty}\frac{(-1)^n b^{5n+1}}{5n+1}$

$\boxed{6}$ $\sum_{n=2}^{\infty}\frac{x^n}{(\ln n)^n}$

a) $|a_n|^{1/n}$ $\left|\frac{x^n}{(\ln n)^n}\right|^{1/n} \to \frac{x}{(\ln n)}$ $\lim_{n\to\infty}\frac{x}{\ln n} = 0$

$p = 0 < 1$ $\therefore$ conv. for all values

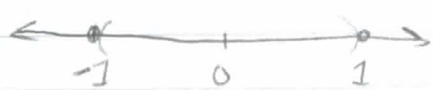


For the group I told the n was an x the answer is:

b.) $\sum_{n=2}^{\infty}\frac{x^n}{(\ln x)^n}$

$|a_n|^{1/n}$ $\left|\frac{x^n}{(\ln x)^n}\right|^{1/n} = \frac{x}{\ln x}$ $\lim_{n\to\infty}\frac{x}{\ln x} = \frac{\infty}{\infty}$

L'Hopitals $\frac{1}{1/x} = |x| < 1$



then check endpoints... (however, pay more attention to (a.))