HAND IN PART

Prof. Girardi	Math 142	Spring 2013	04.18.2013	Exam 3

MARK BOX		
PROBLEM	POINTS	
0	20	
1	20	
2–11	60	
%	100	

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

(1) The MARK BOX above indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) You may **not** use an electronic device, a calculator, books, personal notes.

(3) On Problem 0, fill in the blanks. As you were warned, if you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.

(4) For the do by hand problems, to receive credit you must:
- (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
- (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
- (c) if no such line/box is provided, then box your answer.

(5) For the multiple choice problems, please.
- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
- Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(6) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(7) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
§ 11.9–11.11 and 6.1–6.3 .

0. Fill in: the boxes in problem **0A** and the lines in problem **0B**.

0A. Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0.

Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0.

Let $y = R_N(x)$ be the N^{th}-order Taylor remainder of $y = f(x)$ about x_0.

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0.

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$P_N(x) =$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$P_N(x) =$

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$P_\infty(x) =$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$P_\infty(x) =$

We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$R_N(x) =$	for some c between		and		.

The formula for c_n is

$c_n =$

A Maclaurin series is a Taylor series about the center $x_0 =$ ________.

0B. Volume of Revolutions. Let's say we revolve some region in the xy-plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

- **In parts a, fill in the blanks with:** x **or** y**.**
- **In parts b and c, fill in the blanks with a formula involving *some of*:**
  2 , π **, radius , radius$_{\text{big}}$, radius$_{\text{little}}$, average radius , height , and/or thickness** .

►. **Disk/Washer Method**. Let's find the volume of this solid of revolution using the disk or washer method.

a. If the axis of revolution is:

- the x-axis, or parallel to the x-axis, then we partition the _____ –axis.
- the y-axis, or parallel to the y-axis, then we partition the _____ –axis.

b. If we use the **disk method**, then the volume of a typical disk is:

____ .

If we use the **washer method**, then the volume of a typical washer is:

____ .

c. If we partition the z-axis, where z is either x or y, the Δz is the ____________________ .

►. **Shell Method**. Let's find the volume of this solid of revolution using the shell method.

a. If the axis of revolution is:

- the x-axis, or parallel to the x-axis, then we partition the _____ –axis.
- the y-axis, or parallel to the y-axis, then we partition the _____ –axis.

b. If we use the **shell method**, then the volume of a typical shell is:

____ .

c. If we partition the z-axis, where z is either x or y, the Δz is the ____________________ .

1. Justify your answers below. (Problem source: textbook page 744).

1a. Express e^{-x^2} as an infinite series, in closed form (i.e., with $\sum$ sign and no $\ldots$) .

$e^{-x^2} = \sum_{n=0}^{\infty}$

1b. Express $\int e^{-x^2}\,dx$ as an infinite series, in closed form (i.e., with $\sum$ sign and no $\ldots$) .

$\int e^{-x^2}\,dx = C + \sum_{n=0}^{\infty}$

1c. Approximate $\int_0^1 e^{-x^2}\,dx$ by a finite sum (you need to do the arithmetic since you do not have a calculator on hand, just leave your answer as a sum of a finite number of numbers) where the accuracy (i.e. error) is less than or equal to $\frac{1}{11(5!)}$. Hint: Alternating Series Remainder test.

$\int_0^1 e^{-x^2}\,dx \approx$

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 2 – 11

Instructions.

- Indicate (by circling) your solution to each problem.
- You may choice up to 3 answers for each problem. The scoring is as follows. For a problem with precisely one answer marked and the answer is correct, 6 points. For a problem with precisely two answers marked, one of which is correct, 3 points. For a problem with precisely three answers marked, one of which is correct, 1 point. All other cases, 0 points.

Your Solutions						
PROBLEM						points
2	2a	2b	2c	2d	2e	
3	3a	3b	3c	3d	3e	
4	4a	4b	4c	4d	4e	
5	5a	5b	5c	5d	5e	
6	6a	6b	6c	6d	6e	
7	7a	7b	7c	7d	7e	
8	8a	8b	8c	8d	8e	
9	9a	9b	9c	9d	9e	
10	10a	10b	10c	10d	10e	
11	11a	11b	11c	11d	11e	

STATEMENT OF MULTIPLE CHOICE PROBLEMS

INSTRUCTIONS for MULTIPLE CHOICE PROBLEMS

- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with problems 1–10, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- Select at most one response for each problem.
- The scoring is: 8 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

2. Using a known (commonly used) Taylor series, find the Tayor series for $f(x) = x\cos(4x)$ about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n+1}}{n!}$ b. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$ c. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

d. $\sum_{n=0}^{\infty} \dfrac{(-1)^{n+1} 4^{2n} x^{2n+1}}{(2n)!}$ e. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{n} x^{2n+1}}{(2n)!}$

3. Using a known (commonly used) Taylor series, evaluate $\int \tan^{-1}(t^2)\,dt$ as a power series.

a. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+3}}{(2n+1)(4n+3)}$ b. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+3}}{(4n+3)}$ c. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+2}}{(2n+3)}$

d. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{2n+2}}{(2n+1)}$ e. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{2n+3}}{(2n+3)}$

4. Find the 2^{nd} order Taylor polynomial for $f(x) = \sqrt[3]{x}$ about the center $x_0 = 8$.

a. $2 + \frac{x}{12} - \frac{x^2}{9(2^5)}$ b. $2 + \frac{(x-8)}{12} + \frac{(x-8)^2}{9(2^5)}$ c. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^5)}$ d. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^4)}$

e. none of these

5. Find the Taylor series for $f(x) = (1-5x)^{-3}$ about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty}(-1)^n \frac{5^n(n+1)(n+2)}{2}x^n$ b. $\sum_{n=0}^{\infty} \frac{5^n(n+1)(n+2)}{2}x^n$ c. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{5^n}{n!}x^n$

d. $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2}x^n$ e. none of these

6. Consider the function $f(x) = e^x$ over the interval $(-1, 3)$. The 4^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{4} \frac{x^n}{n!}.$$

The 4^{th} order Remainder term $R_4(x)$ is defined by $R_4(x) = f(x) - P_4(x)$ and so $e^x \approx P_4(x)$ where the approximation is within an error of $|R_4(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_4(x)|$ that is valid for each $x \in (-1, 3)$.

a. $\frac{(e^{-1})(3^4)}{4!}$ b. $\frac{(e^3)(3^4)}{4!}$ c. $\frac{(e^{-1})(3^5)}{5!}$ d. $\frac{(e^3)(3^5)}{5!}$ e. none of these

7. Express the area of the region enclosed by $y = x^2$ and $y = 4x - x^2$ as an integral.

a. $\int_0^4 \left[\left(4x - x^2\right) - x^2\right] \, dx$ b. $\int_0^4 \left[x^2 - \left(4x - x^2\right)\right] \, dx$ c. $\int_0^2 \left[\left(4x - x^2\right) - x^2\right] \, dx$
d. $\int_0^2 \left[x^2 - \left(4x - x^2\right)\right] \, dx$ e. none of these

Problem Source: § 6.1 Exercise # 12.

8. Let R be the region bounded by the curves

$$y = 1 - x^2 \qquad \text{and} \qquad y = 0 \; .$$

Express as an integral the volume of the solid generated by revolving R about the x-axis.

a. $\pi \int_0^1 \sqrt{1-y} \, dy$ b. $2\pi \int_0^1 y\sqrt{1-y} \, dy$ c. $\pi \int_0^1 (1-x^2)^2 \, dx$ d. $\pi \int_{-1}^1 (1-x^2)^2 \, dx$
e. none of these

Problem Source: § 6.2 Exercise # 2.

9. Let R be the region bounded by the curves

$$y = \frac{x^2}{4} \qquad \text{and} \qquad y = 5 - x^2 \; .$$

Express as integral(s) the volume of the solid generated by revolving R about the x-axis.

a. $\pi \int_{-2}^2 \left[\left(5 - x^2\right) - \left(\frac{x^2}{4}\right)\right]^2 \, dx$ b. $\pi \int_{-2}^2 \left[\left(5 - x^2\right)^2 - \left(\frac{x^2}{4}\right)^2\right] \, dx$ c. $2\pi \int_0^5 2y\sqrt{5-y} \, dy$
d. $2\pi \int_0^1 y\sqrt{4y} \, dy \; + \; 2\pi \int_0^1 y\sqrt{5-y} \, dy$ e. none of these

Problem Source: § 6.2 Exercise # 8.

10. Let R be the region bounded by the curves

$$y = \frac{1}{x} \qquad \text{and} \qquad y = 0 \qquad \text{and} \qquad x = 1 \qquad \text{and} \qquad x = 2 \; .$$

Express as integral(s) the volume of the solid generated by revolving R about the y-axis.

a. $2\pi \int_1^2 \frac{1}{x} \, dx$ b. $2\pi \int_1^2 x \, dx$ c. $2\pi \int_1^2 1 \, dx$ d. $\pi \int_0^1 \left[\left(\frac{1}{y}\right)^2 - (1)^2\right] \, dy$
e. none of these

Problem Source: § 6.3 Exercise # 3.

11. Let R be the region bounded by the curves

$$y = x \qquad \text{and} \qquad y = 4x - x^2 \; .$$

Express as integral(s) the volume of the solid generated by revolving R about the line $x = 7$.

a. $2\pi \int_0^3 x \left[x - \left(4x - x^2\right)\right] \, dx$ b. $2\pi \int_0^3 x \left[\left(4x - x^2\right) - x\right] \, dx$
c. $2\pi \int_0^3 (7 - x) \left[x - \left(4x - x^2\right)\right] \, dx$ d. $2\pi \int_0^3 (7 - x) \left[\left(4x - x^2\right) - x\right] \, dx$
e. none of these

Problem Source: § 6.3 Exercise # 22.