1. Commonly Used Taylor Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

1a $$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}.$$

1b $$\int e^{-x^2}\,dx = \sum_{n=0}^{\infty} \int \frac{(-1)^n x^{2n}}{n!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(n!)}$$

1c $$\int_0^1 e^{-x^2}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)n!}\Bigg|_{x=0}^{x=1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(n!)}$$

$$= 1 - \frac{1}{3(1!)} + \frac{1}{5(2!)} - \frac{1}{7(3!)} + \frac{1}{9(4!)} - \frac{1}{11(5!)} + \frac{1}{13(6!)} - + \cdots$$

So By the alternating series Remainder Test

$$\int_0^1 e^{-x^2}\,dx \approx 1 - \frac{1}{3\cdot 1!} + \frac{1}{5\cdot 2!} - \frac{1}{7\cdot 3!} + \frac{1}{9(4!)}$$

within $\frac{1}{11\cdot(5!)}$