[redacted]

Prof. Girardi   Math 142   Fall 2012   09.27.12   Exam 1

MARK BOX		
PROBLEM	POINTS	
1	25	
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
TOTAL	90	
%	100	

NAME: Key

class PIN: 17

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  (b) if a line/box is provided, then:
    — show you work BELOW the line/box
    — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use an electronic device, a calculator, books, personal notes.
(4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) If you do not make at least 12.5 out of 25 points on Problem 1, then your score for the entire exam will be whatever you made on Problem 1.
(6) This exam covers (from *Calculus* (ET) by Stewart 6^{th} ed.): Sections 7.1 – 7.5, 7.8, 11.1 .

Hints:

(1) **You can check your answers to the indefinite integrals by differentiating.**
(2) **For more partial credit, box your $u - du$ substitutions.**

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

1. Fill in the blanks (each worth 1 point).

1a. $\int \frac{du}{u}$ = ln $|u| + C$

1b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u \, du$ = $\frac{a^u}{\ln a}$ $+C$

1c. $\int \cos u \, du$ = sin u $+C$

1d. $\int \sec^2 u \, du$ = tan u $+C$

1e. $\int \sec u \tan u \, du$ = sec u $+C$

1f. $\int \sin u \, du$ = − cos u $+C$

1g. $\int \csc^2 u \, du$ = − cot u $+C$

1h. $\int \csc u \cot u \, du$ = − csc u $+C$

1i. $\int \tan u \, du$ = $-\ln|\cos u| + C$ or $\ln|\sec u|$ $+C$

1j. $\int \cot u \, du$ = $\ln|\sin u| + C$ or $-\ln|\csc u|$ $+C$

1k. $\int \sec u \, du$ = $\ln|\sec u + \tan u| + C$ or $-\ln|\sec u - \tan u|$ $+C$

1l. $\int \csc u \, du$ = $-\ln|\csc u + \cot u| + C$ or $\ln|\csc u - \cot u|$ $+C$

1m. If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} du$ = $\sin^{-1}(u/a)$ $+C$

1n. If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} du$ = $\frac{1}{a}\tan^{-1}(u/a)$ $+C$

1o. If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} du$ = $\frac{1}{a}\sec^{-1}(|u|/a)$ $+C$

1p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do long division

1q. Integration by parts formula: $\int u \, dv$ = $uv - \int v \, du$

1r. Trig substitution: (recall that the *integrand* is the function you are integrating) if the integrand involves $a^2 - u^2$, then one makes the substitution u = $a \sin\theta$

1s. Trig substitution: if the integrand involves $a^2 + u^2$, then one makes the substitution u = $a \tan\theta$

1t. Trig substitution: if the integrand involves $u^2 - a^2$, then one makes the substitution u = $a \sec\theta$

1u. trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta)$ = $2\sin\theta\cos\theta$.

1v. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) = \frac{1}{2}$ $(1 + \cos 2\theta)$.

1w. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) = \frac{1}{2}$ $(1 - \cos 2\theta)$.

1x. trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is $1 + \tan^2\theta = \sec^2\theta$

1y. $\arcsin(-\frac{\sqrt{2}}{2})$ = $-\pi/4$ **RADIANS.** (your answer should be an angle)

2. $\int (\sin x)(\sec x)\, dx = -\ln|\cos x| + C \overset{\text{or}}{=} \ln|\sec x| + C$ $+ C$

③ $\int (\sin x)(\sec x)\, dx = \int \frac{\sin x}{\cos x}\, dx = -\int \frac{du}{u}$

$u = \cos x$

$du = -\sin x\, dx$

$= -\ln|u| + C = \boxed{-\ln|\cos x| + C}$

$\overset{\text{or}}{=} +\ln(|\cos x|)^{-1} + C = \ln \frac{1}{|\cos x|} + C = \boxed{\ln|\sec x| + C}$

3. $\int x \tan^2 x \, dx = x\tan x - \frac{x^2}{2} + \ln|\cos x|$ $+ \mathrm{C}$

3 $\int x \tan^2 x \, dx = x \tan x - x^2 - \int (\tan x - x)\, dx$

$u = x \qquad dv = \tan^2 x \, dx = (\sec^2 x - 1)\, dx$

$du = dx \qquad v = (\tan x - x)$

$= x \tan x - x^2 - \ln|\sec x| + \frac{x^2}{2} + C$

$= \boxed{x \tan x + \frac{x^2}{2} + \ln|\cos x| + C}$

4. $\int \ln(1+x)\,dx = (x+1)\ln|1+x| - x \qquad + C$

Hint: bring to the other side idea.

[redacted] 4. $\int \ln(1+x)\,dx = x\ln(1+x) - \int \frac{x}{1+x}\,dx \overset{LD}{=} x\ln(1+x) - \int \frac{1+x-1}{1+x}\,dx$

$u = \ln(1+x) \qquad dv = dx$

$du = \frac{dx}{1+x} \qquad v = x$

$= x\ln(1+x) - \int\left(1 - \frac{1}{1+x}\right)dx$

$= x\ln(1+x) - x + \ln(1+x) + C$

$= \boxed{(x+1)\ln(1+x) - x + C}$

or

$u = \ln(1+x) \qquad dv = dx$

$du = \frac{1}{1+x}\,dx \qquad v = 1+x$

$\int \ln(1+x)\,dx = (1+x)\ln(1+x) - \int \frac{1+x}{1+x}\,dx$

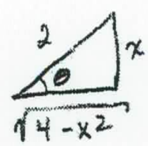
$= (1+x)\ln(1+x) - \int dx$

$= \boxed{(1+x)\ln(1+x) - x + C}$

5. $\boxed{\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx = 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} \qquad + \mathrm{C}}$

[redacted] $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

$x = 2\sin\theta \longrightarrow \sin\theta = \frac{x}{2}$



$dx = 2\cos\theta\, d\theta$

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\cos\theta$

$$\int \frac{x^2\,dx}{\sqrt{4-x^2}} = \int \frac{4\sin^2\theta \;\; 2\cos\theta\, d\theta}{2\cos\theta} = 4\int \sin^2\theta\, d\theta$$

$$= 4\cdot\frac{1}{2}\int(1-\cos 2\theta)\,d\theta = 2\int d\theta - 2\,\frac{1}{2}\int(\cos(2\theta))\,(2\,d\theta)$$

$$= 2\theta - \sin 2\theta + c = 2\theta - 2\sin\theta\cos\theta + c$$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) - 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + c$$

$$= \boxed{2\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} + c}$$

6.

$$\boxed{\int \frac{x^4+2x+2}{x^4(x+1)}\,dx = \frac{2x^{-3}}{-3} + \ln|x+1| \qquad + C}$$

Hint $x^4 = (x-0)^4$

$$\frac{x^4+2x+2}{x^4(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+1}$$ ← Note $x^4 = (x-0)^4$

$$\Rightarrow \frac{x^4+2x+2}{x^4(x+1)} = \frac{Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4}{x^4(x+1)}$$

$$\Rightarrow x^4 + 2x + 2 = Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4$$

$\boxed{x = 0 \Rightarrow 2 = D}$ $\boxed{x = -1 \Rightarrow 1 = E}$

x^4 : $1 = A + E \longrightarrow A = 0$

x^3 : $0 = A + B$

x^2 : $0 = B + C$ $\longrightarrow B = 0$

x : $2 = C + D \longrightarrow C = 0$

constant : $2 = D$

$$\int \frac{x^4+2x+2}{x^5+x^4}\,dx = \int \left(\frac{2}{x^4} + \frac{1}{x+1}\right) dx$$

$$= \int 2x^{-4}\,dx + \int \frac{dx}{x+1} = \boxed{\frac{2x^{-3}}{-3} + \ln|x+1| + c}$$

7. $\int_1^\infty \frac{1}{(3x+1)^4}\,dx = \frac{1}{576}$

Warning: write your solution in proper form.

$$\int_1^\infty \frac{1}{(3x+1)^4}\,dx$$

$$\lim_{c\to\infty}\int_1^c \frac{1}{(3x+1)^4}\,dx = \lim_{c\to\infty} -\frac{1}{9(3x+1)^3}\Big|_1^c$$

$$= 0-\left(-\frac{1}{9(3+1)^3}\right)$$

$$= 0-\left(-\frac{1}{9\cdot 64}\right)$$

$$= 0+\frac{1}{576}$$

$$\boxed{=\frac{1}{576}}$$

$$\int \frac{1}{(3x+1)^4}\,dx \qquad u=3x+1 \quad du=3\,dx$$

$$\frac{1}{3}\int \frac{1}{u^4}\,du$$

$$\frac{1}{3}\int u^{-4}\,du$$

$$= \frac{1}{3}\,\frac{u^{-3}}{-3}+C$$

$$= -\frac{1}{9(3x+1)^3}+C$$

~~All~~ are $\frac{\text{poly}}{\text{poly}}$ so divide thru by n (highest power)

8 ~~9~~. For the following **SEQUENCES** in 1–5:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

$\div n^3$ (8a) ~~a~~.

$$\boxed{\lim_{n\to\infty} \frac{5n^2+4\sqrt{n}}{6n^3+7n^2+1} = 0}$$

$$\lim_{n\to\infty} \frac{5n^2+4\sqrt{n}}{6n^3+7n^2+1} = \lim_{n\to\infty} \frac{\frac{5n^2}{n^3}+\frac{4n^{1/2}}{n^3}}{\frac{6n^3}{n^3}+\frac{7n^2}{n^3}+\frac{1}{n^3}} =$$

$$\lim_{n\to\infty} \frac{\frac{5}{n}+\frac{4}{n^{5/2}}}{6+\frac{7}{n}+\frac{1}{n^3}} = \frac{0+0}{6+0+0} = 0$$

$\div n^8$ (8b) ~~b~~.

$$\boxed{\lim_{n\to\infty} \frac{5n^8+4\sqrt{n}}{6n^3+7n^2+1} = \infty \quad \text{or} \quad \text{DNE}}$$

$$\lim_{n\to\infty} \frac{5n^8+4\sqrt{n}}{6n^3+7n^2+1} = \lim_{n\to\infty} \frac{\frac{5n^8}{n^8}+\frac{4n^{1/2}}{n^8}}{\frac{6n^3}{n^8}+\frac{7n^2}{n^8}+\frac{1}{n^8}} =$$

$$\lim_{n\to\infty} \frac{5+\frac{4}{n^{15/2}}}{\frac{6}{n^5}+\frac{7}{n^6}\ \frac{1}{n^8}} = \cancel{\frac{5+0}{0+0+0}} = \cancel{\frac{5}{0}} = \cancel{+\infty} = \infty$$

$\div n^3$ (8c) ~~c~~.

$$\boxed{\lim_{n\to\infty} \frac{5n^3+4\sqrt{n}}{6n^3+7n^2+1} = \frac{5}{6}}$$

$$\lim_{n\to\infty} \frac{5n^3+4\sqrt{n}}{6n^3+7n^2+1} = \lim_{n\to\infty} \frac{\frac{5n^3}{n^3}+\frac{4n^{1/2}}{n^3}}{\frac{6n^3}{n^3}+\frac{7n^2}{n^3}+\frac{1}{n^3}} =$$

$$\lim_{n\to\infty} \frac{5+\frac{4}{n^{5/2}}}{6+\frac{7}{n}+\frac{1}{n^3}} = \frac{5+0}{6+0+0} = \frac{5}{6}$$