HAND IN PART

Prof. Girardi	Math 142	Spring 2013	04.18.2013	Exam 3

MARK BOX		
PROBLEM	POINTS	
0	20	
1	20	
2–11	60	
%	100	

NAME: Solution Key

PIN: ______________________________

INSTRUCTIONS

(1) The MARK BOX above indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) You may **not** use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) For the do by hand problems, to receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer.

(5) For the multiple choice problems, please.
   - First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
   - Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
   - Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(6) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(7) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
§ 11.9–11.11 and 6.1–6.3 .

0. Fill in: the boxes in problem **0A** and the lines in problem **0B**.

0A. Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0.

Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0.

Let $y = R_N(x)$ be the N^{th}-order Taylor remainder of $y = f(x)$ about x_0.

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0.

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$$P_N(x) \;==\; f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \cdots + \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N$$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$$P_N(x) \;=\; \sum_{n=0}^{N} \frac{f^{(n)}(x_0)}{n!}\,(x - x_0)^n$$

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$$P_\infty(x) \;=\; f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \ldots$$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$$P_\infty(x) \;=\; \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}\,(x - x_0)^n$$

We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) \;=\; \frac{f^{(N+1)}(c)}{(N+1)!}\,(x - x_0)^{(N+1)}$$

for some c between $\boxed{x}$ and $\boxed{x_0}$.

The formula for c_n is

$$c_n \;=\; \frac{f^{(n)}(x_0)}{n!}$$

A Maclaurin series is a Taylor series about the center $\boxed{x_0 = 0}$.

0B. Volume of Revolutions. Let's say we revolve some region in the xy-plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

- **In parts a, fill in the blanks with:** x **or** y**.**
- **In parts b and c, fill in the blanks with a formula involving *some of*:**
  2 , π , radius , $\text{radius}_{\text{big}}$, $\text{radius}_{\text{little}}$, average radius , height , and/or thickness .

► **Disk/Washer Method**. Let's find the volume of this solid of revolution using the disk or washer method.

a. If the axis of revolution is:

- the x-axis, or parallel to the x-axis, then we partition the $\underline{\;x\;}$–axis.
- the y-axis, or parallel to the y-axis, then we partition the $\underline{\;y\;}$–axis.

b. If we use the **disk method**, then the volume of a typical disk is:

$$\underline{\pi\ (\text{radius})^2\ (\text{height})}\,.$$

If we use the **washer method**, then the volume of a typical washer is:

$$\underline{\pi\ (\text{radius}_{\text{big}})^2\ (\text{height})\ -\ \pi\ (\text{radius}_{\text{little}})^2\ (\text{height})\ \stackrel{\text{or}}{=}\ \pi\ \left[(\text{radius}_{\text{big}})^2 - (\text{radius}_{\text{little}})^2\right]\ (\text{height})}\,.$$

c. If we partition the z-axis, where z is either x or y, the $\Delta z =$ $\underline{\text{ height }}$.

► **Shell Method**. Let's find the volume of this solid of revolution using the shell method.

a. If the axis of revolution is:

- the x-axis, or parallel to the x-axis, then we partition the $\underline{\;y\;}$–axis.
- the y-axis, or parallel to the y-axis, then we partition the $\underline{\;x\;}$–axis.

b. If we use the **shell method**, then the volume of a typical shell is:

$$\underline{2\pi\ (\text{average radius})\ (\text{height})\ (\text{thickness})\ \stackrel{\text{or}}{=}\ 2\pi\ (\text{radius})\ (\text{height})\ (\text{thickness})}\,.$$

c. If we partition the z-axis, where z is either x or y, the $\Delta z =$ $\underline{\text{thickness}\ \stackrel{\text{or}}{=}\ \text{radius}_{\text{big}} - \text{radius}_{\text{little}}}$.

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 2 – 11

Instructions.

- Indicate (by circling) your solution to each problem.
- You may choice up to 3 answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 6 points. For a problem with precisely two answers marked, one of which is correct, 3 points. For a problem with precisely three answers marked, one of which is correct, 1 point. All other cases, 0 points.

Your Solutions						
PROBLEM						points
2	2a	2b	2c	2d	2e	
3	3a	3b	3c	3d	3e	
4	4a	4b	4c	4d	4e	
5	5a	5b	5c	5d	5e	
6	6a	6b	6c	6d	6e	
7	7a	7b	7c	7d	7e	
8	8a	8b	8c	8d	8e	
9	9a	9b	9c	9d	9e	
10	10a	10b	10c	10d	10e	
11	11a	11b	11c	11d	11e	