

**HAND IN PART**

Prof. Girardi                      Math 142                      Fall 2012                      11.27.12                      Practice Exam 3

MARK BOX		
PROBLEM	POINTS	
0	20	
1–10	80	
%	100	

**NAME:** \_\_\_\_\_ Solution Key \_\_\_\_\_

**PIN:** \_\_\_\_\_ 17 \_\_\_\_\_

**INSTRUCTIONS**

- (1) On problem 0, fill in the provided box and/or line.
- (2) Now for problems 1 through 10.
  - First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
  - Once finished with problems 1–10, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- (3) The MARK BOX indicates the problems along with their points.  
Check that your copy of the exam has all of the problems.
- (4) You may **not** use: electronic devices, books, personal notes.
- (5) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- (6) If you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- (7) This exam covers (from *Calculus* by Stewart, 6<sup>th</sup> ed., ET):  
11.9–11.11, 6.1–6.3 .

**Honor Code Statement**

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : \_\_\_\_\_

0. Fill in: the boxes in problem **0A** and the lines in problem **0B**.

**0A.** Taylor/Maclaurin Polynomials and Series

Let  $y = f(x)$  be a function with derivatives of all orders in an interval  $I$  containing  $x_0$ .

Let  $y = P_N(x)$  be the  $N^{\text{th}}$ -order Taylor polynomial of  $y = f(x)$  about  $x_0$ .

Let  $y = R_N(x)$  be the  $N^{\text{th}}$ -order Taylor remainder of  $y = f(x)$  about  $x_0$ .

Let  $y = P_\infty(x)$  be the Taylor series of  $y = f(x)$  about  $x_0$ .

Let  $c_n$  be the  $n^{\text{th}}$  Taylor coefficient of  $y = f(x)$  about  $x_0$ .

In open form (i.e., with ... and without a  $\sum$ -sign)

$$P_N(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \dots + \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N$$

In closed form (i.e., with a  $\sum$ -sign and without ... )

$$P_N(x) = \sum_{n=0}^N \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

In open form (i.e., with ... and without a  $\sum$ -sign)

$$P_\infty(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \dots$$

In closed form (i.e., with a  $\sum$ -sign and without ... )

$$P_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

We know that  $f(x) = P_N(x) + R_N(x)$ . Taylor's BIG Theorem tells us that, for each  $x \in I$ ,

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!} (x - x_0)^{(N+1)}$$

for some  $c$  between

$x$

and

$x_0$

The formula for  $c_n$  is

$$c_n = \frac{f^{(n)}(x_0)}{n!}$$

**0B. Volume of Revolutions.** Let's say we revolve some region in the  $xy$ -plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

- In parts a, fill in the blanks with: **perpendicular or parallel.**
- In parts b, c, d, fill in the blanks with a formula involving *some of*:  **$2$ ,  $\pi$ , radius, radius<sub>big</sub>, radius<sub>little</sub>, average radius, height, and/or thickness.**
- $z$  can be either  $x$  or  $y$ .

► **Disk/Washer Method.** Let's find the volume of this solid of revolution using the disk or washer method.

a. We should partition the coordinate axis (i.e., the  $x$ -axis or the  $y$ -axis) that is parallel to the axis of revolution.

b. If we use the **disk method**, then the volume of a typical disk is:

$$\underline{\pi (\text{radius})^2 (\text{height})} .$$

c. If we use the **washer method**, then the volume of a typical washer is:

$$\underline{\pi (\text{radius}_{\text{big}})^2 (\text{height}) - \pi (\text{radius}_{\text{little}})^2 (\text{height}) \quad \text{or} \quad \pi [(\text{radius}_{\text{big}})^2 - (\text{radius}_{\text{little}})^2] (\text{height})} .$$

d. If we partition the  $z$ -axis, the  $\Delta z =$  height .

► **Shell Method.** Let's find the volume of this solid of revolution using the shell method.

a. We should partition the coordinate axis (i.e., the  $x$ -axis or the  $y$ -axis) that is perpendicular to the axis of revolution.

b. If we use the **shell method**, then the volume of a typical shell is:

$$\underline{2\pi (\text{average radius}) (\text{height}) (\text{thickness}) \quad \text{or} \quad 2\pi (\text{radius}) (\text{height}) (\text{thickness})} .$$

c. If we partition the  $z$ -axis, the  $\Delta z =$  thickness  $\quad \text{or} \quad \text{radius}_{\text{big}} - \text{radius}_{\text{little}}$  .

## TABLE FOR YOUR ANSWERS TO PROBLEMS 1-10

### Instructions.

- Indicate (by circling, boxing, or x-ing) your solution to each problem.
- Select at most one response for each problem.
- The scoring is: 8 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

Your Solutions					
PROBLEM					
1	1a	1b	1c	1d	1e
2	2a	2b	2c	2d	2e
3	3a	3b	3c	3d	3e
4	4a	4b	4c	4d	4e
5	5a	5b	5c	5d	5e
6	6a	6b	6c	6d	6e
7	7a	7b	7c	7d	7e
8	8a	8b	8c	8d	8e
9	9a	9b	9c	9d	9e
10	10a	10b	10c	10d	10e