MARK BOX		
PROBLEM	POINTS	
0	37	
1	7	
2	10	
3-5	15+1	
6	15	
7	15	
%	100	

NAME: ____________________

PIN: ____________________

INSTRUCTIONS

- **On Problem 0**, fill in the blanks and boxes. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- **For problems 1 and 5**, circle your answer on the provided chart. You do **NOT** have to show your work.
- **For problems 6 and 7**, to receive credit you **must** show your work and :
  (1) **work in a logical fashion, show all your work, indicate your reasoning**;
  no credit will be given for an answer that *just appears*;
  such explanations help with partial credit
  (2) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (3) if no such line/box is provided, then box your answer.
- Upon request, you will be given as much (blank) scratch paper as you need.
- Check that your copy of the exam has all of the problems.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. Please, if I forget, remind me to pull up a clock on the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): §11.2–11.7 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the instructions on the exam.

Signature : ____________________

0. Fill-in-the boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$, unless otherwise indicated.

- **Sequences** Fill in the boxes with with the proper range of $r \in \mathbb{R}$.
  - $\lim_{n\to\infty} r^n = 0$ if and only if r satisfies $\boxed{}$.
  - $\lim_{n\to\infty} r^n = 1$ if and only if r satisfies $\boxed{}$.
  - the sequence $\{r^n\}_{n=1}^{\infty}$ diverges to ∞ if and only if r satisfies $\boxed{}$.
  - the sequence $\{r^n\}_{n=1}^{\infty}$ diverges but does not diverge to ∞ if and only if r satisfies $\boxed{}$.

- State the $n^{\textbf{th}}$**-term test** for an arbitrary **SERIES** $\sum a_n$.

$\boxed{}$

- State the **Alternating Series Test** for an alternating series $\sum(-1)^n u_n$ where $u_n > 0$ for each $n \in \mathbb{N}$.

If
  - $\lim_{n\to\infty} u_n =$ $\boxed{}$
  - and $\boxed{}$

then $\sum(-1)^n u_n$ $\boxed{}$.

- By definition, for an arbitrary series $\sum a_n$, (fill in these 3 boxes with converges or diverges).
  - $\sum a_n$ is <u>absolutely convergent</u> if and only if $\sum |a_n|$ $\boxed{}$.
  - $\sum a_n$ is <u>conditionally convergent</u> if and only if $\sum a_n$ $\boxed{}$ and $\sum |a_n|$ $\boxed{}$.

- **Geometric Series**. Fill in the boxes with the proper range of $r \in \mathbb{R}$.
  - The series $\sum r^n$ converges if and only if r satisfies $\boxed{}$.
  - The series $\sum r^n$ diverges if and only if r satisfies $\boxed{}$.

- p**-series**. Fill in the boxes with the proper range of $p \in \mathbb{R}$.
  - The series $\sum \frac{1}{n^p}$ converges if and only if $\boxed{}$.
  - The series $\sum \frac{1}{n^p}$ diverges if and only if $\boxed{}$.

- State the **Integral Test** for a positive-termed series $\sum a_n$.

Let $f\colon [1,\infty) \to \mathbb{R}$ be so that
  - $\boxed{}$
  - f is a $\boxed{}$ function
  - f is a $\boxed{}$ function
  - f is a $\boxed{}$ function.

Then $\sum a_n$ converges if and only if $\boxed{}$ converges.

- State the **Comparison Test** for a positive-termed series $\sum a_n$.

  Let $N_0 \in \mathbb{N}$.

  - If ______ when $n \geq N_0$ and ______, then $\sum a_n$ converges.
  - If ______ when $n \geq N_0$ and ______, then $\sum a_n$ diverges.

  Hint: sing the song to yourself.

- State the **Limit Comparison Test** for a positive-termed series $\sum a_n$.

  Let $b_n > 0$ and $L = \lim_{n\to\infty} \frac{a_n}{b_n}$

  - If ______, then ______.
  - If ______, then ______.
  - If ______, then ______.

- State the **Ratio and Root Tests** for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$.

  Let

  $$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \qquad \text{or} \qquad \rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} .$$

  - If ______ then $\sum a_n$ converges absolutely.
  - If ______ then $\sum a_n$ diverges.
  - If ______ then the test is inconclusive.

- In this Flow Chart, fill in the 3 blank boxes with: absolutely convergent, conditionally convergent, or divergent.

  Does $\sum |a_n|$ converge? $\xrightarrow{\text{if NO}}$ Does $\lim_{n\to\infty} |a_n| = 0$? $\xrightarrow{\text{if NO}}$ $\sum a_n$ is ______

  Does $\sum |a_n|$ converge? if YES $\Downarrow$ $\sum a_n$ is ______

  Does $\lim_{n\to\infty} |a_n| = 0$? if YES $\Downarrow$ Is $\sum a_n$ an alternating series?

  if YES $\Downarrow$ Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?

  if YES $\Downarrow$ $\sum a_n$ is ______

- Fix $r \in \mathbb{R}$ with $r \neq 1$. For $N \geq 22$, let $s_N = \sum_{\mathbf{n=22}}^{N} r^n$. (Note the sum starts at 22). Then s_N can be written as:

  $$s_N = \qquad\qquad .$$

  for all $N \geq 22$. Your answer should NOT contain a "…" nor a "$\sum$" sign.

1. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.

On the next 2, think of the n^{th}-term test for divergence and what if $a_n = \frac{1}{n}$		
T	F	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.
T	F	If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
On the next 2, think of a Theorem from class and what if $b_n = -a_n$.		
T	F	If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.
T	F	If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.
On the next 3, think of the mutually exclusive and exhaustive possibilities for a series.		
T	F	If $\sum \lvert a_n\rvert$ converges, then $\sum a_n$ converges.
T	F	If $\sum \lvert a_n\rvert$ diverges, then $\sum a_n$ diverges.
T	F	If $\sum\ a_n$ diverges, then $\sum \lvert a_n\rvert$ diverges.

2. Circle the behavior of the given series. The abbreviations are:

- AC stands for absolutely convergent
- CC stands for conditionally convergent
- DVG stand for divergent
- NOT stands for none of the others.

You can circle up to **1** answers for each problem. The scoring is as follows.

- For a problem with precisely one answer marked and the answer is correct, 1 points.
- All other cases, 0 points.

Series				
$\sum_{n=1}^{\infty} \frac{1}{n^2}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{n}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$	AC	CC	DVG	NOT
$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	AC	CC	DVG	NOT
$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{1}{e^n}$	AC	CC	DVG	NOT
$\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$	AC	CC	DVG	NOT

Instructions for problems 3–5.

- Indicate (by circling) directly in the table below your solution to problems 3–5.
- You may choice up to **2** answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 2 points.
  - All other cases, 0 points.
- Fill in the "number of solutions circled" column (worth 1 pt).
- You do **NOT** have to show your work for problems 3–5.

Your Solutions							
PROBLEM						# of solutions circled	points
3	3a	3b	3c	3d	3e		
4	4a	4b	4c	4d	4e		
5	5a	5b	5c	5d	5e		

3. By using the Limit Comparison Test, one can show that the formal series

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{(n+1)\ (n+2)\ (n+3)\ (n+4)\ (n+5)}}\ . \tag{3}$$

is:

a. convergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
b. convergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
c. divergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 5/2$.
d. divergent by comparing the series in (3) to the p-series $\sum \left(\frac{1}{n}\right)^p$ with $p = 3/2$.
e. none of the others

4. The formal series

$$\sum_{n=17}^{\infty} \frac{1}{n \ln n}$$

is:

a. convergent by the integral test
b. convergent by the ratio test
c. divergent by the integral test
d. divergent by the ratio test
e. none of the others

5. Consider the formal series

$$\sum_{n=1}^{\infty} \frac{1}{n\ (n+1)} \tag{5}$$

and let

$$s_N = \sum_{n=1}^{N} \frac{1}{n\ (n+1)}\ .$$

Note that the partial fractions decompostion of $\frac{1}{n\ (n+1)}$ is $\frac{1}{n} - \frac{1}{n+1}$.

a. $s_N = 1 - \frac{1}{N+1}$ and the series in (5) converges to 1.
b. $s_N = 1 + \frac{1}{N+1}$ and the series in (5) converges to 1.
c. $s_N = 1 + \frac{1}{N}$ and the series in (5) converges to 1.
d. $s_N = 1 - \frac{1}{N}$ and the series in (5) converges to 1.
e. none of the others

6. In this problem, you must show your work. Let

$$a_n = \frac{n!}{(2n)!}$$

6a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} =$$

6b. Check the correct box and then indicate your reasoning below. **SHOW ALL YOUR WORK.** Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n)!}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

7. Check the correct box and then indicate your reasoning below. **SHOW ALL YOUR WORK**. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n^3}}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent