HAND IN PART

Prof. Girardi	Math 142	Fall 2012	11.27.12	Practice Exam 3

MARK BOX		
PROBLEM	POINTS	
0	20	
1–10	80	
%	100	

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS

(1) On problem 0, fill in the provided box and/or line.
(2) Now for problems 1 through 10.
- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with problems 1–10, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.

(3) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
(4) You may **not** use: electronic devices, books, personal notes.
(5) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
(6) If you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
(7) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
11.9–11.11, 6.1–6.3 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

0. Fill in: the boxes in problem **0A** and the lines in problem **0B**.

0A. Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0.

Let $y = P_N(x)$ be the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0.

Let $y = R_N(x)$ be the N^{th}-order Taylor remainder of $y = f(x)$ about x_0.

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0.

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0.

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$P_N(x) =$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$P_N(x) =$

In open form (i.e., with $\ldots$ and without a $\sum$-sign)

$P_\infty(x) =$

In closed form (i.e., with a $\sum$-sign and without $\ldots$)

$P_\infty(x) =$

We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$R_N(x) =$	for some c between		and		.

The formula for c_n is

$c_n =$

0B. Volume of Revolutions. Let's say we revolve some region in the xy-plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

- **In parts a, fill in the blanks with: perpendicular or parallel.**
- **In parts b, c, d, fill in the blanks with a formula involving *some of*:**
  2 , π , radius , radius$_{\text{big}}$, radius$_{\text{little}}$, average radius , height , and/or thickness.
- z can be either x or y.

►. **Disk/Washer Method**. Let's find the volume of this solid of revolution using the disk or washer method.

a. We should partition the coordinate axis (i.e., the x-axis or the y-axis) that is ________________ to the axis of revolution.

b. If we use the **disk method**, then the volume of a typical disk is:

____ .

c. If we use the **washer method**, then the volume of a typical washer is:

____ .

d. If we partition the z-axis, the $\Delta z =$ ________________________________ .

►. **Shell Method**. Let's find the volume of this solid of revolution using the shell method.

a. We should partition the coordinate axis (i.e., the x-axis or the y-axis) that is ________________ to the axis of revolution.

b. If we use the **shell method**, then the volume of a typical shell is:

____ .

c. If we partition the z-axis, the $\Delta z =$ ________________________________ .

TABLE FOR YOUR ANSWERS TO PROBLEMS 1-10

Instructions.

- Indicate (by circling, boxing, or x-ing) your solution to each problem.
- Select at most one response for each problem.
- The scoring is: 8 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

Your Solutions					
PROBLEM					
1	1a	1b	1c	1d	1e
2	2a	2b	2c	2d	2e
3	3a	3b	3c	3d	3e
4	4a	4b	4c	4d	4e
5	5a	5b	5c	5d	5e
6	6a	6b	6c	6d	6e
7	7a	7b	7c	7d	7e
8	8a	8b	8c	8d	8e
9	9a	9b	9c	9d	9e
10	10a	10b	10c	10d	10e

STATEMENT OF MULTIPLE CHOICE PROBLEMS

INSTRUCTIONS for MULTIPLE CHOICE PROBLEMS

- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with problems 1–10, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- Select at most one response for each problem.
- The scoring is: 8 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

1. Using a known (commonly used) Taylor series, find the Tayor series for $f(x) = x\cos(4x)$ about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n+1}}{n!}$ b. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$ c. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

d. $\sum_{n=0}^{\infty} \dfrac{(-1)^{n+1} 4^{2n} x^{2n+1}}{(2n)!}$ e. $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^{n} x^{2n+1}}{(2n)!}$

2. Using a known (commonly used) Taylor series, evaluate $\int \tan^{-1}(t^2)\,dt$ as a power series.

a. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+3}}{(2n+1)(4n+3)}$ b. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+3}}{(4n+3)}$ c. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{4n+2}}{(2n+3)}$

d. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{2n+2}}{(2n+1)}$ e. $C + \sum_{n=0}^{\infty} \dfrac{(-1)^n t^{2n+3}}{(2n+3)}$

3. Find the 2nd order Taylor polynomial for $f(x) = \sqrt[3]{x}$ about the center $x_0 = 8$.

a. $2 + \frac{x}{12} - \frac{x^2}{9(2^5)}$ b. $2 + \frac{(x-8)}{12} + \frac{(x-8)^2}{9(2^5)}$ c. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^5)}$ d. $2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{9(2^4)}$

e. none of these

4. Find the Taylor series for $f(x) = (1-5x)^{-3}$ about the center $x_0 = 0$.

a. $\sum_{n=0}^{\infty}(-1)^n \frac{5^n(n+1)(n+2)}{2} x^n$ b. $\sum_{n=0}^{\infty} \frac{5^n(n+1)(n+2)}{2} x^n$ c. $\sum_{n=0}^{\infty}(-1)^{n+1} \frac{5^n}{n!} x^n$

d. $\sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$ e. none of these

5. Consider the function $f(x) = e^x$ over the interval $(-1,3)$. The 4th order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_4(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = \sum_{n=0}^{4} \frac{x^n}{n!} .$$

The 4th order Remainder term $R_4(x)$ is defined by $R_4(x) = f(x) - P_4(x)$ and so $e^x \approx P_4(x)$ where the approximation is within an error of $|R_4(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_4(x)|$ that is valid for each $x \in (-1,3)$.

a. $\frac{(e^{-1})(3^4)}{4!}$ b. $\frac{(e^{3})(3^4)}{4!}$ c. $\frac{(e^{-1})(3^5)}{5!}$ d. $\frac{(e^{3})(3^5)}{5!}$ e. none of these

6. Express the area of the region enclosed by $y = x^2$ and $y = 4x - x^2$ as an integral.

a. $\int_0^4 \left[\left(4x - x^2\right) - x^2\right]\, dx$ b. $\int_0^4 \left[x^2 - \left(4x - x^2\right)\right]\, dx$ c. $\int_0^2 \left[\left(4x - x^2\right) - x^2\right]\, dx$
d. $\int_0^2 \left[x^2 - \left(4x - x^2\right)\right]\, dx$ e. none of these

Problem Source: § 6.1 Exercise # 12.

7. Let R be the region bounded by the curves

$$y = 1 - x^2 \qquad \text{and} \qquad y = 0\ .$$

Express as an integral the volume of the solid generated by revolving R about the x-axis.

a. $\pi \int_0^1 \sqrt{1-y}\, dy$ b. $2\pi \int_0^1 y\sqrt{1-y}\, dy$ c. $\pi \int_0^1 (1-x^2)^2\, dx$ d. $\pi \int_{-1}^1 (1-x^2)^2\, dx$
e. none of these

Problem Source: § 6.2 Exercise # 2.

8. Let R be the region bounded by the curves

$$y = \frac{x^2}{4} \qquad \text{and} \qquad y = 5 - x^2\ .$$

Express as integral(s) the volume of the solid generated by revolving R about the x-axis.

a. $\pi \int_{-2}^2 \left[\left(5 - x^2\right) - \left(\frac{x^2}{4}\right)\right]^2 dx$ b. $\pi \int_{-2}^2 \left[\left(5 - x^2\right)^2 - \left(\frac{x^2}{4}\right)^2\right] dx$ c. $2\pi \int_0^5 2y\sqrt{5-y}\, dy$
d. $2\pi \int_0^1 y\sqrt{4y}\, dy \ + \ 2\pi \int_0^1 y\sqrt{5-y}\, dy$ e. none of these

Problem Source: § 6.2 Exercise # 8.

9. Let R be the region bounded by the curves

$$y = \frac{1}{x} \qquad \text{and} \qquad y = 0 \qquad \text{and} \qquad x = 1 \qquad \text{and} \qquad x = 2\ .$$

Express as integral(s) the volume of the solid generated by revolving R about the y-axis.

a. $2\pi \int_1^2 \frac{1}{x}\, dx$ b. $2\pi \int_1^2 x\, dx$ c. $2\pi \int_1^2 1\, dx$ d. $\pi \int_0^1 \left[\left(\frac{1}{y}\right)^2 - (1)^2\right] dy$
e. none of these

Problem Source: § 6.3 Exercise # 3.

10. Let R be the region bounded by the curves

$$y = x \qquad \text{and} \qquad y = 4x - x^2\ .$$

Express as integral(s) the volume of the solid generated by revolving R about the line $x = 7$.

a. $2\pi \int_0^3 x\left[x - \left(4x - x^2\right)\right]\, dx$ b. $2\pi \int_0^3 x\left[\left(4x - x^2\right) - x\right]\, dx$
c. $2\pi \int_0^3 (7 - x)\left[x - \left(4x - x^2\right)\right]\, dx$ d. $2\pi \int_0^3 (7 - x)\left[\left(4x - x^2\right) - x\right]\, dx$
e. none of these

Problem Source: § 6.3 Exercise # 22.