HAND IN PART

MARK BOX		
PROBLEM	POINTS	
1-20	100	
%	100	

NAME: ____________________

PIN: ____________________

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND IN PART. Hand in only this part.
  (2) STATEMENT OF MULTIPLE CHOICE PROBLEMS. Do not hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- **For the Multiple Choice** problems, circle your answer(s) on the provided chart. No need to show work. The STATEMENT OF MULTIPLE CHOICE PROBLEMS will not be collected.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, electronic devices, any device with which you can connect to the internet, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13^{th} ed., ET): §8.1–8.5, 8.7–8.8, 10.1–10.10, 11.1–11.5 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ____________________

* Indicate (by circling) directly in the table below your solution to each problem.
* You may choice up to **2** answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 2 points.
  - For a problem with nothing marked (i.e., left blank) 1 point.
  - All other cases, 0 points.
* Fill in the "number of solutions circled" column. (Worth a total of 1 point of extra credit.)

Table for Your Muliple Choice Solutions							Do Not Write Below			
PROBLEM						number of solutions circled	1	2	B	x
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
11	11a	11b	11c	11d	11e					
12	12a	12b	12c	12d	12e					
13	13a	13b	13c	13d	13e					
14	14a	14b	14c	14d	14e					
15	15a	15b	15c	15d	15e					
16	16a	16b	16c	16d	16e					
17	17a	17b	17c	17d	17e					
18	18a	18b	18c	18d	18e					
19	19a	19b	19c	19d	19e					
20	20a	20b	20c	20d	20e					

5	2	1	0
Extra Credit:			

STATEMENT OF MULTIPLE CHOICE PROBLEMS
These sheets of paper are not collected.

- Hint. For a typical (i.e. not improper) definite integral problems $\int_a^b f(x)\,dx$.
  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.
  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.

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1. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \sin^2 x \, \cos x \, dx$$

a. 0

b. $\frac{\sqrt{2}}{2}$

c. $\frac{1}{3}\left(\frac{\pi}{2}\right)^3$

d. $\frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3$

e. None of the others.

2. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{4}} \sin^2 x \, dx$$

a. $\dfrac{\pi}{8} + \dfrac{1}{4}$

b. $\dfrac{\pi}{8} - \dfrac{1}{4}$

c. $\dfrac{\pi}{8} + \dfrac{1}{2}$

d. $\dfrac{\pi}{8} - \dfrac{1}{2}$

e. None of the others.

3. Evaluate the integral

$$\int_{x=0}^{x=\sqrt{\frac{\pi}{2}}} x \sin\left(x^2\right)\, dx$$

a. $\dfrac{1}{3}$

b. $\dfrac{1}{3}\sqrt{\dfrac{\pi}{2}}$

c. $\dfrac{1}{2}$

d. $\dfrac{1}{2}\sqrt{\dfrac{\pi}{2}}$

e. None of the others.

4. Evaluate the integral

$$\int_0^{\pi} e^{5x} \cos 3x \, dx$$

a. $\dfrac{5}{29}\left(e^{5\pi} - 1\right)$

b. $\dfrac{5}{29}\left(e^{5\pi} + 1\right)$

c. $\dfrac{3}{29}\left(e^{5\pi} - 1\right)$

d. $\dfrac{3}{29}\left(e^{5\pi} + 1\right)$

e. None of the others.

5. Evaluate the integral

$$\int_{-1}^{0} \frac{1}{\left(x^2 + 2x + 2\right)^2}\, dx \qquad .$$

Hint. Complete the square: $x^2 + 2x + 2 = (x \pm\ ?\ )^2 \pm\ ??$.

a. $\dfrac{\pi}{4} + \dfrac{1}{2}$

b. $\dfrac{\pi}{8} + \dfrac{1}{2}$

c. $\dfrac{\pi}{8} - \dfrac{1}{4}$

d. $\dfrac{\pi}{8} + \dfrac{1}{4}$

e. None of the others.

6. Let $y = p(x)$ be a polynomial of degree 5.

What is the form of the partial fraction decomposition of

$$\frac{p(x)}{(x^2-1)\,(x^2+1)^2} \ ?$$

Here A, B, C, D, E and F are constants.

a. $\dfrac{A}{x^2-1} + \dfrac{B}{(x^2+1)^2}$

b. $\dfrac{Ax+B}{x^2-1} + \dfrac{Cx+D}{(x^2+1)^2}$

c. $\dfrac{Ax+B}{x^2-1} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$

d. $\dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$

e. None of the others.

7. Evaluate the integral

$$\int_1^3 \frac{5x^2+3x-2}{x^3+2x^2}\,dx \quad .$$

a. $3\ln 5 - \ln 3 - \frac{2}{3}$

b. $3\ln 5 - \ln 3 - \frac{8}{3}$

c. $\ln 5 - \frac{2}{3}$

d. $\frac{2}{3} - \ln 5$

e. None of the others.

8. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^3}\,dx \ .$$

a. 0

b. $\frac{1}{4}$

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

9. Evaluate the integral

$$\int_{x=-\infty}^{x=\infty} \frac{1}{1+x^2}\, dx \ .$$

a. 0

b. π

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. None of the others.

10. Let c be a real number. Evaluate, if it exists, the limit of the sequence

$$\lim_{n\to\infty} \left(1+\tfrac{c}{n}\right)^n \quad .$$

a. 1

b. c

c. e^{-c}

d. e^c

e. None of the others.

11. Evaluate

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

Hint: Telescoping Series, use PFD.

a. 2

b.$\frac{1}{2}$

c. 1

d. 4

e. None of the others.

12. Consider the formal series $\sum_{n=1}^{\infty} a_n$ where

$$a_n \;=\; (-1)^n \frac{(n+1)!}{(2n)!}$$

and let

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \; .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = \frac{1}{2}$.

b. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = 0$.

c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$

d. $\sum_{n=1}^{\infty} a_n$ diverges

e. None of the others.

13. Consider the formal series $\sum_{n=1}^{\infty} a_n$ where

$$a_n \;=\; \frac{\sqrt{n+2}}{2n^2+n+1} \; .$$

a. converges, as can be shown using the Limit Comparison Test and comparing it to $b_n = n^{-\frac{3}{2}}$

b. diverges, as can be shown using the Limit Comparison Test and comparing it to $b_n = n^{-\frac{3}{2}}$

c. converges, as can be shown using the Limit Comparison Test and comparing it to $b_n = \frac{1}{n}$

d. diverges, as can be shown using the Limit Comparison Test and comparing it to $b_n = \frac{1}{n}$

e. None of the others.

14. What is the LARGEST interval (so you have to check your endpoints) for which the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+15)^n}{4^n}$$

is absolutely convergent?

a. $\left(\frac{11}{5}, \frac{19}{5}\right)$

b. $\left[\frac{11}{5}, \frac{19}{5}\right]$

c. $\left(\frac{-19}{5}, \frac{-11}{5}\right)$

d. $\left[\frac{-19}{5}, \frac{-11}{5}\right]$

e. None of the others.

15. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{2}{3-x}$$

about the center $x_0 = 0$ and state when this Taylor series is valid.

a. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n x^n$, valid for $|x| < 1$

b. $\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n$, valid for $|x| < 3$

c. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 1$

d. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 3$

e. None of the others.

16. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{1}{(1-x)^4}$$

about the center $x_0 = 0$ which is valid for $|x| < 1$.

Hint. Start with the Geometric Series (Prof. Girardi sometimes called him the work moose) and differentiate (as many times as needed). Be careful and don't forget the chain rule:

$$D_x(1-x)^{-1} \;=\; (-1)(1-x)^{-2}\,D_x(1-x) \;=\; (-1)(1-x)^{-2}\,(-1) \;=\; (1-x)^{-2}\,.$$

a. $\displaystyle\sum_{n=0}^{\infty}(n)(n-1)(n-2)\,x^n$

b. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\,(n)(n-1)(n-2)\,x^n$

c. $\displaystyle\sum_{n=0}^{\infty}\frac{(n+3)(n+2)(n+1)}{6}\,x^n$

d. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\,\frac{(n+3)(n+2)(n+1)}{6}\,x^n$

e. None of the others.

17. Consider the function

$$f(x) = e^{-x}$$

over the interval $(7, 9)$. The 5^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_5(x) \;=\; \sum_{n=0}^{5}\frac{(-x)^n}{n!} \;=\; 1 \;-\; x \;+\; \frac{x^2}{2!} \;-\; \frac{x^3}{3!} \;+\; \frac{x^4}{4!} \;-\; \frac{x^5}{5!}\,.$$

The 5^{th} order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $e^{-x} \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (7, 9)$.

a. $\dfrac{(e^{-7})(9^5)}{5!}$

b. $\dfrac{(e^{-9})(9^5)}{5!}$

c. $\dfrac{(e^{-7})(9^6)}{6!}$

d. $\dfrac{(e^{-9})(9^6)}{6!}$

e. None of the others.

18. The point with a polar coordinate representation $\left({}^{-}1, \frac{\pi}{4} \right)$ also has a polar coordinate representation $(\ {}^{+}1, \theta\ )$ where θ is

a. $\frac{\pi}{4}$.

b. $\frac{3\pi}{4}$.

c. $\frac{5\pi}{4}$.

d. $\frac{7\pi}{4}$.

e. None of the others.

19. Express the arc length of the *heart* traced out by the curve given by (in polar coordinates)

$$r = 2 + 2\sin\theta$$

as an integral.

a. $\displaystyle\int_{\theta=0}^{\theta=2\pi} (2 + 2\sin\theta)\ d\theta$

b. $\displaystyle\int_{\theta=0}^{\theta=2\pi} (2 + 2\sin\theta)^2\ d\theta$

c. $\displaystyle\int_{\theta=0}^{\theta=2\pi} \sqrt{(2 + 2\sin\theta)^2 + (2\cos\theta)^2}\ d\theta$

d. $\displaystyle\frac{1}{2}\int_{\theta=0}^{\theta=2\pi} \sqrt{(2 + 2\sin\theta)^2 + (2\cos\theta)^2}\ d\theta$

e. None of the others.

20. Prof. Girardi likes

a. the number 17

b. moose

c. colored chalk

d. mathematics

e. All of the above.

Good Luck in your math fun to come!