MARK BOX		
PROBLEM	POINTS	
1 a–k	35	
2	10	
3	10	
4	10	
5	10	
6	10	
7	15	
%	100	

NAME: ______________________

PIN: ______________________

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) If you do not make at least 17.5 out of the 35 points on Problem 1, then your score for the entire exam will be whatever you made on Problem 1.

(6) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): 11.2–11.8 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

1. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

1a. $n^{\textbf{th}}$**-term test** for an arbitrary series $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ __________________ .

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ __________________
- diverges if and only if $|r|$ __________________

1c. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p __________________
- diverges if and only if p __________________

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f($_________$)$ for each $n \in \mathbb{N}$
- f is a ____________________________________ function
- f is a ____________________________________ function
- f is a ____________________________________ function .

Then $\sum a_n$ converges if and only if ____________________________________ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$. (Fill in the blanks with a_n and/or b_n.)

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum$ _________ converge, then $\sum$ _________ converge.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum$ _________ diverge, then $\sum$ _________ diverge.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If _________ $< L <$ _________, then $\sum a_n$ converges if and only if __________________ .

1g. Ratio and Root Tests for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$.

Let $\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ or $\rho = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$.

- If ρ _________ then $\sum a_n$ converges absolutely.
- If ρ _________ then $\sum a_n$ diverges.
- If ρ _________ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum(-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ _________

then $\sum(-1)^n a_n$ ___________________________

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is <u>absolutely convergent</u> if and only if $\sum |a_n|$ ____________________
- $\sum a_n$ is <u>conditionally convergent</u> if and only if $\sum a_n$ __________ and $\sum |a_n|$ __________
- $\sum a_n$ is <u>divergent</u> if and only if $\sum a_n$ ____________________

1j. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

Does $\sum |a_n|$ converge? Since $|a_n| \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.	if NO $\Rightarrow$	Does $\lim_{n\to\infty} |a_n| = 0$?	if NO $\Rightarrow$	$\sum a_n$ is ________________
if YES $\Downarrow$		if YES $\Downarrow$		
$\sum a_n$ is ______________________________		Is $\sum a_n$ an alternating series?		
		if YES $\Downarrow$		
		Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?		
		if YES $\Downarrow$		
		$\sum a_n$ is ______________________________		

1k. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

T F If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

T F If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.

T F If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $S_N = \sum_{n=1}^{N} r^n$, then $S_N = \dfrac{r - r^N}{1 - r}$, for when $r \neq 1$ since we don't like to divide by zero.

2. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{4n^7 - n^6 + 21}{11n^8 - n + 17}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

You may use, without showing, that for $n \geq 1$

$$a_n := \frac{4n^7 - n^6 + 21}{11n^8 - n + 17} > 0 \, .$$

Hint. What would be easier: CT or LCT?

4. Let

$$a_n = \frac{n!}{(2n-1)!}$$

4a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} =$$

4b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n-1)!}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{3n^2+8n-1}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

7. Geometric Series. (On this page, you should do basic algebra but you do NOT have to do any fancy arithmetic (eg, just making up numbers, you can leave $(\frac{17}{18})^{171}$ as just that.) Let, for $N \geq 51$,

$$s_N \;=\; \sum_{n=51}^{N} 2\frac{3^{n+1}}{5^n}\;.$$

7a. Do some algebra to write s_N as $\sum_{n=51}^{N}\; c\, r^n$ for an appropriate constant c and ratio r.

$$s_N \;=\; \sum_{n=51}^{N}$$

7b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation $\sum$ sign nor some dots . . . .).

$$s_N \;=$$

7c. Does $\sum_{n=51}^{\infty} 2\frac{3^{n+1}}{5^n}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=51}^{\infty} 2\frac{3^{n+1}}{5^n}$$