

MARK BOX		
PROBLEM	POINTS	
0	15	
1-9	45	
10	10	
11	10	
12	10	
13	10	
%	100	

NAME: \_\_\_\_\_

PIN: \_\_\_\_\_

### INSTRUCTIONS

- **On Problem 0**, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- **For multiple choice problems 1– 9**, circle your answer(s) on the provided chart. No need to show work. The STATEMENT OF MULTIPLE CHOICE PROBLEMS will not be collected.
- **For problems  $\geq 10$** , to receive credit you **MUST** show ALL your work and :
  - (1) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;**  
such explanations help with partial credit
  - (2) if a line/box is provided, then:
    - show you work BELOW the line/box
    - put your answer on/in the line/box
  - (3) if no such line/box is provided, then box your answer.
- Upon request, you will be given as much (blank) scratch paper as you need.
- Check that your copy of the exam has all of the problems.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13<sup>th</sup> ed., ET): §8.1–8.5, 8.7, 8.8 .

### Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : \_\_\_\_\_

**0. Fill in the blanks (each worth 1 point).**

. If  $u \neq 0$ , then  $\int \frac{du}{u} =$  \_\_\_\_\_  $+ C$

.  $\int \cos u \, du =$  \_\_\_\_\_  $+ C$

.  $\int \sin u \, du =$  \_\_\_\_\_  $+ C$

.  $\int \tan u \, du =$  \_\_\_\_\_  $+ C$

.  $\int \sec u \, du =$  \_\_\_\_\_  $+ C$

.  $\int \sec^2 u \, du =$  \_\_\_\_\_  $+ C$

.  $\int \sec u \tan u \, du =$  \_\_\_\_\_  $+ C$

. Integration by parts formula:  $\int u \, dv =$  \_\_\_\_\_

. Partial Fraction Decomposition. If one wants to integrate  $\frac{f(x)}{g(x)}$  where  $f$  and  $g$  are polynomials and  $[\text{degree of } f] \geq [\text{degree of } g]$ , then one must first do \_\_\_\_\_

. Trig. Substitution ( $a > 0$ ): (recall that the *integrand* is the function you are integrating)

▷ if the integrand involves  $a^2 + u^2$ , then one makes the substitution  $u =$  \_\_\_\_\_

▷ if the integrand involves  $a^2 - u^2$ , then one makes the substitution  $u =$  \_\_\_\_\_

▷ if the integrand involves  $u^2 - a^2$ , then one makes the substitution  $u =$  \_\_\_\_\_

. Trig. Formulas:

▷  $\sin(2\theta) =$  \_\_\_\_\_ (your answer should involve trig functions of  $\theta$ , and not of  $2\theta$ )

▷  $\sin^2(\theta) =$  \_\_\_\_\_ ( $\cos(2\theta)$  or  $\sin(2\theta)$  should appear in your answer)

.  $\arctan(-1) =$  \_\_\_\_\_ **RADIANS**. (your answer should be an angle)

## TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choose up to **2** answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 3 points.
  - For a problem with nothing marked (i.e., left blank) 1 point.
  - All other cases, 0 points.
- Fill in the “number of solutions circled” column. (Worth a total of 1 point of extra credit.)

Your Solutions							Do Not Write Below			
							points			
PROBLEM						number of solutions circled	5	3	1	0
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
							Extra Credit:			

10.  $\int \sin x \sec x \, dx =$  + C

11.  $\int \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} dx =$  + C

Hint:  $x^3 + 2x^2 - 3x = x(x + 3)(x - 1)$ .

12.  $\int \frac{1}{\sqrt{x^2 + 8x + 25}} dx =$  + C

13. You can think of this problem as finding an approximation of  $\ln 2$  since  $\int_1^2 \frac{dx}{x} = \ln 2$ . Let

$$f(x) = \frac{1}{x} \quad \text{and} \quad [a, b] = [1, 2] \quad \text{and} \quad n = 4 .$$

13a. Find the Trapezoidal Rule approximation  $T_n$  of  $\int_a^b f(x) dx$  with  $n$  steps.

► Express your answer as a fraction or a sum of fractions (i.e., you do not have to perform grade-school math) and not as a decimal.

ANSWER:  $T_n =$

optional table		$\Delta x =$	
$i$	$x_i$	$f(x_i)$	weight
0			
1			
2			
3			
4			

13b. Find a good upper bound for  $\max_{a \leq x \leq b} |f''(x)|$ . ANSWER:  $\max_{a \leq x \leq b} |f''(x)| \leq$

► Your answer should be a number.

13c. The Trapezoidal Rule Error Theorem gives that  $\left| T_n - \int_a^b f(x) dx \right| \leq$

► Express your answer as a fraction, not a decimal.

13d. Find the smallest integer  $n$  so that the Trapezoidal Rule Error Theorem guarantees that

$$\left| T_n - \int_a^b f(x) dx \right| \leq 10^{-3}. \quad \text{ANSWER: } n = \boxed{\phantom{000}} .$$

**STATEMENT OF MULTIPLE CHOICE PROBLEMS**

Hint:  $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$  and  $\ln b + \ln a = \ln(ba)$  and  $\ln(a^r) = r \ln a$  if  $a, b > 0$  and  $r \in \mathbb{R}$ .

1. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{1}{x^2 + 1} dx .$$

- a.  $\ln \sqrt{2}$
- b.  $\ln 2$
- c.  $\frac{\pi}{4}$
- d.  $\frac{\pi}{2}$
- e. None of the others.

2. The integral

$$\int \frac{x}{x^2 + 1} dx$$

can be evaluated the following way.

- a. Trig. Substitution using the  $x = \sin \theta$ .
- b. A simple  $u$ - $du$  substitution with  $u = x^2 + 1$ .
- c. The integrand is not in its Partial Fraction Decomposition (PDF) so find the integrand's PDF, for which long division is not necessary.
- d. The integrand is not in its Partial Fraction Decomposition (PDF) so find the integrand's PDF, for which long division is necessary.
- e. None of the others.

3. Evaluate the integral

$$\int_0^4 \frac{x}{x + 9} dx .$$

- a.  $4 - 9 \ln(13) + 9 \ln(9)$
- b.  $13 - 9 \ln(4) + \ln(3)$
- c.  $\frac{1}{9 \ln(13)} - \ln(3)$
- d.  $4 - 13 \ln(9) + 3 \ln(18)$
- e. None of the others.



4. Evaluate the integral

$$\int_{x=1}^{x=e} \ln x \, dx .$$

- a.  $\frac{1}{e} - 1$
- b.  $1 - \frac{1}{e}$
- c.  $2e - 1$
- d. 1
- e. None of the others.

5. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^3 x \sin^4 x \, dx .$$

- a.  $\frac{4}{45}$
- b.  $\frac{14}{45}$
- c.  $\frac{2}{35}$
- d.  $\frac{12}{35}$
- e. None of the others.

6. Let  $y = p(x)$  be a polynomial of degree 5.

What is the form of the partial fraction decomposition of

$$\frac{p(x)}{(x^2 - 1)(x^2 + 1)^2} ?$$

Here  $A, B, C, D, E$  and  $F$  are constants.

- a.  $\frac{A}{x^2 - 1} + \frac{B}{(x^2 + 1)^2}$
- b.  $\frac{Ax + B}{x^2 - 1} + \frac{Cx + D}{(x^2 + 1)^2}$
- c.  $\frac{Ax + B}{x^2 - 1} + \frac{Cx + D}{x^2 + 1} + \frac{Ex + F}{(x^2 + 1)^2}$
- d.  $\frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 1} + \frac{Ex + F}{(x^2 + 1)^2}$
- e. None of the others.

7. Evaluate the integral

$$\int_{x=-\infty}^{x=\infty} \frac{1}{1+x^2} dx .$$

- a. 0
- b.  $\pi$
- c. diverges to infinity
- d. does not exist but also does not diverge to infinity
- e. None of the others.

8. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^3} dx .$$

- a. 0
- b.  $\frac{1}{4}$
- c. diverges to infinity
- d. does not exist but also does not diverge to infinity
- e. None of the others.

9. Investigate the convergence of

$$\int_{x=1}^{x=\infty} \frac{1-e^{-x}}{x} dx .$$

- a. The integral converges by the Limit Comparison Test, comparing the integrand with  $g(x) = \frac{1}{x}$ .
- b. The integral diverges by the Limit Comparison Test, comparing the integrand with  $g(x) = \frac{1}{x}$ .
- c. The integral converges by the Direct Comparison Test, comparing the integrand with  $g(x) = \frac{1}{x}$ .
- d. The integral diverges by the Direct Comparison Test, comparing the integrand with  $g(x) = \frac{1}{x}$ .
- e. None of the others.