1. Commonly Used Taylor Series Fill in the LAST COLUMN of the below table.

$y = f(x)$	Maclaurin series of $y = f(x)$ in open (with "...") form	Maclaurin series of $y = f(x)$ in closed (with $\sum$-sign) form	Largest set of x's for which the Maclaurin series converges to the function.
$\frac{1}{1-x}$			
$\cos x$			
$\sin x$			
e^x			
$\ln(1+x)$			

2. Using a known (commonly used) Taylor series, find the Tayor series for

$$f(x) = \frac{1}{1+2x^3}$$

about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{3n}$ b. $\sum_{n=0}^{\infty} 2^n x^{3n}$
c. $\sum_{n=0}^{\infty} (-1)^n 2^n x^n$ d. $\sum_{n=0}^{\infty} 2^n x^n$
e. None of the others.

3. Find a power series reprsentation of

$$f(x) = \frac{x}{16+x^2}$$

and state the LARGEST interval for which it is valid.

a. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-4, 4)$
b. $f(x) = \sum_{n=0}^{\infty} \frac{1}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-4, 4)$
c. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-16, 16)$
d. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-1, 1)$
e. None of the others.

4. Find the 2^{nd} order Taylor polynomial for

$$f(x) = \sqrt[3]{x}$$

about the center $x_0 = 8$.

a. $2 + \frac{x}{12} - \frac{x^2}{9(2^5)}$ b. $2 + \frac{(x-8)}{12} + \frac{(x-8)^2}{9(2^5)}$
c. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^5)}$ d. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^4)}$
e. None of the others.

5. Suppose that the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ is 9. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?
a. 1 b. 3 c. 9 d. 81 e. None of the others.

6. Find a power series representation of the function $y = f(t)$ where

$$f(t) \; = \; \int_0^t \frac{1}{1+x^7}\, dx$$

and say for which values of t it is valid.

a. $\sum_{n=0}^{\infty} \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$ b. $\sum_{n=0}^{\infty} (-1)^n \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$

c. $\sum_{n=1}^{\infty} \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$ d. $\sum_{n=1}^{\infty} (-1)^n \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$

e. None of the others.

7. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n6^n} \; (2x-1)^n \, .$$

The center is $x_0 =$ ____________________ and the radius of convergence is $R =$ ____________________ . As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

$\longleftrightarrow$

8. Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of the function

$$f(x) = e^{-5x}$$

about center $x_0 = 0$. Define the function remainder $y = R_N(x)$ by $R_N(x) \; := \; f(x) - P_N(x)$.

For each of **8a** and **8b**, below the answer box, you must work in a logical fashion, indicating your reasoning.

8a. For $|R_N(x)|$, find a good upper bound that Taylor's Remainder Theorem guarantees is valid for each $x \in \mathbb{R}$.
Hint. In your answer, you can have N and x and c but you need to indicate what you know about c.

Answer: $|R_N(x)| \; \leq$

Work:

8b. For $|R_N(x)|$, find a good upper bound that Taylor's Remainder Theorem guarantees is valid for each $x \in (2,3)$.
Hint. In your answer, you can have N but cannot have x nor c.

Answer: $|R_N(x)| \; \leq$

Work: