| Prof. Girardi | Math 142 | Spring 2014 | 6 May 2014 | Final Exam |

MARK BOX		
PROBLEM	POINTS POSSIBLE	YOUR SCORE
1–25	100	

NAME: _____ solution key _____

PIN: _____

INSTRUCTIONS

- The MARK BOX above indicates the problems along with their points.
  Check that your copy of the exam has all of the problems.
- This exam comes in two parts.
  (1) <u>HAND IN PART</u>. Hand in <u>only</u> this part, which includes a table for indicating your solutions to the multiple choice problems.
  (2) <u>STATEMENT OF MULTIPLE CHOICE PROBLEMS</u> part. Do <u>not</u> hand in this part. You can take this part home to learn from and to check your answers once the solutions are posted.
- Upon request, you will be given as much (blank) scratch paper as you need.
- During the exam, the use of unauthorized materials is prohibited. Unauthorized materials include: electronic devices, books, and personal notes. Unauthorized materials (including cell phones) must be in a secured (e.g. zipped up, snapped closed) bag placed completely under your desk or, if you did not bring such a bag, given to Prof. Girardi to hold for you during the exam (and they will be returned when you leave the exam). This means no electronic devices (such as cell phones) allowed in your pockets. Please, if I forget, remind me to pull up a clock on the projector screen.
- During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- This exam covers (from *Calculus* by Stewart, 6$^{\text{th}}$ ed., ET):
  §7.1–7.5, 7.8, 11.1 – 11.11, 6.1 – 6.3, 10.3–10.4 .

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choice up to **2** answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 4 points. For a problem with precisely two answers marked, one of which is correct, 2 points. All other cases, 0 points.
- Fill in the "number of solutions circled" column.

Your Solutions						number of solutions circled	Do Not Write Below points		
PROBLEM							4	2	0
1	1a	(1b)	1c	1d	1e				
2	2a	2b	2c	(2d)	2e				
3	3a	3b	3c	(3d)	3e				
4	4a	(4b)	4c	4d	4e				
5	(5a)	5b	5c	5d	5e				
6	6a	6b	6c	(6d)	6e				
7	7a	(7b)	7c	7d	7e				
8	8a	8b	(8c)	8d	8e				
9	9a	(9b)	9c	9d	9e				
10	10a	(10b)	10c	10d	10e				
11	11a	(11b)	11c	11d	11e				
12	(12a)	12b	12c	12d	12e				
13	13a	13b	13c	(13d)	13e				
14	14a	(14b)	14c	14d	14e				
15	15a	(15b)	15c	15d	15e				
16	16a	(16b)	16c	16d	16e				
17	(17a)	17b	17c	17d	17e				
18	18a	(18b)	18c	18d	18e				
19	19a	(19b)	19c	19d	19e				
20	20a	20b	(20c)	20d	20e				
21	21a	(21b)	21c	21d	21e				
22	22a	22b	22c	(22d)	22e				
23	23a	23b	(23c)	23d	23e				
24	24a	(24b)	24c	24d	24e				
25	(25a)	25b	25c	25d	25e				

STATEMENT OF MULTIPLE CHOICE PROBLEMS

- Throughout this exam, unless otherwise stated, follow the common calculus practice of measuring angles in radians (and not degrees).
- Hint for a typical (i.e. not improper) definite integral problems $\int_a^b f(x)\,dx$. First do the indefinite integral, say you get $\int f(x)\,dx = F(x) + C$. To check if you did this part correctly, you can use the Fundemental Theorem of Calculus (i.e. $F'(x)$ should be $f(x)$). Once you are confident that your indefinite integral is correct, use the indefinite integral to find the definite integral.

1. Evaluate the integral

$$\int_{x=0}^{x=1} x e^x \, dx .$$

   a. 0   b. 1   c. e d. $2e - 1$ e. None of the others.

2. Evaluate the integral

$$\int_{x=0}^{x=1} \arcsin(x) \, dx .$$

   a. 0   b. 1   c. $\frac{\pi}{2}$ d. $\frac{\pi}{2} - 1$ e. None of the others.

3. Evaluate the integral

$$\int_{x=0}^{x=1} \sin^4 x \, dx .$$

   a. 1   b. π c. $1 + \sin 2 + \sin 4$ d. $\frac{3}{8} - \frac{1}{4}\sin 2 + \frac{1}{32}\sin 4$ e. None of the others.

4. Evaluate the integral

$$\int_{x=5}^{x=10} \frac{\sqrt{x^2 - 25}}{x} \, dx$$

AND specify the initial substitution.

   a. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$.   b. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sec\theta$

   c. $(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$.   d. $5(\sqrt{3} - \frac{\pi}{3})$ using the initial substitute $x = 5\sin\theta$

   e. None of the others.

5. Evaluate the integral

$$\int_{x=1}^{x=3} \frac{5x^2 + 3x - 2}{x^3 + 2x^2} \, dx .$$

   a. $3\ln 5 - \ln 3 - \frac{2}{3}$ b. $3\ln 5 - \ln 3 - \frac{8}{3}$ c. $\ln 5 - \frac{2}{3}$ d. $\frac{2}{3} - \ln 5$ e. None of the others.

6. Evaluate the integral

$$\int_{x=-1}^{x=1} \frac{1}{x^3} \, dx .$$

   a. 0   b. $\frac{1}{4}$ c. diverges to infinity   d. does not exist but also does not diverge to infinity

   e. None of the others.

7. Let R be the (2-dimensional) region bounded by the ellipse (where $a, b > 0$)

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 .$$

Revolve R about the x-axis, which will generate a 3-dimensional ellipsoid $\mathcal{E}$ with equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{b}\right)^2 = 1 .$$

Express as an integral the volume of this $\mathcal{E}$. Make use of symmetry.

a. $2\pi \int_0^a \left[b^2 - \frac{b^2}{a^2}x^2\right]^2 dx$ b. $2\pi \int_0^a \left[b^2 - \frac{b^2}{a^2}x^2\right] dx$

c. $2\pi \int_0^a \left[b - \frac{b}{a}x\right]^2 dx$ d. $2\pi \int_0^a x \left[b^2 - \frac{b^2}{a^2}x^2\right] dx$ e. None of the others.

8. Let R be the region bounded by the curves

$$y = \sqrt{x} \quad \text{and} \quad y = x^3 .$$

Express as integral(s) the volume of the solid generated by revolving R about the x-axis.

a. $\pi \int_0^1 \left[\sqrt{x} - x^3\right]^2 dx$ b. $\pi \int_0^1 \left[x - x^6\right]^2 dx$ c. $\pi \int_0^1 \left[x - x^6\right] dx$ d. $2\pi \int_0^1 \left[x - x^6\right] dx$

e. None of the others.

9. Let R be the region in the first quadrant bounded by the curves

$$y = x^2 \quad \text{and} \quad y = 2 - x^2 .$$

Express as integral(s) the volume of the solid generated by revolving R about the y-axis.

a. $2\pi \int_0^1 \left[(2 - x^2) - x^2\right] dx$ b. $2\pi \int_0^1 x \left[(2 - x^2) - x^2\right] dx$

c. $2\pi \int_0^1 x \left[(2 - x^2) - x^2\right]^2 dx$ d. $2\pi \int_0^1 x \left[x^2 - (2 - x^2)\right] dx$ e. None of the others.

10. Let R be the region in bounded by the curves

$$y = \sqrt{x} \quad \text{and} \quad y = x^3 .$$

Express as integral(s) the volume of the solid generated by revolving R about the line $x = $ -1.

a. $2\pi \int_0^1 (x) \left(x^{\frac{1}{2}} - x^3\right) dx$ b. $2\pi \int_0^1 (x + 1) \left(x^{\frac{1}{2}} - x^3\right) dx$

c. $2\pi \int_0^1 (x - 1) \left(x^{\frac{1}{2}} - x^3\right) dx$ d. $2\pi \int_0^1 (x + 1) \left(x^{\frac{1}{2}} - x^3\right)^2 dx$ e. None of the others.

11. A ball is dropped from the height of 30 feet. Each time it hits the floor it rebounds to $\frac{5}{6}$ its previous height. Find the total distance the ball travels (in feet).

a. 300   b. 330   c. 350   d. 360   e. None of the others.

12. Give a formula, without ...'s and without $\sum$-sign, for

$$s_n = \sum_{k=2}^{n} \frac{2}{k^2 - 1} ,$$

which is valid for $n \geq 3$. Use this formula to find

$$s = \sum_{k=2}^{\infty} \frac{2}{k^2 - 1} .$$

a. $s_n = \frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$ and $s = \frac{3}{2}$ b. $s_n = \frac{1}{1} + \frac{1}{2} - \frac{1}{n+1}$ and $s = \frac{3}{2}$

c. $s_n = \frac{1}{1} - \frac{1}{2} + \frac{1}{n} - \frac{1}{n+1}$ and $s = \frac{1}{2}$ d. $s_n = \frac{1}{1} - \frac{1}{2} + \frac{1}{n}$ and $s = \frac{1}{2}$ e. None of the others.

Some abbreviations (as used in class).

- AC - absolutely convergent
- CC - conditionally convergent
- DVG - divergent
- LCT - limit comparison test
- AST - alternating series test

13. Consider the following two formal series.

$$\text{Series A is} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \ .$$

$$\text{Series B is} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \ .$$

a. Both series are AC   b. Both series are DVG   c. Series A is AC and Series B is CC

d. Series A is CC and Series B is AC   e. None of the others.

14. Let (don't overlook that cube root sign in the denominator)

$$u_n \ = \ \frac{1}{\sqrt[3]{(n+2)(n+5)(n+7)}} \ .$$

Consider the following two formal series.

$$\text{Series A is} \qquad \sum_{n=1}^{\infty} u_n \ .$$

$$\text{Series B is} \qquad \sum_{n=1}^{\infty} (-1)^n \, u_n \ .$$

a. Series B is AC.

b. Series B is CC since

- Series A is DVG, as can be shown by the LCT using $\sum \frac{1}{n}$
- Seris B converges, as can be shown by the AST.

c. Series B is CC since

- Series A is DVG, as can be shown by the LCT using $\sum \frac{1}{\sqrt[3]{n}}$
- Seris B converges, as can be shown by the AST.

d. Series B is DVG.   e. None of the others.

15. Consider the formal series
$$\sum_{n=1}^{\infty} (-1)^n \, \frac{n!}{(3n)!} \; .$$

Let
$$a_n \;=\; (-1)^n \, \frac{n!}{(3n)!} \qquad \text{and} \qquad \rho \;=\; \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \; .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = \frac{1}{3}$.

b. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = 0$.

c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$.

d. $\rho > 1$ so by the Ratio Test $\sum_{n=1}^{\infty} a_n$ diverges.   e. None of the others.

16. What is the LARGEST interval for which the formal power series
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \, (x-3)^n$$

is convergent (either AC or CC, so, in other words, its interval of convergence)?

a. $(2,4)$ b. $(2,4]$ c. $(-4,-2)$ d. $(-4,-2]$ e. None of the others.

17. Find a power series reprsentation of
$$f(x) \;=\; \frac{x}{x^2 + 16}$$

and state the LARGEST interval for which it is valid.

a. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-4,4)$

b. $f(x) = \sum_{n=0}^{\infty} \frac{1}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-4,4)$

c. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-16,16)$

d. $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{16^{n+1}} x^{2n+1}$, valid precisely when $x \in (-1,1)$

e. None of the others.

18. Find a power series representation of the function $y = f(t)$ where
$$f(t) \;=\; \int_0^t \frac{1}{1+x^7} \, dx$$

and say for which values of t it is valid.

a. $\sum_{n=0}^{\infty} \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$ b. $\sum_{n=0}^{\infty} (-1)^n \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$

c. $\sum_{n=1}^{\infty} \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$ d. $\sum_{n=1}^{\infty} (-1)^n \frac{t^{7n+1}}{7n+1}$, valid for $t \in (-1,1)$

e. None of the others.

19. Suppose that the interval of convergence of the series $\sum_{n=1}^{\infty} c_n (x - x_0)^n$ is $(x_0 - R, x_0 + R]$. What can be said about the series at $x_0 + R$?

a. It must be absolutely convergent.   b. It must be conditionally convergent.

c. It must be divergent.   d. Nothing can be said.   e. None of the others.

20. Find the 2$^{\text{nd}}$ order Taylor polynomial for $f(x) = \sqrt[3]{x}$ about the center $x_0 = 8$.

a. $2 + \frac{x}{12} - \frac{x^2}{9(2^5)}$ b. $2 + \frac{(x-8)}{12} + \frac{(x-8)^2}{9(2^5)}$ c. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^5)}$ d. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^4)}$

e. None of the others.

21. Find the Taylor series for $f(x) = (1 - 5x)^{-3}$ about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty} (-1)^n \frac{5^n (n+1)(n+2)}{2} x^n$ b. $\sum_{n=0}^{\infty} \frac{5^n (n+1)(n+2)}{2} x^n$ c. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{5^n}{n!} x^n$

d. $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$ e. None of the others.

22. Consider the function $f(x) = e^x$ over the interval $(-1, 3)$. The 4th order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{4} \frac{x^n}{n!} .$$

The 4th order Remainder term $R_4(x)$ is defined by $R_4(x) = f(x) - P_4(x)$ and so $e^x \approx P_4(x)$ where the approximation is within an error of $|R_4(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_4(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\frac{(e^{-1})(3^4)}{4!}$ b. $\frac{(e^3)(3^4)}{4!}$ c. $\frac{(e^{-1})(3^5)}{5!}$ d. $\frac{(e^3)(3^5)}{5!}$ e. None of the others.

23. In Polar Coordinates, a point (r, θ) also has which of the following representations?

a. $(r, \theta + \pi)$ b. $(-r, \theta)$ c. $(-r, \theta + \pi)$

d. It has no other representation.   e. None of the others.

24. Express the polar equation

$$r = 2 \sin \theta$$

as a Cartesion equation.

a. $x^2 + (y - 2)^2 = 2$ b. $x^2 + (y - 1)^2 = 1$ c. $(x - 1)^2 + y^2 = 1$ d. $(x - 2)^2 + y^2 = 2$

e. None of the others.

25. Express the area enclosed by $r = 5 - 5 \sin \theta$ as on integral.

a. $\frac{1}{2} \int_0^{2\pi} [5 - 5 \sin \theta]^2 \, d\theta$ b. $\int_0^{2\pi} [5 - 5 \sin \theta]^2 \, d\theta$ c. $\frac{1}{2} \int_0^{2\pi} [5 - 5 \sin \theta] \, d\theta$ d. $\frac{1}{2} \int_0^{2\pi} [5^2 - 5^2 \sin^2 \theta] \, d\theta$

e. None of the others.