HAND IN PART

Prof. Girardi	Math 142	Spring 2014	03.20.2014	Exam 2

MARK BOX		
PROBLEM	POINTS	
0	35	
1	12	
2	33	
3–6	20	
%	100	

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

(1) The MARK BOX above indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) You may **not** use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) Problems 3–6 are multiple choice.
- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
- Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(5) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(6) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): §11.2– 11.8 .

0. Fill-in-the boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$, unless otherwise indicated.

0a. Sequences (Afterall, this is needed for Geometric Series!) For probelm 0a, fill in the 4 boxes as follows.

- If the limit exists in $\mathbb{R}$ (so the limit is a <u>finite</u> number), then $=$ *the specific real number of the limit* (e.g, fill in the box with $\boxed{=17}$) .
- If the limit does not exist in R (eg, if the limit is ∞ or the sequence oscillates), then just fill in the box with $\boxed{DNE}$.

Let $-\infty < r < \infty$. (Warning, don't confuse sequences with series.)

- If $|r| < 1$, then $\lim_{n\to\infty} r^n$ $\boxed{=0}$.
- If $|r| > 1$, then $\lim_{n\to\infty} r^n$ $\boxed{\text{DNE}}$.
- If $r = 1$, then $\lim_{n\to\infty} r^n$ $\boxed{=1}$.
- If $r =$ -1, then $\lim_{n\to\infty} r^n$ $\boxed{\text{DNE}}$.

0b. (Fill in the 5 boxes.) Fix $r \in \mathbb{R}$ with $r \neq 1$. For $N \geq 17$, let $s_N = \sum_{\mathbf{n=17}}^{N} r^n$ (Note the sum starts at 17). Then s_N can be written as:

$$s_N = \boxed{\frac{r^{17} - r^{N+1}}{1-r}} .$$

for all $N \geq 17$. Your answer should NOT contain a $\sum$ sign nor . . . .

0c. $n^{\mathbf{th}}$**-term test** for an arbitrary series $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ $\boxed{\text{diverges}}$.

0d. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$ (hint: look at the previous questions)

- converges if and only if $|r|$ $\boxed{<1}$.
- diverges if and only if $|r|$ $\boxed{\geq 1}$.

0e. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p $\boxed{>1}$.
- diverges if and only if p $\boxed{\leq 1}$.

0f. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$. Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f(\boxed{n})$ for each $n \in \mathbb{N}$
- f is a $\boxed{\text{continuous}}$ function
- f is a $\boxed{\text{positive}}$ function
- f is a $\boxed{\text{decreasing (nonincreasing is also ok)}}$ function.

Then $\sum a_n$ converges if and only if $\boxed{\int_{x=1}^{x=\infty} f(x)\,dx}$ converges.

0g. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$. (Fill in the blanks with a_n and/or b_n.)

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum \boxed{b_n}$ converge, then $\sum \boxed{a_n}$ converge.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum \boxed{b_n}$ diverge, then $\sum \boxed{a_n}$ diverge.

Hint: sing the song to yourself.

0h. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$. Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If $\boxed{0} < L < \boxed{\infty}$, then $\sum a_n$ converges if and only if $\boxed{\sum b_n \text{ converges}}$.

0i. Ratio and Root Tests for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$. Let

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \qquad \text{or} \qquad \rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}} .$$

- If ρ $\boxed{< 1}$ then $\sum a_n$ converges absolutely.
- If ρ $\boxed{> 1}$ then $\sum a_n$ diverges.
- If ρ $\boxed{= 1}$ then the test is inconclusive (in other words, the test fails).

0j. Alternating Series Test for an alternating series $\sum (-1)^n u_n$ where $u_n > 0$ for each $n \in \mathbb{N}$. If

- u_n $\boxed{>}$ u_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} u_n = \boxed{0}$

then $\sum (-1)^n u_n$ $\boxed{\text{converges}}$.

0k. By definition, for an arbitrary series $\sum a_n$, (fill in these 4 boxes with converges or diverges).

- $\sum a_n$ is <u>absolutely convergent</u> if and only if $\sum |a_n|$ $\boxed{\text{congverges}}$.
- $\sum a_n$ is <u>conditionally convergent</u> if and only if $\sum a_n$ $\boxed{\text{converges}}$ and $\sum |a_n|$ $\boxed{\text{diverges}}$.
- $\sum a_n$ is <u>divergent</u> if and only if $\sum a_n$ $\boxed{\text{diverges}}$.

0l. Fill in the 3 blank boxes with absolutely convergent, conditionally convergent, or divergent) on the following FLOW CHART from class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditionally convergent, or divergent.

Does $\sum \lvert a_n \rvert$ converge? Since $\lvert a_n \rvert \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.	if NO $\Rightarrow$	Does $\lim_{n\to\infty} \lvert a_n \rvert = 0$?	if NO $\Rightarrow$	$\sum a_n$ is $\boxed{\text{divergent}}$
if YES $\Downarrow$		if YES $\Downarrow$		
$\sum a_n$ is $\boxed{\text{absolutely convergent}}$		Is $\sum a_n$ an alternating series?		
		if YES $\Downarrow$		
		Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?		
		if YES $\Downarrow$		
		$\sum a_n$ is $\boxed{\text{conditionally convergent}}$		

1. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.
Scoring: 2 pts for correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

On the next 2, think of the n^{th}-term test for divergence and what if $a_n = \frac{1}{n}$		
T	F	If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
T	F	If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.
On the next 2, think of a Theorem from class and what if $b_n = -a_n$.		
T	F	If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.
T	F	If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.
On the next 2, think of a Theorem from class and what if $f(x) = \sin(\pi x)$.		
T	F	If a function $f\colon [0,\infty) \to \mathbb{R}$ satisfies that $\lim_{x\to\infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n, that $\lim_{n\to\infty} a_n = L$.
T	F	If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n\to\infty} a_n = L$ and $f\colon [0,\infty) \to \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n, then $\lim_{x\to\infty} f(x) = L$.

2. Circle the behavior of the given series. The abbreviations are:

- AC stands for absolutely convvergent
- CC stands for conditionally convergent
- DVG stand for divergent
- NOT stands for none of these.

You can circle up to **2** answers for each problem. The scoring is as follows.

- For a problem with precisely one answer marked and th answer is correct, 4 points.
- For a problem with precisely two answers marked, on of which is correct, 2 points.
- All other cases, 0 points.
- 1 point for filling in the column of # of solutions circled.

Series					# of solutions circled	points
$\sum_{n=1}^{\infty} \frac{1}{n^2}$	AC	CC	DVG	NOT		
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$	AC	CC	DVG	NOT		
$\sum_{n=1}^{\infty} \frac{1}{n}$	AC	CC	DVG	NOT		
$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$	AC	CC	DVG	NOT		
$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	AC	CC	DVG	NOT		
$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$	AC	CC	DVG	NOT		
$\sum_{n=1}^{\infty} \frac{1}{e^n}$	AC	CC	DVG	NOT		
$\sum_{n=1}^{\infty} \frac{(-1)^n}{e^n}$	AC	CC	DVG	NOT		

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 3 – 6

Instructions.

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choice up to 3 answers for each problem. The scoring is as follows.
  - For a problem with precisely one answer marked and the answer is correct, 5 points.
  - For a problem with precisely two answers marked, one of which is correct, 3 points.
  - For a problem with precisely three answers marked, one of which is correct, 1 point.
  - All other cases, 0 points.
- Turn in this Hand In Part of the test.
- As for the Statement of Mulitple Choice Problem of the exam, note the following.
  - Do NOT hand it in. Take it home with you.
  - Use the back blank sides for scratch paper.
- Fill in the "number of solutions circled" column.

Your Solutions							
PROBLEM						# of solutions circled	points
3	3a	3b	3c	3d	3e		
4	4a	4b	4c	4d	4e		
5	5a	5b	5c	5d	5e		
6	6a	6b	6c	6d	6e		
TOTOL POINT							

STATEMENT OF MULTIPLE CHOICE PROBLEMS

3. Evaluate

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

Hint: Telescoping Series, use PFD.

a. 2 b. $\frac{1}{2}$ c. 1 d. 4 e. none of these

For sol'n, see Spring 2013, Exam 2, problem 6.

4. Consider the formal seris $\sum_{n=1}^{\infty} a_n$ where

$$a_n = (-1)^n \frac{(n+1)!}{(2n)!}$$

and let

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = \frac{1}{2}$. b. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = 0$.

c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$ d. $\sum_{n=1}^{\infty} a_n$ diverges e. none of these

For sol'n, see Spring 2013, Exam 2, problem 8.

5. Consider the formal seris $\sum_{n=1}^{\infty} a_n$ where

$$a_n = \frac{\sqrt{n+2}}{2n^2+n+1} .$$

Recall that the Limit Comparison Test was on this exam in problem 0h.

a. converges, as can be shown using the Limit Comparison Test and comparing it to $b_n = n^{-\frac{3}{2}}$

b. diverges, as can be shown using the Limit Comparison Test and comparing it to $b_n = n^{-\frac{3}{2}}$

c. converges, as can be shown using the Limit Comparison Test and comparing it to $b_n = \frac{1}{n}$

d. diverges, as can be shown using the Limit Comparison Test and comparing it to $b_n = \frac{1}{n}$

e. none of these

This was homework problem: §11.4, # 21.

6. What is the LARGEST interval (so you have to check your endpoints) for which the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+15)^n}{4^n}$$

is absolutely convergent?

a. $\left(\frac{11}{5}, \frac{19}{5}\right)$ b. $\left[\frac{11}{5}, \frac{19}{5}\right]$ c. $\left(\frac{-19}{5}, \frac{-11}{5}\right)$ d. $\left[\frac{-19}{5}, \frac{-11}{5}\right]$ e. none of these

For sol'n, see Spring 2013, Final Exam, problem 22.