

HAND IN PART

MARK BOX		
PROBLEM	POINTS	
0	25	
1-7	35	
8	10	
9	10	
10	10	
11	10	
%	100	

NAME: KEYPIN: 017**INSTRUCTIONS**

- On Problem 0, fill in the blanks. As you know, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- Problems 1-7 are multiple choice.
 - First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
 - Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
 - You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).
- For the remaining problems, to receive credit you must:
 - (1) **work in a logical fashion, show all your work, indicate your reasoning;**
no credit will be given for an answer that just appears;
such explanations help with partial credit
 - (2) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (3) if no such line/box is provided, then box your answer.
- Upon request, you will be given as much (blank) scratch paper as you need.
- Check that your copy of the exam has all of the problems.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. Please, if I forget, remind me to pull up a clock on the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Stewart, 6th ed., ET):
§7.1-7.5, 7.8, 11.1 .

0. Fill in the blanks (each worth 1 point).

- $\int \cos u \, du = \underline{\sin u} + C$
- $\int \sin u \, du = \underline{-\cos u} + C$
- trig formula ... since $\cos^2 \theta + \sin^2 \theta = 1$, we know that the corresponding relationship between tangent (i.e., \tan) and secant (i.e., \sec) is $\underline{1 + \tan^2 \theta = \sec^2 \theta}$.
- Integration by parts formula: $\int u \, dv = \underline{uv - \int v \, du}$
- If a is a constant and $a > 0$ then $\int \frac{1}{a^2 + u^2} \, du = \underline{\frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + C}$
- Trig. Substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u = \underline{a \tan \theta}$
- If $u \neq 0$, then $\int \frac{du}{u} = \underline{\ln |u|} + C$
- Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polynomials and $[\text{degree of } f] \geq [\text{degree of } g]$, then one must first do long division
- If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u \, du = \underline{\frac{a^u}{\ln a} + C}$
- $\int \tan u \, du = \underline{\ln |\sec u| \text{ or } -\ln |\cos u|} + C$
- $\int \cot u \, du = \underline{-\ln |\csc u| \text{ or } \ln |\sin u|} + C$
- $\int \sec^2 u \, du = \underline{\tan u} + C$
- $\int \sec u \tan u \, du = \underline{\sec u} + C$
- $\int \csc^2 u \, du = \underline{-\cot u} + C$
- $\int \csc u \cot u \, du = \underline{-\csc u} + C$
- $\int \sec u \, du = \underline{\ln |\sec u + \tan u| \text{ or } -\ln |\sec u - \tan u|} + C$
- $\int \csc u \, du = \underline{-\ln |\csc u + \cot u| \text{ or } \ln |\csc u - \cot u|} + C$
- If a is a constant and $a > 0$ then $\int \frac{1}{\sqrt{a^2 - u^2}} \, du = \underline{\sin^{-1} \left(\frac{u}{a} \right) + C}$
- If a is a constant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2 - a^2}} \, du = \underline{\frac{1}{a} \sec^{-1} \left(\frac{|u|}{a} \right) + C}$
- Trig. Substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u = \underline{a \sin \theta}$
- Trig. Substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u = \underline{a \sec \theta}$
- Trig. Formula ... your answer should involve trig functions of θ , and not of 2θ : $\sin(2\theta) = \underline{2 \sin \theta \cos \theta}$.
- Trig. Formula ... $\cos(2\theta)$ or $\sin(2\theta)$ should appear in your answer $\cos^2(\theta) = \underline{\frac{1 + \cos(2\theta)}{2}}$
- Trig. Formula ... $\cos(2\theta)$ or $\sin(2\theta)$ should appear in your answer $\sin^2(\theta) = \underline{\frac{1 - \cos(2\theta)}{2}}$
- $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = \underline{\frac{-\pi}{3}}$ **RADIANS.** (your answer should be an angle)

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choose up to **2** answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 5 points. For a problem with precisely two answers marked, one of which is correct, 2 points. All other cases, 0 points.
- Fill in the “number of solutions circled” column. (Worth 2 points of extra credit.)

Your Solutions							Do Not Write Below		
							points		
PROBLEM						number of solutions circled	5	2	0
1	1a	1b	1c	1d	1e				
2	2a	2b	2c	2d	2e				
3	3a	3b	3c	3d	3e				
4	4a	4b	4c	4d	4e				
5	5a	5b	5c	5d	5e				
6	6a	6b	6c	6d	6e				
7	7a	7b	7c	7d	7e				
							Extra Credit:		

8. For the following **SEQUENCES**:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE (which is equivalent to saying it diverges).

Put your **ANSWER IN** the box and show your **WORK BELOW** the box.

8a.

$$\lim_{n \rightarrow \infty} \frac{5n^2 + 4\sqrt{n}}{6n^2 + 7n + 1} = \frac{5}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{\frac{5n^2 + 4n^{1/2}}{n^2}}{\frac{6n^2 + 7n + 1}{n^2}} = \lim_{n \rightarrow \infty} \frac{5 + \frac{4}{n^{3/2}}}{6 + \frac{7}{n} + \frac{1}{n^2}} = \frac{5 + 0}{6 + 0 + 0}$$

8b.

$$\lim_{n \rightarrow \infty} \frac{-5n^8 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = -\infty \text{ or DNE.}$$

divide num. & den. by n^8 (highest power we see) and do the algebra. $= n^8$

$$\lim_{n \rightarrow \infty} \frac{-5 + \frac{4}{n^{15/2}}}{\frac{6}{n^5} + \frac{7}{n^6} + \frac{1}{n^8}} = \frac{-5 + 0}{0 + 0 + 0} = \frac{-5}{0} = -\infty = -\infty$$

\uparrow
 $a_n < 0$ for big n .

8c.

$$\lim_{n \rightarrow \infty} \frac{5n^3 + 4\sqrt{n}}{6n^8 + 7n^2 + 1} = 0$$

divide num. & den. by n^8 (highest power) and do the algebra. $= n^8$

$$= \lim_{n \rightarrow \infty} \frac{\frac{5}{n^5} + \frac{4}{n^{15/2}}}{6 + \frac{7}{n^6} + \frac{1}{n^8}} = \frac{0 + 0}{6 + 0 + 0} = 0$$

9a. Complete the square by filling in each of the two lines with a (positive or negative) number.

$$x^2 - 6x + 13 = (x + \underline{-3})^2 + \underline{4}.$$

$$x^2 - 6x + 13 = \underbrace{x^2 - 6x + 9}_{(x-3)^2} + \underbrace{-9 + 13}_4$$

9b.

$$\int \frac{1}{\sqrt{x^2 - 6x + 13}} dx = \ln \left| \sqrt{x^2 - 6x + 13} + x - 3 \right| + C$$

$$\text{or } \ln \left| \frac{\sqrt{x^2 - 6x + 13}}{2} + \frac{x - 3}{2} \right| + C$$

From Stewart, Calculus ET, 2nd Ed., § 7.3 Exercise 7.3

For the solution, change t to x .

$$x - 3 = 2 \tan \theta$$

24. $t^2 - 6t + 13 = (t^2 - 6t + 9) + 4 = (t - 3)^2 + 2^2.$

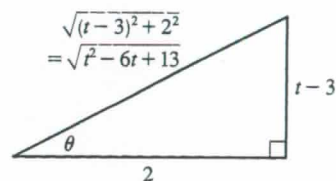
Let $t - 3 = 2 \tan \theta$, so $dt = 2 \sec^2 \theta d\theta$. Then

$$\int \frac{dt}{\sqrt{t^2 - 6t + 13}} = \int \frac{1}{\sqrt{(2 \tan \theta)^2 + 2^2}} 2 \sec^2 \theta d\theta$$

$$= \int \frac{2 \sec^2 \theta}{2 \sec \theta} d\theta = \int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C_1 \quad [\text{by Formula 7.2.1}]$$

$$= \ln \left| \frac{\sqrt{t^2 - 6t + 13}}{2} + \frac{t - 3}{2} \right| + C_1$$

$$= \ln |\sqrt{t^2 - 6t + 13} + t - 3| + C \quad \text{where } C = C_1 - \ln 2$$



10.

$$\int \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} dx = \frac{1}{2} \ln(x^2 + 1) + \tan^{-1} x + \frac{-1}{2(x^2 + 1)} + C$$

From the 100 Integrals Handout.

Circled # 71.

$$\textcircled{71} \quad \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} = \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)^2} = \frac{Ax + B}{(x^2 + 1)^1} + \frac{Cx + D}{(x^2 + 1)^2}$$

$$\Rightarrow \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)^2} = \frac{(Ax + B)(x^2 + 1) + (Cx + D)}{(x^2 + 1)^2}$$

$$\Rightarrow x^3 + x^2 + 2x + 1 = (Ax + B)(x^2 + 1) + (Cx + D)$$

$$x^3 : 1 = A$$

$$x^2 : 1 = B$$

$$x : 2 = A + C \rightarrow C = 1$$

$$\text{constant} : 1 = B + D \rightarrow D = 0$$

$$\int \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} dx = \int \left(\frac{x + 1}{x^2 + 1} + \frac{x}{(x^2 + 1)^2} \right) dx$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 + 1} + \int \frac{dx}{x^2 + 1} + \frac{1}{2} \int (x^2 + 1)^{-2} (2x dx)$$

$$= \frac{1}{2} \ln(x^2 + 1) + \tan^{-1}(x) + \frac{1}{2} \frac{(x^2 + 1)^{-1}}{-1} + C.$$

11. Part 11a should help with part 11b.

11a $\int e^{x^2} (2x) dx = e^{x^2} + C$

<u>Way #1</u>	$u = x^2 \Rightarrow du = 2x dx$	$\int e^{x^2} 2x dx = \int e^u du = e^u + C = e^{x^2} + C$
<u>Way #2</u>	$\tilde{u} = e^{x^2} \Rightarrow d\tilde{u} = e^{x^2} (2x) dx$	$\int e^{x^2} 2x dx = \int d\tilde{u} = \tilde{u} + C = e^{x^2} + C$

11b. We cannot integrate the functions $y = e^{x^2}$ and $y = x^2 e^{x^2}$ with techniques we have learned thus far (in fact, they do not have elementary antiderivatives). But we can integrate $y = (2x^2 + 1) e^{x^2}$ with techniques we know thus far. Evaluate $\int (2x^2 + 1) e^{x^2} dx$.

$$\int (2x^2 + 1) e^{x^2} dx = x e^{x^2} + C$$

Exercise from textbook: § 7.5 # 81

81. The function $y = 2xe^{x^2}$ does have an elementary antiderivative, so we'll use this fact to help evaluate the integral.

$$\int (2x^2 + 1) e^{x^2} dx = \int 2x^2 e^{x^2} dx + \int e^{x^2} dx$$

$$= \left[\int x (2xe^{x^2}) dx \right] + \int e^{x^2} dx$$

$$\left[\begin{array}{l} u = x, \quad dv = 2xe^{x^2} dx \\ du = dx, \quad v = e^{x^2} \end{array} \right]$$

$$= \left[xe^{x^2} - \int e^{x^2} dx \right] + \int e^{x^2} dx$$

$$= xe^{x^2} + C$$

Part (a) suggests that to integrate $\int x (2xe^{x^2}) dx$, try parts with $dv = 2x e^{x^2}$

To check $D_x (xe^{x^2}) = x (e^{x^2} (2x)) + (1)(e^{x^2})$
 $= e^{x^2} (2x^2 + 1) \checkmark$

MC 1

STATEMENT OF MULTIPLE CHOICE PROBLEMS

1. Evaluate the integral

$$\int_0^1 \frac{x}{x^2+9} dx.$$

Hint: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$.

- a. $\ln\left(\frac{10}{9}\right)$ (b) $\frac{1}{2} \ln\left(\frac{10}{9}\right)$ c. $\frac{1}{2} \ln(10) - \ln(9)$ d. $2\left(\frac{10}{9}\right)$ e. None of the others.

$$\int \frac{x}{x^2+9} dx = \frac{1}{2} \int \frac{2x dx}{x^2+9} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2+9| + C$$

$$\begin{array}{l} u = x^2+9 \\ du = 2x dx \end{array}$$

Check $D_x \frac{1}{2} \ln|x^2+9| = \frac{1}{2} \frac{1}{x^2+9} \cdot 2x = \frac{x}{x^2+9} \checkmark$

$$\begin{aligned} \text{So } \int_0^1 \frac{x}{x^2+9} dx &= \frac{1}{2} \ln|x^2+9| \Big|_{x=0}^{x=1} = \frac{1}{2} \ln 10 - \frac{1}{2} \ln 9 \\ &= \frac{1}{2} [\ln 10 - \ln 9] = \frac{1}{2} \ln\left(\frac{10}{9}\right) \end{aligned}$$

2. Evaluate the integral

$$\int_0^4 \frac{x}{x+9} dx.$$

Hint: do you have (strictly) bigger bottoms?

- (a) $4 - 9\ln(13) + 9\ln(9)$ b. $13 - 9\ln(4) + \ln(3)$ c. $\frac{1}{9\ln(13)} - \ln(3)$ d. $4 - 13\ln(9) + 3\ln(18)$
e. None of the others.

$$\int \frac{x}{x+9} dx = \int \frac{1x - 9}{x+9} dx = x - 9 \ln|x+9| + C$$

$$\begin{array}{l} \frac{x}{x+9} = \frac{x+9}{x+9} - \frac{9}{x+9} \\ \text{Long} \\ \text{Division} \\ \text{(Fake)} \\ = 1 - \frac{9}{x+9} \end{array}$$

Check $D_x [x - 9 \ln|x+9|] = 1 - \frac{9}{x+9}$
 $= \frac{x+9}{x+9} - \frac{9}{x+9} = \frac{x}{x+9} \checkmark$

$$\begin{aligned} \text{So } \int_0^4 \frac{x}{x+9} dx &= [x - 9 \ln|x+9|] \Big|_{x=0}^{x=4} \\ &= [4 - 9 \ln|13|] - [0 - 9 \ln|9|] \\ &= 4 - 9 \ln(13) + 9 \ln(9) \end{aligned}$$

3. Evaluate

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$$

- a. $\frac{3}{10}$ b. $\frac{7}{10}$ c. $\frac{8}{15}$ (d) $\frac{2}{15}$ e. None of the others.

$$\int \sin^2 x \cos^3 x \, dx = \int (\sin^2 x) (1 - \sin^2 x) \cos x \, dx$$

$$u = \sin x$$

$$du = \cos x \, dx$$

$$= \int u^2 (1 - u^2) \, du = \int (u^2 - u^4) \, du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx = \left. \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} \right|_{x=0}^{x=\pi/2} = \frac{1}{3} - \frac{1}{5} = \frac{5-3}{15} = \frac{2}{15}$$

4. Evaluate the integral

$$\int_{x=5}^{x=10} \frac{\sqrt{x^2 - 25}}{x} \, dx$$

AND specify the initial substitution.

- a. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sec \theta$.
 c. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sin \theta$.
 e. None of the others.
 (b) $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sec \theta$
 d. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5 \sin \theta$

- First, Evaluate $\int \frac{\sqrt{x^2 - 25}}{x} \, dx$, assuming that $x \geq 5$.

Solution. The integrand involves a radical of the form $\sqrt{x^2 - a^2}$ with $a = 5$, so from Table 8.4.1 we make the substitution

$$x = 5 \sec \theta, \quad 0 \leq \theta < \pi/2$$

$$\frac{dx}{d\theta} = 5 \sec \theta \tan \theta \quad \text{or} \quad dx = 5 \sec \theta \tan \theta \, d\theta$$

Thus,

$$\begin{aligned} \int \frac{\sqrt{x^2 - 25}}{x} dx &= \int \frac{\sqrt{25 \sec^2 \theta - 25}}{5 \sec \theta} (5 \sec \theta \tan \theta) d\theta \\ &= \int \frac{5 |\tan \theta|}{5 \sec \theta} (5 \sec \theta \tan \theta) d\theta \\ &= 5 \int \tan^2 \theta d\theta \quad \tan \theta \geq 0 \text{ since } 0 \leq \theta < \pi/2 \\ &= 5 \int (\sec^2 \theta - 1) d\theta = 5 \tan \theta - 5\theta + C \end{aligned}$$

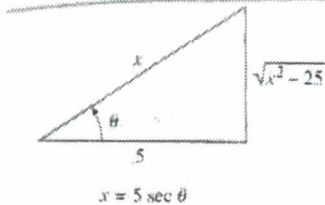


Figure 8.4.5

To express the solution in terms of x , we will represent the substitution $x = 5 \sec \theta$ geometrically by the triangle in Figure 8.4.5, from which we obtain

$$\tan \theta = \frac{\sqrt{x^2 - 25}}{5}$$

From this and the fact that the substitution can be expressed as $\theta = \sec^{-1}(x/5)$, we obtain

$$\int \frac{\sqrt{x^2 - 25}}{x} dx = \sqrt{x^2 - 25} - 5 \sec^{-1}\left(\frac{x}{5}\right) + C$$

• Check $D_x \left[(x^2 - 25)^{1/2} - 5 \sec^{-1}\left(\frac{x}{5}\right) \right]$

$$\begin{aligned} &= \frac{1}{2} (x^2 - 25)^{-1/2} (2x) - 5 \frac{1}{\frac{x}{5} \sqrt{\left(\frac{x}{5}\right)^2 - 1}} \cdot \frac{1}{5} \\ &= \frac{x}{(x^2 - 25)^{1/2}} - \frac{1}{\frac{x}{5} \sqrt{\frac{x^2}{25} - \frac{25}{25}}} \quad (\text{know } x \geq 5) \\ &= \frac{x}{(x^2 - 25)^{1/2}} - \frac{25}{x(x^2 - 25)^{1/2}} = \frac{(x^2 - 25)}{x(x^2 - 25)^{1/2}} \cdot \frac{(x^2 - 25)^{1/2}}{(x^2 - 25)^{1/2}} \\ &= \frac{(x^2 - 25)(x^2 - 25)^{1/2}}{x(x^2 - 25)} = \frac{\sqrt{x^2 - 25}}{x} \quad \checkmark \end{aligned}$$

• $\int_5^{10} \frac{\sqrt{x^2 - 25}}{x} dx = \left. \sqrt{x^2 - 25} - 5 \sec^{-1}\left(\frac{x}{5}\right) \right|_{x=5}^{x=10}$

$$\begin{aligned} &= \left[\sqrt{100 - 25} - 5 \sec^{-1} 2 \right] - \left[0 - 5 \sec^{-1} 1 \right] \\ &= \sqrt{75} - 5 \cdot \frac{\pi}{3} = 5\sqrt{3} - 5\left(\frac{\pi}{3}\right) = 5\left(\sqrt{3} - \frac{\pi}{3}\right) \end{aligned}$$

MC4

5. Evaluate the integral

$$\int_{x=1}^{x=3} \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx.$$

- (a) $3 \ln 5 - \ln 3 - \frac{2}{3}$ b. $3 \ln 5 - \ln 3 - \frac{8}{3}$ c. $\ln 5 - \frac{2}{3}$ d. $\frac{2}{3} - \ln 5$ e. None of the others.

• $\frac{5x^2 + 3x - 2}{x^3 + 2x^2} = \frac{5x^2 + 3x - 2}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$. Multiply by $x^2(x+2)$ to

get $5x^2 + 3x - 2 = Ax(x+2) + B(x+2) + Cx^2$. Set $x = -2$ to get $C = 3$, and take

$x = 0$ to get $B = -1$. Equating the coefficients of x^2 gives $5 = A + C \Rightarrow A = 2$. So

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx = \int \left(\frac{2}{x} - \frac{1}{x^2} + \frac{3}{x+2} \right) dx = 2 \ln|x| + \frac{1}{x} + 3 \ln|x+2| + C.$$

• Check $D_x [2 \ln|x| + x^{-1} + 3 \ln|x+2|] = \frac{2}{x} - 1x^{-2} + \frac{3}{x+2}$
 $= \frac{2}{x} - \frac{1}{x^2} + \frac{3}{x+2} = \frac{2x(x+2) - (x+2) + 3x^2}{x^2(x+2)} = \frac{5x^2 + 3x - 2}{x^3 + 2x^2} \checkmark$

• $\left[3 \ln|x+2| + 2 \ln|x| + \frac{1}{x} \right] \Big|_{x=1}^{x=3} =$

$$[3 \ln 5 + 2 \ln 3 + \frac{1}{3}] - [3 \ln 3 + \underbrace{2 \ln 1}_{=0} + 1] =$$

$$3 \ln 5 - \ln 3 - \frac{2}{3}.$$

6. Evaluate the integral

$$\int_{x=-\infty}^{x=+\infty} \frac{dx}{1+x^2}$$

- a. $+\infty$ b. $-\infty$ (c) π d. does not exist (i.e., divergent) e. None of the others.

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^0 \frac{dx}{1+x^2} + \int_0^{\infty} \frac{dx}{1+x^2}$$

$$= \lim_{t \rightarrow -\infty} \int_t^0 \frac{dx}{1+x^2} + \lim_{t \rightarrow \infty} \int_0^t \frac{dx}{1+x^2}$$

$$= \lim_{t \rightarrow -\infty} [\tan^{-1} x]_t^0 + \lim_{t \rightarrow \infty} [\tan^{-1} x]_0^t$$

$$= \lim_{t \rightarrow -\infty} (-\tan^{-1} t) + \lim_{t \rightarrow \infty} \tan^{-1} t = -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2} = \pi.$$

from Simmons, Calc. p412.

7. Let $y = f(x)$ be an odd function (i.e., $f(-x) = -f(x)$) and $\int_0^{\infty} f(x) dx = +\infty$. What is $\int_{-\infty}^{+\infty} f(x) dx$?
 a. $+\infty$ b. $-\infty$ c. 0 **(d.) does not exist (i.e., divergent)** e. None of the others.

MC 5

In 7, $\int_{-\infty}^{\infty} f(x) dx$ DNE, see below class notes.

DEFINITION OF AN IMPROPER INTEGRAL OF TYPE I

If $\int_a^t f(x) dx$ exists for every number $t \geq a$, then

$$\int_a^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$$

provided this limit exists (as a finite number).

(b) If $\int_t^b f(x) dx$ exists for every number $t \leq b$, then

$$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$$

provided this limit exists (as a finite number).

The improper integrals $\int_a^{\infty} f(x) dx$ and $\int_{-\infty}^b f(x) dx$ are called **convergent** if the corresponding limit exists and **divergent** if the limit does not exist. (DNE)

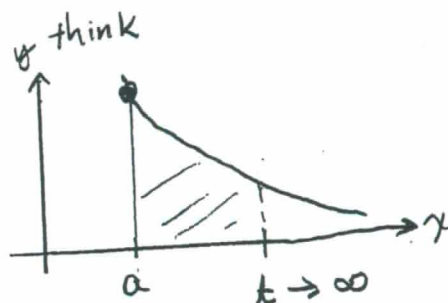
(c) If both $\int_a^{\infty} f(x) dx$ and $\int_{-\infty}^a f(x) dx$ are convergent, then we define

\equiv

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^a f(x) dx + \int_a^{\infty} f(x) dx$$

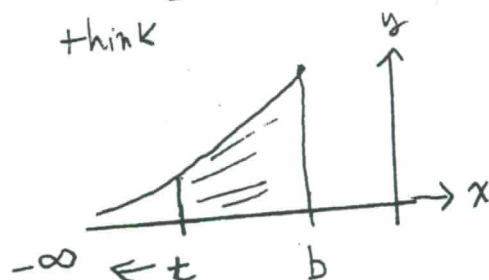
In part (c) any real number a can be used (see Exercise 74).

(a) For $\int_a^{\infty} f(x) dx$,

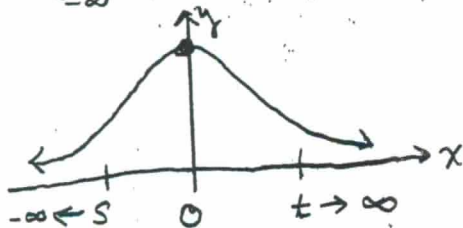


(b) For $\int_{-\infty}^b f(x) dx$,

think



(c) For $\int_{-\infty}^{\infty} f(x) dx$, think



$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^{\infty} f(x) dx$$

$$= \lim_{s \rightarrow -\infty} \int_s^0 f(x) dx + \lim_{t \rightarrow \infty} \int_0^t f(x) dx$$

(*) If one, or both, of these limits DNE then $\int_{-\infty}^{\infty} f(x) dx$ DNE.