HAND IN PART

MARK BOX		
PROBLEM	POINTS	
0	15	
1-12	60	
13	10	
14	15	
%	100	

NAME: _____

PIN: _____

INSTRUCTIONS

- This exam comes in two parts.
  (1) HAND IN PART. Hand in only this part.
  (2) STATEMENT OF MULTIPLE CHOICE PROBLEMS. Do not hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- **On Problem 0**, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- **For multiple choice problems 1–12**, circle your answer(s) on the provided chart. No need to show work. The STATEMENT OF MULTIPLE CHOICE PROBLEMS will not be collected.
- **For problems > 12**, to receive credit you **MUST**:
  (1) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (2) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (3) if no such line/box is provided, then box your answer.
- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: books, electronic devices, any device with which you can connect to the internet, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. At a student's request, I will project my watch upon the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Thomas, 13th ed., ET): §8.1–8.5 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : _____

0. Fill in the blanks (each worth 1 point). (Recall that the *integrand* is the function you are integrating.)

0.1 $\int \frac{du}{u} =$ _____ +C

0.2 $\int \cos u \, du =$ _____ +C

0.3 $\int \sin u \, du =$ _____ +C

0.4 $\int \sec^2 u \, du =$ _____ +C

0.5 $\int \tan u \, du =$ _____ +C

0.6 $\int \cot u \, du =$ _____ +C

0.7 If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} \, du =$ _____ +C

0.8 If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} \, du =$ _____ +C

0.9 Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do _____

0.10 Integration by parts formula: $\int u \, dv =$ _____

0.11 Trig. Substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u =$ _____

0.12 Trig. Substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u =$ _____

0.13 Trig. Substitution:
if the integrand involves $a^2 - u^2$, then one makes the substitution $u =$ _____

0.14 Trig. Formula. (your answer should involve trig functions of θ, and not of 2θ): $\sin(2\theta) =$ _____

0.15 Trig Formula. Since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is _____

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choice up to **2** answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 3 points.
  - For a problem with nothing marked (i.e., left blank) 1 point.
  - All other cases, 0 points.
- Fill in the "number of solutions circled" column. (Worth a total of 1 point of extra credit.)

Your Solutions							Do Not Write Below			
PROBLEM						number of solutions circled	1	2	B	x
1	1a	1b	1c	1d	1e					
2	2a	2b	2c	2d	2e					
3	3a	3b	3c	3d	3e					
4	4a	4b	4c	4d	4e					
5	5a	5b	5c	5d	5e					
6	6a	6b	6c	6d	6e					
7	7a	7b	7c	7d	7e					
8	8a	8b	8c	8d	8e					
9	9a	9b	9c	9d	9e					
10	10a	10b	10c	10d	10e					
11	11a	11b	11c	11d	11e					
12	12a	12b	12c	12d	12e					
							Extra Credit:			

13. $\int \frac{x^2}{\sqrt{9-x^2}}\, dx \; =$ $+ \text{C}$

14. $\int \frac{x+4}{x^2+2x+5}\,dx \;=$ $+ \text{C}$

STATEMENT OF MULTIPLE CHOICE PROBLEMS
These sheets of paper are not collected.

- Hint. For a definite integral problems $\int_a^b f(x)\,dx$.
  (1) First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$.
  (2) Next check if you did the indefininte integral correctly by using the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$).
  (3) Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$.
- Hint. Throughout this exam, unless otherwise stated, follow the common calculus practice of measuring angles in radians (not degrees).

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1. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{1}{x^2+1}\,dx\ .$$

a. $\ln\sqrt{2}$

b. $\ln 2$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{2}$

e. None of the others.

2. The integral

$$\int \frac{x}{x^2+1}\,dx$$

can be evaluated the following way.

a. Trig. Substitution using the $x = \sin\theta$.

b. A simple u-du substitution with $u = x^2 + 1$.

c. The integrand is not in its Partial Fraction Decomposition (PDF) so find the integrand's PDF, for which long division is not necessary.

d. The integrand is not in its Partial Fraction Decomposition (PDF) so find the integrand's PDF, for which long division is necessary.

e. None of the others.

3. Evaluate the integral

$$\int_0^4 \frac{x}{x+9}\,dx\ .$$

a. $4 - 9\ln(13) + 9\ln(9)$

b. $13 - 9\ln(4) + \ln(3)$

c. $\frac{1}{9\ln(13)} - \ln(3)$

d. $4 - 13\ln(9) + 3\ln(18)$

e. None of the others.

4. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^3 x\ dx\ .$$

a. $\frac{-1}{3}$

b. $\frac{-2}{3}$

c. $\frac{1}{3}$

d . $\frac{2}{3}$

e. None of the others.

5. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^2 x\ dx\ .$$

a. $\frac{\pi}{4}$

b. π

c. $\frac{3\pi}{4}$

d. 2π

6. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^3 x \, \sin^4 x \, dx \, .$$

a. $\dfrac{4}{45}$
b. $\dfrac{14}{45}$
c. $\dfrac{2}{35}$
d. $\dfrac{12}{35}$
e. None of the others.

7. Evaluate the integral

$$\int_{x=1}^{x=e} \ln x \, dx \, .$$

a. 0
b. 1
c. e
d. $2e$
e. None of the others.

8. Evaluate the integral

$$\int_{x=0}^{x=\pi} e^{3x} \cos 2x \, dx \, .$$

a. 0
b. $\frac{1}{5}\left(3e^{3\pi}\right)$
c. $\frac{1}{5}\left(3e^{3\pi} - 3\right)$
d. $\frac{1}{13}\left(3e^{3\pi} - 3\right)$
e. None of the others.

9. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{1}{\sqrt{4+x^2}}\, dx\ .$$

Do not overlook the square root sign in the denominator.

a. $\frac{1}{2}\arctan\frac{1}{2}$

b. $\arctan\frac{1}{2}$

c. $\ln\left|\frac{\sqrt{5}}{2}+\frac{1}{2}\right|$

d. $\ln\left|\frac{\sqrt{5}}{2}+\frac{1}{2}\right| - \ln|2|$

e. None of the others.

10. Evaluate the integral

$$\int_{x=0}^{x=\frac{\sqrt{3}}{2}} \frac{4x^2}{(1-x^2)^{3/2}}\, dx$$

AND specify the initial substitution.

a. $\ln\left|(4\sqrt{3}-\frac{4\pi}{3})\right|$ using the initial substitute $x=\sin\theta$.

b. $(4\sqrt{3}-\frac{4\pi}{3})$ using the initial substitute $x=\sin\theta$

c. $\ln\left|(4\sqrt{3}-\frac{4\pi}{3})\right|$ using the initial substitute $x=\sec\theta$.

d. $(4\sqrt{3}-\frac{4\pi}{3})$ using the initial substitute $x=\sec\theta$

e. None of the others.

11. Let $y=p(x)$ be a polynomial of degree 5.
What is the form of the partial fraction decomposition of

$$\frac{p(x)}{(x^2-1)\,(x^2+1)^2}\ ?$$

Here A, B, C, D, E and F are constants.

a. $\dfrac{A}{x^2-1}+\dfrac{B}{(x^2+1)^2}$

b. $\dfrac{Ax+B}{x^2-1}+\dfrac{Cx+D}{(x^2+1)^2}$

c. $\dfrac{Ax+B}{x^2-1}+\dfrac{Cx+D}{x^2+1}+\dfrac{Ex+F}{(x^2+1)^2}$

d. $\dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{Cx+D}{x^2+1}+\dfrac{Ex+F}{(x^2+1)^2}$

e. None of the others.

12. Evaluate the integral

$$\int_{x=1}^{x=3} \frac{5x^2 + 3x - 2}{x^3 + 2x^2}\, dx \ .$$

a. $3\ln 5 - \ln 3 - \frac{2}{3}$

b. $3\ln 5 - \ln 3 - \frac{8}{3}$

c. $\ln 5 - \frac{2}{3}$

d. $\frac{2}{3} - \ln 5$

e. None of the others.