

HAND IN PART

Prof. Girardi Math 142 Fall 2012 11.29.12 Exam 3

MARK BOX		
PROBLEM	POINTS	
0A & 0B	16	
1–14	84	
%	100	

NAME: _____

PIN: _____

INSTRUCTIONS

- (1) On problem 0, fill in the provided box and/or line.
- (2) Now for the multiple choice problems 1 through 14.
 - First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
 - Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- (3) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (4) You may **not** use: electronic devices, books, personal notes.
- (5) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: put your pencil down and raise your hand.
- (6) If you do not make at least half of the points on Problem 0, then your score for the entire exam will be whatever you made on Problem 0.
- (7) This exam covers (from *Calculus* by Stewart, 6th ed., ET):
11.9–11.11, 6.1–6.3 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

0. Fill in: the boxes in problem 0A and the lines in problem 0B.

0A. Taylor/Maclaurin Polynomials and Series

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0 .

Let $y = P_N(x)$ be the N^{th} -order Taylor polynomial of $y = f(x)$ about x_0 .

Let $y = R_N(x)$ be the N^{th} -order Taylor remainder of $y = f(x)$ about x_0 .

Let $y = P_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0 .

Let c_n be the n^{th} Taylor coefficient of $y = f(x)$ about x_0 .

In open form (i.e., with ... and without a \sum -sign)

$$P_N(x) =$$

In closed form (i.e., with a \sum -sign and without ...)

$$P_N(x) =$$

In open form (i.e., with ... and without a \sum -sign)

$$P_\infty(x) =$$

In closed form (i.e., with a \sum -sign and without ...)

$$P_\infty(x) =$$

We know that $f(x) = P_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) = \text{for some } c \text{ between } \boxed{} \text{ and } \boxed{} .$$

The formula for c_n is

$$c_n =$$

A Maclaurin series is a Taylor series about the center $x_0 = \boxed{}$.

0B. Volume of Revolutions. Let's say we revolve some region in the xy -plane around an axis of revolution so we get a solid of revolution. Next we want to find the volume of this solid of revolution.

-
- **In parts a**, fill in the blanks with: x or y .
 - **In parts b and c**, fill in the blanks with a formula involving *some of*: 2 , π , radius, radius_{big}, radius_{little}, average radius, height, and/or thickness.
-

► **Disk/Washer Method.** Let's find the volume of this solid of revolution using the disk or washer method.

a. If the axis of revolution is:

- the x -axis, or parallel to the x -axis, then we partition the _____ -axis.
- the y -axis, or parallel to the y -axis, then we partition the _____ -axis.

b. If we use the **disk method**, then the volume of a typical disk is:

_____ .

If we use the **washer method**, then the volume of a typical washer is:

_____ .

c. If we partition the z -axis, where z is either x or y , the Δz is the _____ .

► **Shell Method.** Let's find the volume of this solid of revolution using the shell method.

a. If the axis of revolution is:

- the x -axis, or parallel to the x -axis, then we partition the _____ -axis.
- the y -axis, or parallel to the y -axis, then we partition the _____ -axis.

b. If we use the **shell method**, then the volume of a typical shell is:

_____ .

c. If we partition the z -axis, where z is either x or y , the Δz is the _____ .

TABLE FOR YOUR ANSWERS TO MULTIPLE CHOICE PROBLEMS 1 – 14

Instructions.

- Indicate (by circling, boxing, or x-ing) your solution to each problem.
- Select at most one response for each problem.
- The scoring is: 6 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

Your Solutions					
PROBLEM					
1	1a	1b	1c	1d	1e
2	2a	2b	2c	2d	2e
3	3a	3b	3c	3d	3e
4	4a	4b	4c	4d	4e
5	5a	5b	5c	5d	5e
6	6a	6b	6c	6d	6e
7	7a	7b	7c	7d	7e
8	8a	8b	8c	8d	8e
9	9a	9b	9c	9d	9e
10	10a	10b	10c	10d	10e
11	11a	11b	11c	11d	11e
12	12a	12b	12c	12d	12e
13	13a	13b	13c	13d	13e
14	14a	14b	14c	14d	14e

STATEMENT OF MULTIPLE CHOICE PROBLEMS
INSTRUCTIONS for MULTIPLE CHOICE PROBLEMS

- First, indicate (to yourself) your answers directly on the STATEMENT OF MULTIPLE CHOICE PROBLEMS part.
- Once finished with the multiple choice problems, go back to the HAND IN PART and indicate your answers on the table provided. You can take the STATEMENT OF MULTIPLE CHOICE PROBLEMS part home with you.
- Select at most one response for each problem.
- The scoring is: 6 points for a correct answer, 0 points for an incorrect answer, and 1 point for a blank answer.

1. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{2}{3-x}$$

about the center $x_0 = 0$ and state when this Taylor series is valid. Hint, by simple algebra,

$$f(x) = \frac{2}{3-x} = \left(\frac{2}{3}\right) \left(\frac{1}{1-\frac{x}{3}}\right).$$

- a. $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n x^n$, valid for $|x| < 1$ b. $\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n$, valid for $|x| < 3$
 c. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 1$ d. $\sum_{n=0}^{\infty} \frac{2}{3^{n+1}} x^n$, valid for $|x| < 3$ e. none of these
2. Using a known (commonly used) Taylor series, find the Taylor series for

$$f(x) = \frac{1}{(1-x)^4}$$

about the center $x_0 = 0$ which is valid for $|x| < 1$. Hint. Start with the Taylor series expansion

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \quad \text{valid for } |x| < 1$$

and differentiate (as many times as needed). Be careful and don't forget the chain rule:

$$D_x(1-x)^{-1} = (-1)(1-x)^{-2} D_x(1-x) = (-1)(1-x)^{-2}(-1) = (1-x)^{-2}.$$

- a. $\sum_{n=0}^{\infty} \frac{(n)(n-1)(n-2)}{6} x^{n-3}$ b. $\sum_{n=0}^{\infty} (n)(n-1)(n-2) x^n$
 c. $\sum_{n=0}^{\infty} \frac{(n+3)(n+2)(n+1)}{6} x^n$ d. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+3)(n+2)(n+1)}{6} x^n$ e. none of these
3. Using a known (commonly used) Taylor series, evaluate $\int \tan^{-1}(t^2) dt$ as a power series.
- a. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(4n+3)}$ b. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(2n+1)(4n+3)}$ c. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+2}}{(2n+3)}$
 d. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+2}}{(2n+1)}$ e. $C + \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+3}}{(2n+3)}$

4. Find the 3rd order Taylor polynomial for $f(x) = \frac{1}{x}$ about the center $x_0 = 2$.
- a. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$ b. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}(x-2)^2 - \frac{3}{8}(x-2)^3$
 c. $\frac{1}{2} + \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 + \frac{1}{16}(x-2)^3$ d. $\frac{1}{2} - \frac{1}{4}x + \frac{1}{4}x^2 - \frac{3}{8}x^3$ e. none of these

5. Find the Taylor series for $f(x) = x^4 - 3x^2 + 1$ about the center $x_0 = 1$.
- a. $(x-1)^4 - 3(x-1)^2 + 1$ b. $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4$
 c. $-1 - 2(x-1) + 6(x-1)^2 + 24(x-1)^3 + 24(x-1)^4$ d. $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4 + 2(x-1)^5$
 e. none of these

6. Find the Taylor series for

$$f(x) = \frac{1}{x^2}$$

about the center $x_0 = 1$.

- a. $\sum_{n=0}^{\infty} (-1)^n (n+1)! x^n$ b. $\sum_{n=0}^{\infty} (-1)^n (n+1)! (x-1)^n$
 c. $\sum_{n=0}^{\infty} (-1)^n (n+1) (x-1)^n$ d. $\sum_{n=0}^{\infty} (-1)^{n+1} (n+1) (x-1)^n$ e. none of these
7. Consider the function

$$f(x) = e^{-x}$$

over the interval $(7, 9)$. The 5th order Taylor polynomial of $y = f(x)$ about the center $x_0 = 0$ is

$$P_5(x) = \sum_{n=0}^5 \frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}.$$

The 5th order Remainder term $R_5(x)$ is defined by $R_5(x) = f(x) - P_5(x)$ and so $e^{-x} \approx P_5(x)$ where the approximation is within an error of $|R_5(x)|$. Using Taylor's (BIG) Theorem, find a good upper bound for $|R_5(x)|$ that is valid for each $x \in (-1, 3)$.

- a. $\frac{(e^{-7})(9^5)}{5!}$ b. $\frac{(e^{-9})(9^5)}{5!}$ c. $\frac{(e^{-7})(9^6)}{6!}$ d. $\frac{(e^{-9})(9^6)}{6!}$ e. none of these
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The region R for problems 8–10

Let R be the region in the first quadrant enclosed by

$$2y = 2 - x \quad \text{and} \quad \text{the } x\text{-axis} \quad \text{and} \quad \text{the } y\text{-axis} .$$

Note that the line $2y = 2 - x$ goes through the points $(0, 1)$ and $(2, 0)$.

- 8.** Using the disk/washer method, express as an integral the volume of the solid generated by revolving R about the x -axis.
- a. $\pi \int_0^2 (1 - \frac{x}{2}) dx$ b. $\pi \int_0^2 (1 - \frac{x}{2})^2 dx$ c. $\pi \int_0^1 (2 - 2y) dy$ d. $\pi \int_0^1 (2 - 2y)^2 dy$ e. none of these
- 9.** Using the shell method, express as an integral the volume of the solid generated by revolving R about the x -axis.
- a. $2\pi \int_0^1 y(2 - 2y) dy$ b. $2\pi \int_0^2 y(2 - 2y) dy$ c. $\pi \int_0^1 (2 - 2y)^2 dy$ d. $2\pi \int_0^2 y(2 - 2y)^2 dy$
e. none of these
- 10.** Using the shell method, express as an integral the volume of the solid generated by revolving R about the line $y = 3$.
- a. $2\pi \int_0^1 (3 - y)(2 - 2y) dy$ b. $2\pi \int_0^3 (3 - y)(2 - 2y) dy$ c. $\pi \int_0^1 (3 + y)(2 - 2y) dy$
d. $2\pi \int_0^3 (3 + y)(2 - 2y) dy$ e. none of these
-

The region R for problems 11–14

Let R be the region in the first quadrant enclosed by

$$y = x^2 \quad \text{and} \quad y = x + 2 \quad \text{and} \quad \text{the } y\text{-axis} .$$

Hint: in the first quadrant, the parabola $y = x^2$ and the line $y = x + 2$ intercept at the point $(2, 4)$.

- 11.** Express the area of R as integral(s) with respect to x .
- a. $\int_0^2 [(x^2) + (x + 2)] dx$ b. $\int_0^2 [(x + 2) + (x^2)] dx$
c. $\int_0^2 [(x^2) - (x + 2)] dx$ d. $\int_0^2 [(x + 2) - (x^2)] dx$ e. none of these
- 12.** Express as an integral, in terms of x , the volume of the solid generated by revolving R about the x -axis.
- a. $\pi \int_0^2 [(x + 2)^2 + (x^2)^2] dx$ b. $\pi \int_0^2 [(x + 2) + (x^2)]^2 dx$
c. $\pi \int_0^2 [(x + 2)^2 - (x^2)^2] dx$ d. $\pi \int_0^2 [(x + 2) - (x^2)]^2 dx$ e. none of these
- 13.** Express the area of R as integral(s) with respect to y .
- a. $\int_0^4 (\sqrt{y} - (y - 2)) dy$ b. $\int_0^4 ((y - 2) - \sqrt{y}) dy$
c. $\int_0^2 \sqrt{y} dy + \int_2^4 [\sqrt{y} - (y - 2)] dy$ d. $\int_0^2 \sqrt{y} dy + \int_2^4 [(y - 2) - \sqrt{y}] dy$ e. none of these
- 14.** Express as integral(s), in terms of y , the volume of the solid generated by revolving R about the x -axis.
- a. $2\pi \int_0^4 y (\sqrt{y} - (y - 2)) dy$ b. $2\pi \int_0^4 y ((y - 2) - \sqrt{y}) dy$
c. $2\pi \int_0^2 y \sqrt{y} dy + \int_2^4 [\sqrt{y} - (y - 2)] dy$ d. $2\pi \int_0^2 y \sqrt{y} dy + \int_2^4 [(y - 2) - \sqrt{y}] dy$
e. none of these
-