HAND IN PART

MARK BOX		
PROBLEM	POINTS POSSIBLE	YOUR SCORE
1–25	100	

NAME: _____

PIN: _____

INSTRUCTIONS

- The MARK BOX above indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- This exam comes in two parts.
  (1) HAND IN PART. Hand in only this part, which includes a table for indicating your solutions to the multiple choice problems.
  (2) STATEMENT OF MULTIPLE CHOICE PROBLEMS part. Do not hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. Please, if I forget, remind me to pull up a clock on the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): §7.1–7.5, 7.8, 11.1 – 11.11, 6.1 – 6.3, 10.3–10.4 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choice up to **2** answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 4 points. For a problem with precisely two answers marked, one of which is correct, 1 points. All other cases, 0 points.
- Fill in the "number of solutions circled" column.

TABLE FOR YOUR ANSWERS							Do Not Write Below		
							points		
PROBLEM						number of solutions circled	4	1	0
1	1a	1b	1c	1d	1e				
2	2a	2b	2c	2d	2e				
3	3a	3b	3c	3d	3e				
4	4a	4b	4c	4d	4e				
5	5a	5b	5c	5d	5e				
6	6a	6b	6c	6d	6e				
7	7a	7b	7c	7d	7e				
8	8a	8b	8c	8d	8e				
9	9a	9b	9c	9d	9e				
10	10a	10b	10c	10d	10e				
11	11a	11b	11c	11d	11e				
12	12a	12b	12c	12d	12e				
13	13a	13b	13c	13d	13e				
14	14a	14b	14c	14d	14e				
15	15a	15b	15c	15d	15e				
16	16a	16b	16c	16d	16e				
17	17a	17b	17c	17d	17e				
18	18a	18b	18c	18d	18e				
19	19a	19b	19c	19d	19e				
20	20a	20b	20c	20d	20e				
21	21a	21b	21c	21d	21e				
22	22a	22b	22c	22d	22e				
23	23a	23b	23c	23d	23e				
24	24a	24b	24c	24d	24e				
25	25a	25b	25c	25d	25e				

STATEMENT OF MULTIPLE CHOICE PROBLEMS

- Hint for a typical (i.e. not improper) definite integral problems $\int_a^b f(x)\,dx$. First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$. To check if you did this part correctly, you can use the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$). Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.
- Hint: $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$ and $\ln(a^r) = r \ln a$ if $a, b > 0$ and $r \in \mathbb{R}$.

1. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{1}{x^2+1}\,dx\ .$$

a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. $\ln\sqrt{3}$
d. $\ln 3$
e. None of the others.

2. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{x}{x^2+1}\,dx\ .$$

a. $\frac{\pi}{4}$
b. $\frac{\pi}{2}$
c. $\ln\sqrt{3}$
d. $\ln 3$
e. None of the others.

3. Evaluate the integral

$$\int_{x=0}^{x=e} \ln x \; dx \; .$$

a. $\frac{1}{e} - 1$
b. $1 - \frac{1}{e}$
c. $2e - 1$
d. 1
e. None of the others.

4. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^3 x \, \sin^4 x \; dx \; .$$

a. $\frac{4}{45}$
b. $\frac{14}{45}$
c. $\frac{2}{35}$
d. $\frac{12}{35}$
e. None of the others.

5. Evaluate the integral

$$\int_{x=2}^{x=3} \frac{4x^2 + 13x - 9}{x^3 + 2x^2 - 3x} \; dx \; .$$

a. $\ln \frac{45}{4}$
b. $\ln \frac{45}{12}$
c. $\ln \frac{15}{4}$
d. $\ln \frac{81}{10}$
e. None of the others.

6. Evaluate the integral

$$\int_{x=0}^{x=1} \frac{1}{\sqrt{x^2+8x+25}}\, dx\ .$$

Hint: $x^2+8x+25 = (x+4)^2+9$.

a. $\ln \frac{39}{29}$
b. $\ln \frac{\sqrt{34}+5}{9}$
c. $\frac{1}{3}\ln \frac{39}{29}$
d. $\frac{1}{3}\ln \frac{\sqrt{34}+5}{9}$
e. None of the others.

7. Evaluate the integral

$$\int_{x=0}^{x=\frac{\pi}{2}} \cos^4 x\ dx\ .$$

a. $\frac{16\pi-2}{32}$
b. $\frac{16\pi+2}{32}$
c. $\frac{6\pi-1}{32}$
d. $\frac{6\pi+1}{32}$
e. None of the others.

8.Evaluate the integral

$$\int_{x=0}^{x=\frac{3\pi}{2}} e^x \cos x\ dx\ .$$

a. $\frac{1+e^{3\pi/2}}{2}$
b. $\frac{1-e^{3\pi/2}}{2}$
c. $\frac{-1+e^{3\pi/2}}{2}$
d. $\frac{-1-e^{3\pi/2}}{2}$
e. None of the others.

9. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^{2/3}}\, dx\ .$$

a. 0
b. $\frac{2}{3}$
c. diverges to infinity
d. does not exist but also does not diverge to infinity
e. None of the others.

10. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^{3}}\, dx\ .$$

a. 0
b. $\frac{1}{4}$
c. diverges to infinity
d. does not exist but also does not diverge to infinity
e. None of the others.

11. Find

$$\lim_{n\to\infty} \frac{\sqrt{25n^8+5n^7-n^2+1}}{3n^4+5n^2-n-2}\ .$$

a. 0
b. $\frac{25}{3}$
c. $\frac{5}{3}$
d. ∞
e. None of the others.

12. For what value $r \in \mathbb{R}$ does

$$\sum_{n=2}^{\infty} r^n \;=\; \frac{1}{4} \;?$$

a. $\frac{1}{3}$
b. $\frac{1}{4}$
c. $\frac{1}{5}$
d. $\frac{\sqrt{17}\,-1}{8}$
e. None of the others.

13. The formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}} \;.$$

a. is absolutely convergent, as can be shown by the limit comparison test (LCT) with $b_n = \frac{1}{n^2}$.
b. is conditionally convergent as can by shown by using only the alternating series test (AST) and not other tests.
c. converges conditionally as can be shown by using the LCT with $b_n = \frac{1}{n}$ as well as the AST.
d. diverges.
e. None of the others.

14. The formal series

$$\sum_{n=2}^{\infty} \frac{1}{n\ (\ln n)^2} \;.$$

a. converges, as can be shown by the limit comparison test using a p-series.
b. diverges, as can be shown by the limit comparison test using a p-series.
c. converges, as can be shown by the integral test.
d. diverges, as can be shown by the integral test.
e. None of the others.

15. Consider the formal series

$$\sum_{n=2}^{\infty} \left(\frac{2n+3}{3n+2}\right)^n .$$

a. The series converges by the Root Test.
b. The series diverges by the Root Test.
c. The Root Test is inconclusive.
d. The Root Test cannot be applies.
e. None of the others.

16. Let c be a natural number (i.e., $c \in \{1, 2, 3, 4, \ldots\}$). The series

$$\sum_{n=1}^{\infty} \frac{(n!)^6}{(cn)!}$$

a. converges when $c < 6$ and diverges when $c \geq 6$
b. converges when $c \leq 6$ and diverges when $c > 6$
c. diverges when $c < 6$ and converges when $c \geq 6$
d. diverges when $c \leq 6$ and converges when $c > 6$
e. None of the others.

17. What is the LARGEST set for which the formal power series

$$\sum_{n=17}^{\infty} \frac{x^n}{n!}$$

is convergent (either absolutely or conditionally , so, in other words, its interval of convergence)?
a. $\{0\}$
b. $(-\infty, +\infty)$
c. $[-1, 1)$
d. $(-1, 1]$
e. None of the others.

18. Using a known (commonly used) Taylor series, find the Tayor series about the center $x_0 = 0$ for

$$f(x) = \frac{1}{1-2x^3} \ .$$

a. $\sum_{n=0}^{\infty} (-1)^n x^n$
b. $\sum_{n=0}^{\infty} (-1)^n 2^n x^n$
c. $\sum_{n=0}^{\infty} (-1)^n x^{3n}$
d. $\sum_{n=0}^{\infty} (-1)^n 2^n x^{3n}$
e. None of the others.

19. Using a known (commonly used) Taylor series, find the Tayor series about the center $x_0 = 1$ for

$$f(x) = \frac{1}{3-2x} \ .$$

a. $\sum_{n=0}^{\infty} 2^n (x-1)^n$
b. $\sum_{n=0}^{\infty} (-1)^n 2^n (x-1)^n$
c. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{3}\right) 2^n (x-1)^n$
d. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{3}\right)^n (x-1)^n$
e. None of the others.

20. Find the 3rd order Taylor polynomial for $f(x) = \frac{1}{x}$ about the center $x_0 = 2$.
a. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$
b. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}(x-2)^2 - \frac{3}{8}(x-2)^3$
c. $\frac{1}{2} + \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 + \frac{1}{16}(x-2)^3$
d. $\frac{1}{2} - \frac{1}{4}x + \frac{1}{4}x^2 - \frac{3}{8}x^3$
e. None of the others.

21. Consider the function

$$f(x) = e^{-x} \ .$$

The 5^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_5(x) \ = \ \sum_{n=0}^{5} \frac{(-x)^n}{n!} \ = \ 1 \ - \ x \ + \ \frac{x^2}{2!} \ - \ \frac{x^3}{3!} \ + \ \frac{x^4}{4!} \ - \ \frac{x^5}{5!} \ .$$

The 5^{th} order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $e^{-x} \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\frac{e(3^5)}{5!}$

b. $\frac{(3^5)}{(e^3)\ (5!)}$

c. $\frac{e(3^6)}{6!}$

d. $\frac{(3^6)}{(e^3)\ (6!)}$

e. None of the others.

22. Express the polar equation

$$r = 2\sin\theta$$

in Cartesion equations.

a. $x^2 + (y-2)^2 = 2$

b. $x^2 + (y-1)^2 = 1$

c. $(x-1)^2 + y^2 = 1$

d. $(x-2)^2 + y^2 = 2$

e. None of the others.

23. Express the area enclosed by $r = 5 - 5\sin\theta$ as an integral.

a. $\frac{1}{2}\int_0^{2\pi}[5 - 5\sin\theta]^2 \ d\theta$

b. $\int_0^{2\pi}[5 - 5\sin\theta]^2 \ d\theta$

c. $\frac{1}{2}\int_0^{2\pi}[5 - 5\sin\theta] \ d\theta$

d. $\frac{1}{2}\int_0^{2\pi}[5^2 - 5^2\sin^2\theta] \ d\theta$

e. None of the others.

24. Let V be the solid of revolution obtained by revolving, about the x-axis, the region bounded by

$$y = x$$
$$y = x^2$$

between $x = 0$ and $x = 1$. Using the shell method express the volume of V as an integral.

a. $2\pi \int_{x=0}^{x=1} \left(y - \sqrt{y}\right)\, dy$

b. $2\pi \int_{x=0}^{x=1} \left(\sqrt{y} - y\right)\, dy$

c. $2\pi \int_{x=0}^{x=1} (y) \left(y - \sqrt{y}\right)\, dy$

d. $2\pi \int_{x=0}^{x=1} (y) \left(\sqrt{y} - y\right)\, dy$

e. None of the others.

25. What is your favorite number?

a. 17 b. 17 c. 17 d. 17 e. 17

Thanks for a wonderful semester. Good Luck in Math 241 and 242.