MARK BOX		
PROBLEM	POINTS	
0	37	
1	7	
2	10	
3-5	15+1	
6	15	
7	15	
%	100	

NAME: KEY

PIN: 17

INSTRUCTIONS

- **On Problem 0**, fill in the blanks and boxes. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- **For problems 1 and 5**, circle your answer on the provided chart. You do **NOT** have to show your work.
- **For problems 6 and 7**, to receive credit you **must** show your work and :
  (1) **work in a logical fashion, show all your work, indicate your reasoning**;
  no credit will be given for an answer that *just appears*;
  such explanations help with partial credit
  (2) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (3) if no such line/box is provided, then box your answer.
- Upon request, you will be given as much (blank) scratch paper as you need.
- Check that your copy of the exam has all of the problems.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. Please, if I forget, remind me to pull up a clock on the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): §11.2–11.7 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ______________________

0. Fill-in-the boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$, unless otherwise indicated.

- **Sequences** Fill in the boxes with with the proper range of $r \in \mathbb{R}$.
  - $\lim_{n\to\infty} r^n = 0$ if and only if r satisfies [$|r| < 1$ also ok: $-1 < r < 1$ or $r \in (-1, 1)$].
  - $\lim_{n\to\infty} r^n = 1$ if and only if r satisfies [$r = 1$].
  - the sequence $\{r^n\}_{n=1}^{\infty}$ diverges to ∞ if and only if r satisfies [$r > 1$ also ok: $r \in (1, \infty)$].
  - the sequence $\{r^n\}_{n=1}^{\infty}$ diverges but does not diverge to ∞ if and only if r satisfies [$r \leq -1$ also ok: $r \in (-\infty, -1]$].

- State the **n^{th}-term test** for an arbitrary **SERIES** $\sum a_n$.

  [If $\lim_{n\to\infty} a_n \neq 0$ (which includes the case that $\lim_{n\to\infty} a_n$ does not exist), then $\sum a_n$ diverges .]

- State the **Alternating Series Test** for an alternating series $\sum(-1)^n u_n$ where $u_n > 0$ for each $n \in \mathbb{N}$.

  If
  - $\lim_{n\to\infty} u_n =$ [0]
  - and [$u_n > u_{n+1}$ for each $n \in \mathbb{N}$ (also ok: u_n's are (strictly) decreasing)]

  then $\sum(-1)^n u_n$ [converges].

- By definition, for an arbitrary series $\sum a_n$, (fill in these 3 boxes with converges or diverges).
  - $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ [converges].
  - $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ [converges] and $\sum |a_n|$ [diverges].

- **Geometric Series**. Fill in the boxes with the proper range of $r \in \mathbb{R}$.
  - The series $\sum r^n$ converges if and only if r satisfies [$|r| < 1$].
  - The series $\sum r^n$ diverges if and only if r satisfies [$|r| \geq 1$].

- **p-series**. Fill in the boxes with the proper range of $p \in \mathbb{R}$.
  - The series $\sum \frac{1}{n^p}$ converges if and only if [$p > 1$].
  - The series $\sum \frac{1}{n^p}$ diverges if and only if [$p \leq 1$].

- State the **Integral Test** for a positive-termed series $\sum a_n$.

  Let $f\colon [1, \infty) \to \mathbb{R}$ be so that
  - [$a_n = f(n)$ for each $n \in \mathbb{N}$]
  - f is a [continuous] function
  - f is a [positive] function
  - f is a [decreasing (nonincreasing is also ok)] function.

  Then $\sum a_n$ converges if and only if [$\int_{x=1}^{x=\infty} f(x)\, dx$] converges.

- State the **Comparison Test** for a positive-termed series $\sum a_n$.

  Let $N_0 \in \mathbb{N}$.

  - If $0 \le a_n \le c_n$ when $n \ge N_0$ and $\sum c_n$ converges, then $\sum a_n$ converges.
  - If $0 \le d_n \le a_n$ when $n \ge N_0$ and $\sum d_n$ diverges, then $\sum a_n$ diverges.

  Hint: sing the song to yourself.

- State the **Limit Comparison Test** for a positive-termed series $\sum a_n$.

  Let $b_n > 0$ and $L = \lim_{n\to\infty} \frac{a_n}{b_n}$

  - If $0 < L < \infty$, then $[\sum b_n$ converges $\Longleftrightarrow$ $\sum a_n$ converges $]$.
  - If $L = 0$, then $[\sum b_n$ converges $\Longrightarrow$ $\sum a_n$ converges $]$.
  - If $L = \infty$, then $[\sum b_n$ diverges $\Longrightarrow$ $\sum a_n$ diverges $]$.

- State the **Ratio and Root Tests** for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$.

  Let

  $$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \qquad \text{or} \qquad \rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} .$$

  - If $\rho < 1$ then $\sum a_n$ converges absolutely.
  - If $\rho > 1$ then $\sum a_n$ diverges.
  - If $\rho = 1$ then the test is inconclusive.

- In this Flow Chart, fill in the 3 blank boxes with: absolutely convergent, conditionally convergent, or divergent.

  Does $\sum |a_n|$ converge? — if NO $\Rightarrow$ Does $\lim_{n\to\infty} |a_n| = 0$? — if NO $\Rightarrow$ $\sum a_n$ is divergent

  if YES $\Downarrow$ (from "Does $\sum |a_n|$ converge?"): $\sum a_n$ is absolutely convergent

  if YES $\Downarrow$ (from "Does $\lim_{n\to\infty} |a_n| = 0$?"): Is $\sum a_n$ an alternating series?

  if YES $\Downarrow$: Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?

  if YES $\Downarrow$: $\sum a_n$ is conditionally convergent

- Fix $r \in \mathbb{R}$ with $r \ne 1$. For $N \ge 22$, let $s_N = \sum_{\mathbf{n=22}}^{N} r^n$. (Note the sum starts at 22). Then s_N can be written as:

  $$s_N = \frac{r^{22} - r^{N+1}}{1-r} .$$

  for all $N \ge 22$. Your answer should NOT contain a "…" nor a "$\sum$" sign.

1. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.

On the next 2, think of the n^{th}-term test for divergence and what if $a_n = \frac{1}{n}$		
T	(F)	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.
(T)	F	If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
On the next 2, think of a Theorem from class and what if $b_n = -a_n$.		
(T)	F	If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.
T	(F)	If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.
On the next 3, think of the mutually exclusive and exhaustive possibilities for a series.		
(T)	F	If $\sum \lvert a_n\rvert$ converges, then $\sum a_n$ converges.
T	(F)	If $\sum \lvert a_n\rvert$ diverges, then $\sum a_n$ diverges.
(T)	F	If $\sum a_n$ diverges, then $\sum \lvert a_n\rvert$ diverges.

2. Circle the behavior of the given series. The abbreviations are:

- AC stands for absolutely convergent
- CC stands for conditionally convergent
- DVG stand for divergent
- NOT stands for none of the others.

You can circle up to **1** answers for each problem. The scoring is as follows.

- For a problem with precisely one answer marked and the answer is correct, 1 points.
- All other cases, 0 points.

Series				
$\sum_{n=1}^{\infty} \frac{1}{n^2}$	(AC)	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$	(AC)	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{n}$	AC	CC	(DVG)	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$	AC	(CC)	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	AC	CC	(DVG)	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$	AC	(CC)	DVG	NOT
$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	AC	CC	(DVG)	NOT
$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$	AC	(CC)	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{e^n}$	(AC)	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$	(AC)	CC	DVG	NOT

Instructions for problems 3–5.

- Indicate (by circling) directly in the table below your solution to problems 3–5.
- You may choice up to **2** answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 2 points.
  - All other cases, 0 points.
- Fill in the "number of solutions circled" column (worth 1 pt).
- You do **NOT** have to show your work for problems 3–5.

Your Solutions **SEE PAGES TO FOLLOW FOR MORE DETAILS ON THE SOLUTIONS.**							
PROBLEM						# of solutions circled	points
3	3a	(3b)	3c	3d	3e		
4	4a	4b	(4c)	4d	4e		
5	(5a)	5b	5c	5d	5e		

3. By using the Limit Comparison Test, one can show that the formal series

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{(n+1)\ (n+2)\ (n+3)\ (n+4)\ (n+5)}}\ . \tag{3}$$

is:

a. convergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
b. convergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
c. divergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
d. divergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
e. none of the others

4. The formal series

$$\sum_{n=17}^{\infty} \frac{1}{n \ln n}$$

is:

a. convergent by the integral test
b. convergent by the ratio test
c. divergent by the integral test
d. divergent by the ratio test
e. none of the others

5. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{1}{n\ (n+1)} \tag{5}$$

and let

$$s_N = \sum_{n=1}^{N} \frac{1}{n\ (n+1)}\ .$$

Note that the partial fractions decompostion of $\frac{1}{n\ (n+1)}$ is $\frac{1}{n} - \frac{1}{n+1}$.

a. $s_N = 1 - \frac{1}{N+1}$ and the series in (5) converges to 1.
b. $s_N = 1 + \frac{1}{N+1}$ and the series in (5) converges to 1.
c. $s_N = 1 + \frac{1}{N}$ and the series in (5) converges to 1.
d. $s_N = 1 - \frac{1}{N}$ and the series in (5) converges to 1.
e. none of the others

3soln. This problem was very similar to one of your quiz problems, whose solution is located at

`http://people.math.sc.edu/girardi/m142/quizzes/LCTquiz151026soln.pdf`

In our problem 3 here,

$$a_n \;=\; \frac{n}{\sqrt{\,(n+1)\;(n+2)\;(n+3)\;(n+4)\;(n+5)}}$$

For n sufficiently big,

$$a_n \;=\; \frac{n}{\sqrt{\,(n+1)\;(n+2)\;(n+3)\;(n+4)\;(n+5)}} \;\overset{\text{when } n \text{ is big}}{\approx}\; \frac{n}{\sqrt{(n)\;(n)\;(n)\;(n)\;(n)}} \;=\; \frac{n}{\sqrt{n^5}} \;=\; \frac{n^1}{n^{5/2}} \;=\; \frac{1}{n^{3/2}}\;.$$

So we let $b_n = \left(\frac{1}{n}\right)^{3/2}$ and compute

$$\begin{aligned}\frac{a_n}{b_n} \;&=\; \frac{n}{\sqrt{\,(n+1)\;(n+2)\;(n+3)\;(n+4)\;(n+5)}}\;\frac{n^{3/2}}{1} \;=\; \frac{n^{5/2}}{[\;(n+1)\;(n+2)\;(n+3)\;(n+4)\;(n+5)\;]^{1/2}}\\ &=\; \left[\frac{n^5}{(n+1)\;(n+2)\;(n+3)\;(n+4)\;(n+5)}\right]^{1/2} \;=\; \left[\frac{n}{(n+1)}\;\frac{n}{(n+2)}\;\frac{n}{(n+3)}\;\frac{n}{(n+4)}\;\frac{n}{(n+5)}\right]^{1/2}\\ &\xrightarrow{n\to\infty} [\;(1)\;\;(1)\;\;(1)\;\;(1)\;\;(1)\;]^{1/2} \;=\; 1\;.\end{aligned}$$

Since $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$, the LCT says the $\sum a_n$ and $\sum b_n$ do the same thing. Since $\sum b_n$ is a p-series with $p = \frac{3}{2} > 1$, the $\sum b_n$ converges. So the $\sum a_n$ converges.

4soln. This was homework §11.3 # 21. It is also within another homework problem: §11.3 #27. See

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-3exercisesALL.pdf`

along with solutions at

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-3solnALL.pdf` .

Basically, apply the integral test. Note that the computation of

$$\int \frac{1}{x\,\ln x}\,dx \;=\; \ln|\ln|u|| \;+\; C\;,$$

as seen by using a u-du subsitution with $u = \ln x$. Let's see what happens if we try the ratio test. Let $a_n = \frac{1}{n\,\ln n}$. Then

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \;=\; \lim_{n\to\infty} \frac{1}{(n+1)\;\ln(n+1)}\;\frac{n\;\ln n}{1} \;=\; \lim_{n\to\infty} \frac{n}{n+1}\;\frac{\ln n}{\ln(n+1)}\;.$$

By L'Hopital's Rule,

$$\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} \;=\; \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n+1}} \;=\; \lim_{n\to\infty} \frac{n+1}{n} \;=\; 1\;.$$

Since $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, the Ratio Test is inconclusive.

5soln. This is Example 6 on page 691 from §11.2 of the textbook (Stewart).

EXAMPLE 6 Show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent, and find its sum.

SOLUTION This is not a geometric series, so we go back to the definition of a convergent series and compute the partial sums.

$$s_n = \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)}$$

We can simplify this expression if we use the partial fraction decomposition

$$\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$$

(see Section 7.4). Thus we have

$$\begin{aligned} s_n &= \sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+1} \right) \\ &= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right) \\ &= 1 - \frac{1}{n+1} \end{aligned}$$

and so
$$\lim_{n \to \infty} s_n = \lim_{n \to \infty} \left(1 - \frac{1}{n+1}\right) = 1 - 0 = 1$$

Therefore the given series is convergent and

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

□

6soln. A solution from a student.

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6. In this problem, you must show your work. Let

$$a_n = \frac{n!}{(2n)!}$$

7/7

6a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)}{(2n+1)(2n+2)}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{n!} = \frac{(2n)!}{(2n+2)!} \cdot \frac{(n+1)!}{n!} = \frac{(2n)!}{(2n)!(2n+1)(2n+2)} \cdot \frac{(n!)(n+1)}{n!}$$

$$= \frac{(n+1)}{(2n+1)(2n+2)}$$

6b. Check the correct box and then indicate your reasoning below. **SHOW ALL YOUR WORK.** Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

4/4

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n)!}$$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

$\sum a_n = (-1)^n \frac{n!}{(2n)!}$, $\sum |a_n| = \frac{n!}{(2n)}$

Ratio test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$= \lim_{n\to\infty} \left|\frac{(n+1)!}{(2n+2)!} \cdot \frac{(2n)!}{n!}\right| = \lim_{n\to\infty} \left|\frac{(2n)!}{(2n+2)!} \cdot \frac{(n+1)!}{n!}\right| = \lim_{n\to\infty} \left|\frac{(n+1)}{(2n+1)(2n+2)}\right| = \lim_{n\to\infty} \left|\frac{(n+1)}{(4n^2+6n+2)}\right| \overset{L.H.}{=}$$

$$= \lim_{n\to\infty} \left|\frac{1}{8n+6}\right| = 0 = \rho \rightarrow \rho = 0 < 1$$ absolutely convergent by the ratio test

Lovely

7soln. This problem is similar to homework §11.4 # 40b(i). See

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-4exercisesALL.pdf`

as well as the 6^{th} page of

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ETfromInstructorManual/Stewart6ET11-4solnALL.pdf`

as well as pages 2 and 4 of

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-4soln.pdf` .

Indeed, let $1 < j < \infty$ and consider the formal series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^j} \qquad \text{and let} \qquad a_n = \frac{\ln n}{n^j} . \tag{7}$$

Homework 40b(i) from §11.4 took $j = 3$. For the exam #7, we have $j = 3/2$. The series in (7) is also Exercise # 30 from §11.3; see

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-3exercisesALL.pdf`

along with solutions at

`http://people.math.sc.edu/girardi/m142/hmwk/Stewart6ET11-3solnALL.pdf` .

Since $\sum \frac{1}{n^p}$ converges when $p > 1$ and since $\ln n$ grows slower than n^q for any $q > 0$, we make an educated guess that the series in (7) converges. We next need to comfirm our educated guess, which we'll do 3 ways (1 way is enough).

LCT Compare with solution to 40b(i) on the 6^{th} page of these solutions, where $j = 3$ and they took $p = 2$.
Let $b_n = \frac{1}{n^p}$. Compute

$$\frac{a_n}{b_n} = \frac{\ln n}{n^j} \, \frac{n^p}{1} = \frac{\ln n}{n^{j-p}}$$

So $\lim_{n\to\infty} \frac{a_n}{b_n}$ is of the form $\frac{\infty}{\infty}$ provided $j - p > 0$. So also let $j - p > 0$ and apply L'Hopital's rule to see that

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\ln n}{n^{j-p}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{(j-p)\, n^{j-p-1}} = \frac{1}{(j-p)} \lim_{n\to\infty} \frac{1}{n^{j-p}} \overset{j-p>0}{=} 0 .$$

Part of the LCT says the

if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

To get the $\sum b_n$ to converges, we need $p > 1$. So we need

$$0 < j - p \qquad \text{and} \qquad 1 < p ,$$

which is equivalent to

$$1 < p < j .$$

So to confirm our educated guess, pick any p so that $1 < p < j$ and apply the LCT as indicated above.

CT Compare with solution to 40b(i) on the 4^{th} page of these solutions, where $j = 3$ and I took $p = 1.5$
For convergence, we want to *bound above by a convergent.* Using the Helpful Intuition, for any $q > 0$,

$$0 \le \frac{\ln n}{n^j} \overset{\text{when } n \text{ is big}}{\le} \frac{n^q}{n^j} = \frac{1}{n^{j-q}} = \frac{1}{n^p} = c_n$$

where we take $p = j - q$ and $c_n = \frac{1}{n^p}$. For $\sum c_n$ to converge, we need $p > 1$. Let's summarize what we need:

$$0 < q \qquad \text{and} \qquad 1 < p \qquad \text{where} \qquad p = j - q . \tag{CT}$$

What does condition (CT) look like when expressed in terms of conditions on p (without any q)? Well, note that if $p = j - q$, then $q = j - p$ and so

$$0 < q \qquad \text{if and only if} \qquad p < j .$$

So condition (CT) becomes

$$1 < p < j .$$

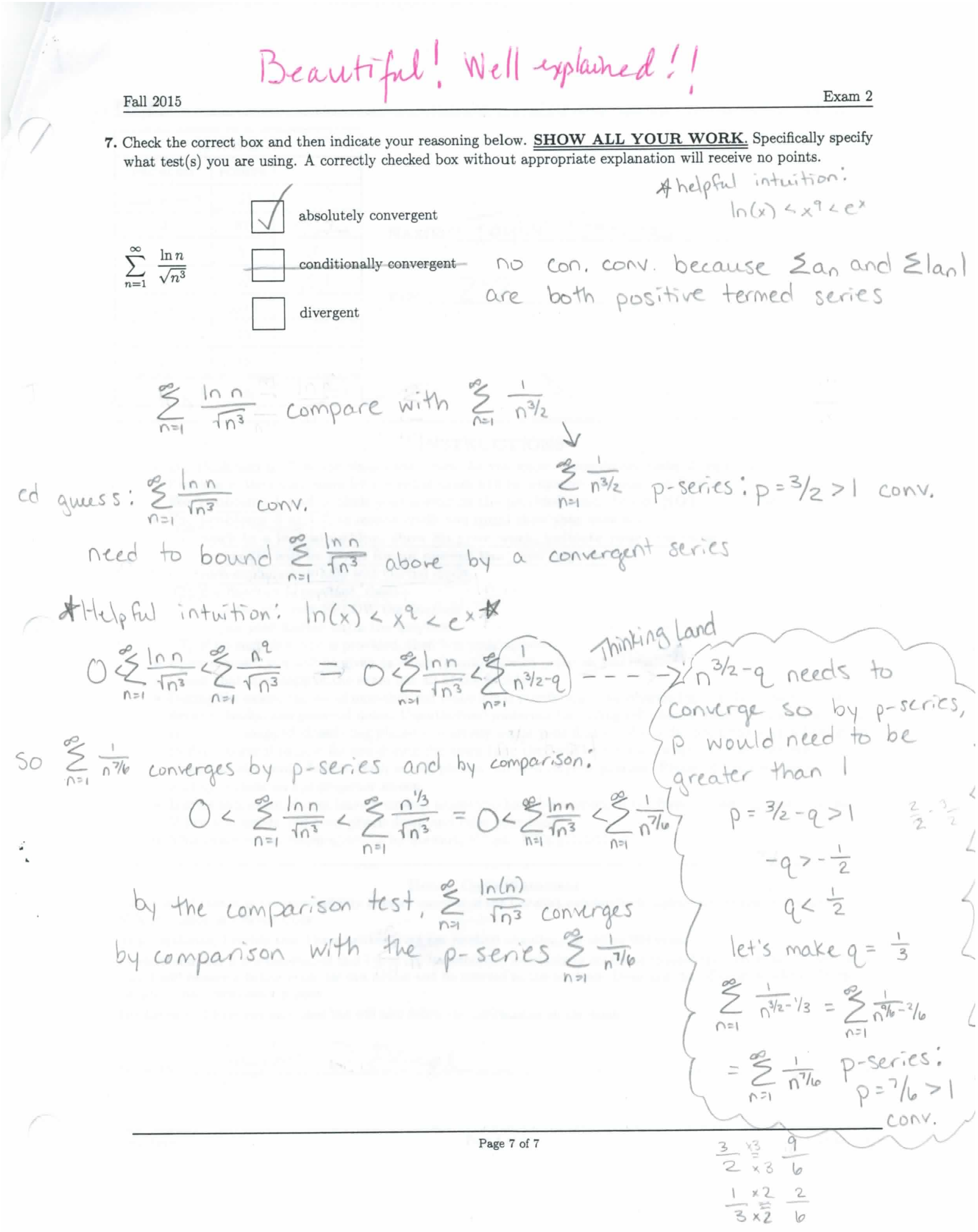
Note that if $1 < p < j$ and $p = j - q$, then

$$0 < q < j - 1 .$$

So to confirm our educated guess, pick any p so that $1 < p < j$ and apply the CT as indicated above.

Integral Test See the solution to Exercise 30 on the page of 6^{th} page of these solutions. Can you compute the needed $\int (\ln x) \left(x^{-3/2}\right) dx$? Hint: parts with $u = \ln x$ and $dv = x^{-3/2} dx$. Check what you get up against the solutions.

A solution from a student. Note that for $j = 3/2$ and her choice of $p = 7/6$, one has $1 < p < j$.



Beautiful! Well explained!!

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7. Check the correct box and then indicate your reasoning below. **SHOW ALL YOUR WORK.** Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}}$$

- [x] absolutely convergent
- [ ] ~~conditionally convergent~~
- [ ] divergent

*helpful intuition: $\ln(x) < x^q < e^x$

no con. conv. because $\sum a_n$ and $\sum |a_n|$ are both positive termed series

$\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}}$ compare with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ p-series: $p = 3/2 > 1$ conv.

cd guess: $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}}$ conv.

need to bound $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}}$ above by a convergent series

Helpful intuition: $\ln(x) < x^q < e^x$

$0 < \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}} < \sum_{n=1}^{\infty} \frac{n^q}{\sqrt{n^3}} = 0 < \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}} < \sum_{n=1}^{\infty} \frac{1}{n^{3/2-q}}$ - - - →

Thinking Land: $n^{3/2-q}$ needs to converge so by p-series, p would need to be greater than 1

$p = 3/2 - q > 1$

$-q > -\frac{1}{2}$

$q < \frac{1}{2}$

let's make $q = \frac{1}{3}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2 - 1/3}} = \sum_{n=1}^{\infty} \frac{1}{n^{9/6 - 2/6}}$

$= \sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$ p-series: $p = 7/6 > 1$ conv.

So $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$ converges by p-series and by comparison:

$0 < \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}} < \sum_{n=1}^{\infty} \frac{n^{1/3}}{\sqrt{n^3}} = 0 < \sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}} < \sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

by the comparison test, $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n^3}}$ converges by comparison with the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

$\frac{3}{2} \frac{\times 3}{\times 3} \frac{9}{6}$

$\frac{1}{3} \frac{\times 2}{\times 2} \frac{2}{6}$

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