HAND IN PART

Prof. Girardi	Math 142	Fall 2012	12.10.12	Final Exam

MARK BOX		
PROBLEM	POINTS	
1–25	25	
%	100	

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

(1) There are 25 equally weighted multiple choice problems.
- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided.
- Hand in the HAND IN PART. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you (so you can check your answers once the solutions are posted).

(2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(3) You may **not** use: electronic devices, books, personal notes.

(4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.

(5) The final exam will cover:
7.1–7.5, 7.8, 11.1–11.11, 6.1–6.3, 10.3–10.4 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 1 – 25

Instructions.

- Indicate (by circling, boxing, or x-ing) your solution to each problem.
- Select at most one response for each problem.
- The scoring is: full points for a correct answer and 0 points for an incorrect-or-blank answer.

Your Solutions					
PROBLEM					
1	1a	1b	1c	1d	1e
2	2a	2b	2c	2d	2e
3	3a	3b	3c	3d	3e
4	4a	4b	4c	4d	4e
5	5a	5b	5c	5d	5e
6	6a	6b	6c	6d	6e
7	7a	7b	7c	7d	7e
8	8a	8b	8c	8d	8e
9	9a	9b	9c	9d	9e
10	10a	10b	10c	10d	10e
11	11a	11b	11c	11d	11e
12	12a	12b	12c	12d	12e
13	13a	13b	13c	13d	13e
14	14a	14b	14c	14d	14e
15	15a	15b	15c	15d	15e
16	16a	16b	16c	16d	16e
17	17a	17b	17c	17d	17e
18	18a	18b	18c	18d	18e
19	19a	19b	19c	19d	19e
20	20a	20b	20c	20d	20e
21	21a	21b	21c	21d	21e
22	22a	22b	22c	22d	22e
23	23a	23b	23c	23d	23e
24	24a	24b	24c	24d	24e
25	25a	25b	25c	25d	25e

STATEMENT OF MULTIPLE CHOICE PROBLEMS

1. Evaluate the integral

$$\int_0^5 \frac{1}{x^2+25}\,dx\ .$$

Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b + \ln a = \ln(ab)$, and $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$, and $r\,(\ln(a)) = \ln(a^r)$.

a. $\frac{\pi}{20}$ b. $\frac{\pi}{4}$ c. $\ln(2)$ d. $\ln\sqrt{2}$ e. None of these

2. Evaluate the integral

$$\int_0^5 \frac{x}{x^2+25}\,dx\ .$$

Hint. If $a, b > 0$ and $r \in \mathbb{R}$, then: $\ln b + \ln a = \ln(ab)$, and $\ln b - \ln a = \ln\left(\frac{b}{a}\right)$, and $r\,(\ln(a)) = \ln(a^r)$.

a. $\ln 2$ b. $\ln\sqrt{2}$ c. $\ln 4$ d. $\frac{\pi}{4}$ e. none of these

3. Evaluate the integral

$$\int_0^5 \frac{x^2}{x^2+25}\,dx\ .$$

a. $\ln 2$ b. $\dfrac{20+5\pi}{4}$ c. $\dfrac{20-5\pi}{4}$ d. $\dfrac{20-25\pi}{4}$ e. none of these

4. Evaluate the integral, for $x > 0$,

$$\int \frac{1-x+2x^2-x^3}{x\left(x^2+1\right)^2}\,dx\ .$$

a. $\ln|x| - \dfrac{\ln(x^2+1)}{2} - \arctan(x) - \dfrac{1}{2(x^2+1)} + C$

b. $\ln|x| + \ln(x^2+1) - \arctan(x) + \dfrac{1}{2(x^2+1)} + C$

c. $\ln|x| - \dfrac{\ln(x^2+1)}{2} - \arctan(x) + C$

d. $\ln|x| - \dfrac{\ln(x^2+1)}{2} - \arctan(x) - \dfrac{1}{x^2+1} + C$

e. none of these

5. Evaluate the integral

$$\int_1^e \ln x\,dx\ .$$

Hints. $\ln 1 = 0$ and $\ln e = 1$.

a. $e^{-1}-1$ b. $1-e^{-1}$ c. $2e-1$ d. 1 e. none of these

6. Evaluate the integral

$$\int_0^1 x^2\,e^x\,dx\ .$$

a. $\frac{3}{2}-e$ b. $e+2$ c. $e-2$ d. $-e$ e. none of these

7. Evaluate the integral

$$\int_0^{\pi/2} e^x\,\sin x\,dx\ .$$

a. $\dfrac{1-e^{\pi/2}}{2}$ b. $\dfrac{1+e^{\pi/2}}{2}$ c. $1-e^{\pi/2}$ d. $1+e^{\pi/2}$ e. none of these

8. Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \sin^2 x \, dx \ .$$

a. 0 b. π c. $\dfrac{\pi}{8} - \dfrac{1}{4}$ d. $\dfrac{\pi}{8} + \dfrac{1}{4}$ e. none of these

9. Evaluate the integral

$$\int_{-1}^{0} \frac{1}{(x^2+2x+2)^2} \, dx \ .$$

Hint. Complete the square: $x^2+2x+2 = (x \pm \ ? \ )^2 \pm \ ??$.

a. $\dfrac{\pi}{4} + \dfrac{1}{2}$ b. $\dfrac{\pi}{8} + \dfrac{1}{2}$ c. $\dfrac{\pi}{8} - \dfrac{1}{4}$ d. $\dfrac{\pi}{8} + \dfrac{1}{4}$ e. none of these

10. Let

$$f(x) \ = \ \frac{-2}{(x-11)^3} \ .$$

Evaluate the integral

$$\int_{10}^{12} f(x) \ dx \ .$$

Hint. The integrand (i.e., the function $y = f(x)$) does not exist at $x = 11$.

a. 0 b. 2 c. diverges to infinity d. does not exist but also does not diverge to infinity
e. none of these

The region R for problems 11–14

Let R be the region in the first quadrant enclosed by

$$y = 7 - x \qquad \text{and} \qquad y = 0 \qquad \text{and} \qquad x = 3 \ .$$

Note that R is the triangle with vertices: $(3,0)$ and $(3,4)$ and $(7,0)$.

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11. Express, as an integral with respect to y, the area of R.

a. $\int_0^4 (7-y) \ dy$ b. $\int_3^7 (7-y) \ dy$ c. $\int_0^4 [(7-y)-3] \ dy$ d. $\int_0^4 [3-(7-y)] \ dy$ e. none of these

12. Express, as an integral with respect to y, the volume of the solid generated by revolving R about the y-axis.

a. $\pi \int_0^4 (7-y)^2 \ dy$ b. $\pi \int_0^4 \left[(7-y)^2 - 3^2\right] \ dy$ c. $\pi \int_0^4 [(7-y)-3]^2 \ dy$ d. $\pi \int_0^4 \left[3^2 - (7-y)^2\right] \ dy$
e. none of these

13. Express, as an integral with respect to y, the volume of the solid generated by revolving R about $x = 3$.

a. $\pi \int_0^4 (7-y)^2 \ dy$ b. $\pi \int_0^4 \left[(7-y)^2 - 3^2\right] \ dy$ c. $\pi \int_0^4 [(7-y)-3]^2 \ dy$ d. $\pi \int_0^4 \left[3^2 - (7-y)^2\right] \ dy$
e. none of these

14. Express, as an integral with respect to y, the volume of the solid generated by revolving R about x-axis.

a. $2\pi \int_0^4 y \ (7-y) \ dy$ b. $2\pi \int_0^4 y \ [(7-y)-3] \ dy$ c. $2\pi \int_0^4 [(7-y)-3] \ dy$
d. $2\pi \int_0^4 y \ [3-(7-y)] \ dy$ e. none of these

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15. Express the polar equation

$$r = (\tan\theta)\ (\sec\theta)$$

as a Cartesian equation.

a. $y = x^2$ b. $y^2 = x$ c. $y = x$ d. $x^2 + y^2 = 1$ e. none of these

16. Express the area enclosed by

$$r = 3\cos\theta$$

as an integral.

a. $\frac{1}{2}\int_0^{2\pi}(3\cos\theta)\ d\theta$ b. $\frac{1}{2}\int_0^{2\pi}(3\cos\theta)^2\ d\theta$ c. $\frac{1}{2}\int_0^{\pi}(3\cos\theta)\ d\theta$ d. $\frac{1}{2}\int_0^{\pi}(3\cos\theta)^2\ d\theta$
e. none of these

17. Evaluate

$$\lim_{n\to\infty}\frac{\sqrt{25n^3+4n^2+n-5}}{7n^{\frac{3}{2}}+6n-1}\ .$$

a. 0 b. ∞ c. $\frac{25}{7}$ d. $\frac{5}{7}$ e. none of these

18. Find **<u>all</u>** real numbers r satisfying that

$$\sum_{n=2}^{\infty} r^n \ = \ \frac{1}{12}\ .$$

a. $\dfrac{1}{12}$ b. $\dfrac{1}{12}$ and $\dfrac{-1}{12}$ c. $\dfrac{1}{4}$ and $\dfrac{-1}{3}$ d. $\dfrac{1}{3}$ and $\dfrac{-1}{4}$ e. none of these

19. Consider the following two series.

Series A is $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$.

Series B is $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$.

a. Both series converge absolutely. b. Both series diverge.
c. Series A converges conditionally and Series B diverges.
d. Series A diverges and Series B converges conditionally. e. none of these

20. Consider the formal series

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{(n+2)(n+7)}}\ .$$

a. This series is absolutely convergent, as can be shown by the limit comparison test (LCT) with $b_n = \frac{1}{n^2}$.
b. This series is conditionally convergent, as can be shown by using only the AST and not other tests.
c. This series converges conditionally, as can be shown by using the LCT with $b_n = \frac{1}{n}$ as well as the AST .
d. This series diverges. e. none of these

21. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \, \frac{n!}{(3n)!} \, .$$

Let

$$a_n \;=\; (-1)^n \, \frac{n!}{(3n)!} \qquad \text{and} \qquad \rho \;=\; \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| \, .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = \frac{1}{3}$.
b. $\sum_{n=1}^{\infty} a_n$ converges absolutely by the Ratio Test because $\rho = 0$.
c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$.
d. $\rho > 1$ so by the Ratio Test $\sum_{n=1}^{\infty} a_n$ diverges. e. none of these

22. What is the LARGEST interval for which the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+15)^n}{4^n}$$

is absolutely convergent?

a. $\left(\frac{11}{5}, \frac{19}{5}\right)$ b. $\left[\frac{11}{5}, \frac{19}{5}\right]$ c. $\left(\frac{-19}{5}, \frac{-11}{5}\right)$ d. $\left[\frac{-19}{5}, \frac{-11}{5}\right]$ e. none of these

23. Using the known (commonly used Talyor) power series expansion

$$\ln(1+x) \;=\; \sum_{n=1}^{\infty} (-1)^{n+1} \, \frac{x^n}{n} \qquad \text{valid for} \quad x \in (-1, 1] \, ,$$

find a power series expansion, and state when it is valid, for $\ln(10-x)$.

a. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(10-x)^n}{n}$ for $x \in (-1, 1]$ b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(10-x)^n}{n}$ for $x \in [-1, 1)$

c. $\sum_{n=1}^{\infty} \frac{(x-9)^n}{-n}$ for $x \in (8, 10]$ d. $\sum_{n=1}^{\infty} \frac{(x-9)^n}{-n}$ for $x \in [8, 10)$ e. none of these

24. Find the 3^{rd} order Taylor polynomial, about the center $x_0 = 1$, for the function $f(x) = x^5 - x^2 + 5$.

a. $5 + 3(x-1) + 9(x-1)^2 + 10(x-1)^3$ b. $5 + 3(x-1) + 18(x-1)^2 + 60(x-1)^3$
c. $5 + 3x + 9x^2 + 10x^3$ d. $5 + 3x + 18x^2 + 60x^3$ e. none of these

25. Consider the function $f(x) = e^{-x}$ as well as the interval $(7, 9)$.
The 5^{th} order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_5(x) \;=\; \sum_{n=0}^{5} \frac{(-x)^n}{n!} \;=\; 1 \;-\; x \;+\; \frac{x^2}{2!} \;-\; \frac{x^3}{3!} \;+\; \frac{x^4}{4!} \;-\; \frac{x^5}{5!} \, .$$

The 5^{th} order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $f(x) \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (7, 9)$.

a. $\frac{(e^{-7})(9^5)}{5!}$ b. $\frac{(e^{-9})(9^5)}{5!}$ c. $\frac{(e^{-0})(9^6)}{6!}$ d. $\frac{(e^{-9})(9^6)}{6!}$ e. none of these