HAND IN PART

Prof. Girardi	Math 142	Spring 2014	02.11.2014	Exam 1

MARK BOX		
PROBLEM	POINTS	
0	30	
1–10	70	
%	100	

NAME: ______________________________________

PIN: ______________________________________

INSTRUCTIONS

(1) The MARK BOX above indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) You may **not** use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) Problems 1–10 are multiple choice.
- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
- Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(5) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(6) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
7.1–7.5, 7.8, 11.1 .

0. Fill in the blanks (each worth 1 point).

a. If $u \neq 0$, then $\int \frac{du}{u} \;=\;$ ________ $|u| + C$

b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u\, du \;=\;$ ______________________________ $+ C$

c. $\int \cos u\, du \;=\;$ ____ $+ C$

d. $\int \sec^2 u\, du \;=\;$ ____ $+ C$

e. $\int \sec u \tan u\, du \;=\;$ ____ $+ C$

f. $\int \sin u\, du \;=\;$ ____ $+ C$

g. $\int \csc^2 u\, du \;=\;$ ____ $+ C$

h. $\int \csc u \cot u\, du \;=\;$ ____ $+ C$

i. $\int \tan u\, du \;=\;$ ____ $+ C$

j. $\int \cot u\, du \;=\;$ ____ $+ C$

k. $\int \sec u\, du \;=\;$ ____ $+ C$

l. $\int \csc u\, du \;=\;$ ____ $+ C$

m. If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}}\, du \;=\;$ ____ $+ C$

n. If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2}\, du \;=\;$ ____ $+ C$

o. If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}}\, du \;=\;$ ____________________________________ $+ C$

p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials

and [degree of f] $\geq$ [degree of g], then one must first do ______________________________

q. Integration by parts formula: $\int u\, dv \;=\;$ ____

r. Trig. Substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u \;=\;$ __________________________

s. Trig. Substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u \;=\;$ __________________________

t. Trig. Substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u \;=\;$ __________________________

u. Trig. Formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) =$ ______________ .

v&w. Trig. Formula ... $\cos(2\theta)$ should appear in your answer & notice the $\frac{1}{2}$ already stuck out front:
$\cos^2(\theta) \;=\; \frac{1}{2}$ (________________________) and $\sin^2(\theta) \;=\; \frac{1}{2}$ (________________________) .

x. trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween

tangent (i.e., tan) and secant (i.e., sec) is ____ .

y. arcsin(-$\frac{1}{2}$) = _________________ **RADIANS**. (your answer should be an angle)

z. By definition, the sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers converges to the real number L provided for each $\epsilon > 0$ there exists a natural number N so that if the natural number n satisfies n_______ N, then $|L - a_n| < \epsilon$.

ä. $\lim\limits_{n\to\infty} \dfrac{5n^{17} + 6n^2 + 1}{7n^{18} + 9n^3 + 5} \;=\;$ _______________

b̈. $\lim\limits_{n\to\infty} \dfrac{-5n^{18} + 6n^2 + 1}{7n^{17} + 9n^3 + 5} \;=\;$ _______________

c̈. $\lim\limits_{n\to\infty} \dfrac{36n^{17} - 6n^2 - 1}{4n^{17} + 9n^3 + 5} \;=\;$ _______________

d̈. $\lim\limits_{n\to\infty} \sqrt{\dfrac{36n^{17} - 6n^2 - 1}{4n^{17} + 9n^3 + 5}} \;=\;$ _______________

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 1 – 10

Instructions.

- Indicate (by circling) directly in the table below your solution to each problem.
- You may choice up to 3 answers for each problem. The scoring is as follows.
    - For a problem with precisely one answer marked and the answer is correct, 7 points.
    - For a problem with precisely two answers marked, one of which is correct, 3 points.
    - For a problem with precisely three answers marked, one of which is correct, 1 point.
    - All other cases, 0 points.
- Turn in this Hand In Part of the test.
- As for the Statement of Mulitple Choice Problem of the exam, note the following.
    - Do NOT hand it in. Take it home with you.
    - Use the back blank sides for scratch paper.
- Fill in the "number of solutions circled" column.

Your Solutions							
PROBLEM						# of solutions circled	points
1	1a	1b	1c	1d	1e		
2	2a	2b	2c	2d	2e		
3	3a	3b	3c	3d	3e		
4	4a	4b	4c	4d	4e		
5	5a	5b	5c	5d	5e		
6	6a	6b	6c	6d	6e		
7	7a	7b	7c	7d	7e		
8	8a	8b	8c	8d	8e		
9	9a	9b	9c	9d	9e		
10	10a	10b	10c	10d	10e		
TOTOL POINT							

STATEMENT OF MULTIPLE CHOICE PROBLEMS

1. Evaluate the integral

$$\int_0^1 x^3\left(x^4+1\right)^2\,dx$$

a. $\frac{7}{12}$ b. $\frac{7}{3}$ c. $\frac{1}{12}$ d. $\frac{1}{3}$ e. none of these

2. Evaluate the integral

$$\int_0^1 \ln(1+x)\,dx\ .$$

a. $2\ln(2)-2$ b. $2\ln(2)-1$ c. $-\frac{1}{2}$ d. $\frac{1}{2}$ e. none of these

3. Evaluate the integral

$$\int_0^\pi x\sin(x)\,dx\ .$$

a. 1 b. 2 c. π d. $-\pi$ e. none of these

4. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^2 x\cos^3 x\,dx\ .$$

a. $-\frac{4}{7}$ b. $\frac{4}{7}$ c. $-\frac{2}{15}$ d. $\frac{2}{15}$ e. none of these

5. Evaluate the integral

$$\int_0^{\pi/4} \sec^3 x\,dx\ .$$

a. $\left[\sqrt{2}-\ln\left(\sqrt{2}\ +\ 1\right)\ \right]$ b. $\frac{1}{2}\left[\sqrt{2}-\ln\left(\sqrt{2}\ +\ 1\right)\ \right]$

c. $\left[\sqrt{2}+\ln\left(\sqrt{2}\ +\ 1\right)\ \right]$ d. $\frac{1}{2}\left[\sqrt{2}+\ln\left(\sqrt{2}\ +\ 1\right)\ \right]$ e. none of these

6. Evaluate the integral

$$\int_1^3 \frac{dx}{x^2\sqrt{x^2+4}}\ .$$

a. $\frac{\sqrt{13}-\sqrt{5}}{8}$ b. $\sqrt{5}-\frac{\sqrt{13}}{3}$ c. $\frac{\sqrt{5}}{4}-\frac{\sqrt{13}}{12}$ d. $\frac{\sqrt{13}}{12}-\frac{\sqrt{5}}{4}$ e. none of these

7. Evaluate the integral

$$\int_2^3 \frac{dx}{x^2-1}\ .$$

Recall: $(\ln a)+(\ln b)=\ln(ab)$ and $(\ln a)-(\ln b)=\ln(\frac{a}{b})$

a. $\ln\left(\frac{3}{2}\right)$ b. $\frac{1}{2}\ln\left(\frac{3}{2}\right)$ c. $\ln\left(\frac{8}{3}\right)$ d. $\ln\left(\frac{2}{3}\right)$ e. none of these

8. Evaluate the integral

$$\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx\ .$$

Hint: Do we have (strictly) bigger bottoms?

Recall: $(\ln a)+(\ln b)=\ln(ab)$ and $(\ln a)-(\ln b)=\ln(\frac{a}{b})$

a. $\ln(\frac{2}{3})+\frac{7}{6}$ b. $\ln(6)+\frac{7}{6}$ c. $\ln(6)+\frac{-1}{6}$ d. $\ln(\frac{2}{3})+\frac{-1}{6}$ e. none of these

9. Evaluate, if it exists, the integral

$$\int_0^4 \frac{2x\,dx}{x^2-4}\ .$$

a. $\ln 3$ b. $\ln 48$ c. diverges to ∞ d. diverges but not to ∞ e. none of these

10. Let c be a real number. Evaluate, if it exists, the limit of the sequence

$$\lim_{n\to\infty} \left(1+\tfrac{c}{n}\right)^n \ .$$

a. 1 b. c c. e^{-c} d. e^{c} e. none of these.