NAME: ______________________________ **PIN**:________

Prof. Girardi	Math 142	Fall 2011	12.09.11	Final Exam

INSTRUCTIONS:

(1) This test consists of 25 multiple choice problems, each worth 4 points. The exam is copied two sided. You should turn in just this top piece of paper, with your answers indicated on the back of this page. You can take home the rest of the exam.

(2) You may **not** use a calculator, books, personal notes.

(3) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(4) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
§: 7.1 - 7.5. 7.8, 11.1 - 11.9, 6.1-6.3, 10.3-10.4 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

Mark your solution with an X.

Your Solution Table					
PROBLEM					
1	X	b	c	d	e
2	X	b	c	d	e
3	X	b	c	d	e
4	a	b	c	X	e
5	a	X	c	d	e
6	X	b	c	d	e
7	a	b	X	d	e
8	a	b	X	d	e
9	a	X	c	d	e
10	a	b	c	X	e
11	a	X	c	d	e
12	a	X	c	d	e
13	X	b	c	d	e
14	a	b	c	X	e
15	a	b	X	d	e
16	a	X	c	d	e
17	X	b	c	d	e
18	a	b	X	d	e
19	a	X	c	d	e
20	a	X	c	d	e
21	a	b	X	d	e
22	a	b	c	X	e
23	X	b	c	d	e
24	a	X	c	d	e
25	X	b	c	d	e

$$\int_{x=0}^{x=1} \frac{x}{x^2+9}\,dx = \frac{1}{2}\int_{u=9}^{u=10} \frac{du}{u} = \frac{1}{2}\ln|u|\,\Bigg|_{u=9}^{u=10}$$

$$= \frac{1}{2}\ln 10 - \frac{1}{2}\ln 9$$

$$\boxed{\begin{array}{l} u = x^2+9 \\ du = 2x\,dx \\ \hline x=0 \Rightarrow u=9 \\ x=1 \Rightarrow u=10 \end{array}}$$

$$\int_0^4 \frac{x}{x+9}\,dx$$

Do not have strictly bigger bottoms so need to do long division.
But it's easy to "fake" long division here:

$$\frac{x}{x+9} = \frac{x+9-9}{x+9} = \frac{x+9}{x+9} - \frac{9}{x+9} = 1 - \frac{9}{x+9}.$$

So

$$\int_0^4 \frac{x}{x+9}\,dx = \int_0^4 \left[1 - \frac{9}{x+9}\right] dx$$

$$= \Big[x - 9 \ln|x+9| \Big] \Big|_{x=0}^{x=4}$$

$$= (4 - 9\ln 13) - (0 - 9 \ln 9)$$

$$= 4 - 9 \ln 13 + 9 \ln 9.$$

$$\int_0^1 \frac{1}{x^p}\,dx = \int_0^1 x^{-p}\,dx \overset{p\neq 1}{=\!=} \left.\frac{x^{-p+1}}{-p+1}\right|_{x=0}^{x=1}$$

$$= \frac{1}{1-p} - 0 = \frac{1}{1-p}.$$

Want $\dfrac{1}{1-p} = 1.25$

$$\frac{1}{1-p} = 1\tfrac{1}{4}$$

$$\frac{1}{1-p} = \frac{5}{4}$$

$$1-p = \frac{4}{5}$$

$$\frac{4}{5} = 1-p$$

$$p = 1 - \frac{4}{5} = \frac{1}{5} = \frac{2}{10} = \boxed{0.2}$$

$$\int_0^4 (x^2+1)\, e^{-x}\, dx$$

//

Step 1 $\displaystyle \int x^2 e^{-x}\, dx = -x^2 e^{-x} - (-2) \int x e^{-x}\, dx$

$u = x^2 \qquad dv = e^{-x}\, dx$

$du = 2x\, dx \qquad v = -e^{-x}$

$$= -x^2 e^{-x} + 2 \int x e^{-x}\, dx$$

$$= -x^2 e^{-x} + 2\left[-x e^{-x} - - \int e^{-x}\, dx\right]$$

$u = x \qquad dv = e^{-x}\, dx$

$du = dx \qquad v = -e^{-x}$

$$= -x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x}\, dx$$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C$$

//

So $\displaystyle \int (x^2+1) e^{-x}\, dx = \int x^2 e^{-x}\, dx + \int e^{-x}\, dx$

$$= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} - e^{-x} + C$$

$$= -x^2 e^{-x} - 2x e^{-x} - 3e^{-x} + C$$

$$= (-e^{-x})(x^2 + 2x + 3).$$

//

So $\displaystyle \int_0^4 (x^2+1) e^{-x}\, dx = (-e^{-x})(x^2+2x+3) \Big|_{x=0}^{x=4}$

$$= (-e^{-4})(16+8+3) - (-1)(3)$$

$$= -27e^{-4} + 3.$$

Similar to an example from class.

$$\boxed{\int \sin^2 x \cos^3 x \, dx = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}$$

$$\cancel{\begin{array}{l} s = \cos x \\ ds = -\sin x \, dx \end{array}} \qquad \boxed{\begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array}}$$

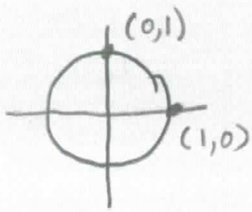
↑ will not work

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x \boxed{\cos x \, dx}$$

$$= \int \sin^2 x \, (1 - \sin^2 x) \boxed{\cos x \, dx}$$

$$= \int t^2 (1 - t^2) \, dt$$

$$= \int (t^2 - t^4) \, dt$$



$$= \frac{t^3}{3} - \frac{t^5}{5} + C$$

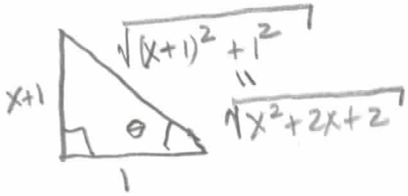
$$= \frac{(\sin x)^3}{3} - \frac{(\sin x)^5}{5} + C$$

So $\displaystyle\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx = \left[\frac{1}{3} - \frac{1}{5} \right] - [0 - 0] = \frac{5-3}{15} = \frac{2}{15}$.

$$\int \frac{dx}{(x^2+2x+2)^2} = \int \frac{\sec^2\theta \, d\theta}{(\sec^2\theta)^2} = \int \frac{\sec^2\theta}{\sec^4\theta}\, d\theta$$

$$x^2+2x+2 = (x+1)^2+1$$

$$x+1 = \tan\theta \quad \rightarrow$$



$$dx = \sec^2\theta \, d\theta$$

$$x^2+2x+2 = (x+1)^2+1 = \tan^2\theta+1 = \sec^2\theta$$

$$= \int \frac{1}{\sec^2\theta}\, d\theta$$

$$= \int \cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C = \frac{1}{2}\left[\theta + \frac{1}{2}\cdot 2\cos\theta\sin\theta\right] + C$$

$$= \frac{1}{2}\theta + \cos\theta \sin\theta + C$$

$$= \frac{1}{2}\arctan(x+1) + \frac{1}{\sqrt{x^2+2x+2}} \cdot \frac{x+1}{\sqrt{x^2+2x+2}} + C$$

$$= \frac{1}{2}\arctan(x+1) + \frac{x+1}{x^2+2x+2} + C,$$

Problem Source: Textbook, § 7.4, # 15

$$\boxed{\int \frac{x^3-2x^2-4}{x^3-2x^2}\,dx = x + \ln|x| - \frac{2}{x} - \ln|x-2| \qquad + C}$$

PFD

Hint: Do we have (Strictly) Bigger Bottoms ?

NO so need to do long division ... but it's easy to "fake" long division on this one (lucky us)

$$\frac{x^3-2x^2-4}{x^3-2x^2} = \frac{x^3-2x^2}{x^3-2x^2} + \frac{-4}{x^3-2x^2} = 1 + \frac{-4}{x^3-2x^2}$$

Find PFD for $\frac{-4}{x^3-2x^2} = \frac{-4}{x^2(x-2)} = \frac{-4}{\underbrace{(x-0)^2}_{(\text{linear term})^2}\underbrace{(x-2)}_{(\text{linear term})^1}}$

$$\frac{-4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} = \frac{Ax(x-2)+B(x-2)+Cx^2}{x^2(x-2)}$$

$$\Rightarrow \boxed{-4 = Ax(x-2)+B(x-2)+Cx^2} \quad \begin{array}{l} \xrightarrow{x=0} -4 = -2B \Rightarrow \boxed{B=2} \\ \xrightarrow{x=2} -4 = C\cdot 2^2 \Rightarrow \boxed{C=-1} \end{array}$$

equate coeff

$x^2: 0 = A + C \xrightarrow{C=-1} \boxed{A = 1}$

$x^1: 0 = -2A + B$

$x^0: -4 = -2B$

$$\int \frac{x^3-2x^2-4}{x^3-2x^2}\,dx = \int \left[1 + \frac{1}{x} + \frac{2}{x^2} + \frac{-1}{x-2}\right] dx$$

$$\Downarrow$$

$$\int 2x^{-2}\,dx = \frac{2x^{-1}}{-1} + C$$

So

$$\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx = \left[4 + \ln 4 - \frac{2}{4} - \ln 2\right] - \left[3 + \ln 3 - \frac{2}{3} - \ln 1\right]$$

$$= \ln 4 - \ln 2 - \ln 3 + 4 - \frac{1}{2} - 3 + \frac{2}{3} = \left(\ln\frac{4}{6}\right) + \frac{7}{6}$$

$$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \int \frac{x^2}{2} \frac{1}{x} \, dx$$

$$\boxed{\begin{array}{ll} u = \ln x & dv = x\,dx \\ du = \frac{dx}{x} & v = \frac{x^2}{2} \end{array}}$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{2 \cdot 2} + C$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

so

$$\int_1^e x \ln x \, dx = \left[\frac{x^2 \ln x}{2} - \frac{x^2}{4} \right]_{x=1}^{x=e}$$

$$= \left[\frac{e^2 \ln e}{2} - \frac{e^2}{4} \right] - \left[\frac{1 \ln 1}{2} - \frac{1}{4} \right]$$

$$= \frac{e^2}{2} - \frac{e^2}{4} - 0 + \frac{1}{4}$$

$$= \left(\frac{e^2}{4} + \frac{1}{4} \right)$$

$$\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \boxed{\frac{1}{36}}$$

Warning: write your sol'n in proper form.

$$\int_1^\infty \frac{dx}{(2x+1)^3} = \lim_{b\to\infty} \frac{1}{2}\int_{x=1}^{x=b} (2x+1)^{-3}\,(2dx)$$

$$= \lim_{b\to\infty} \frac{1}{2}\,\frac{(2x+1)^{-2}}{-2}\,\Bigg|_{x=1}^{x=b}$$

$$= -\frac{1}{4}\lim_{b\to\infty} \frac{1}{(2x+1)^2}\,\Bigg|_{x=1}^{x=b}$$

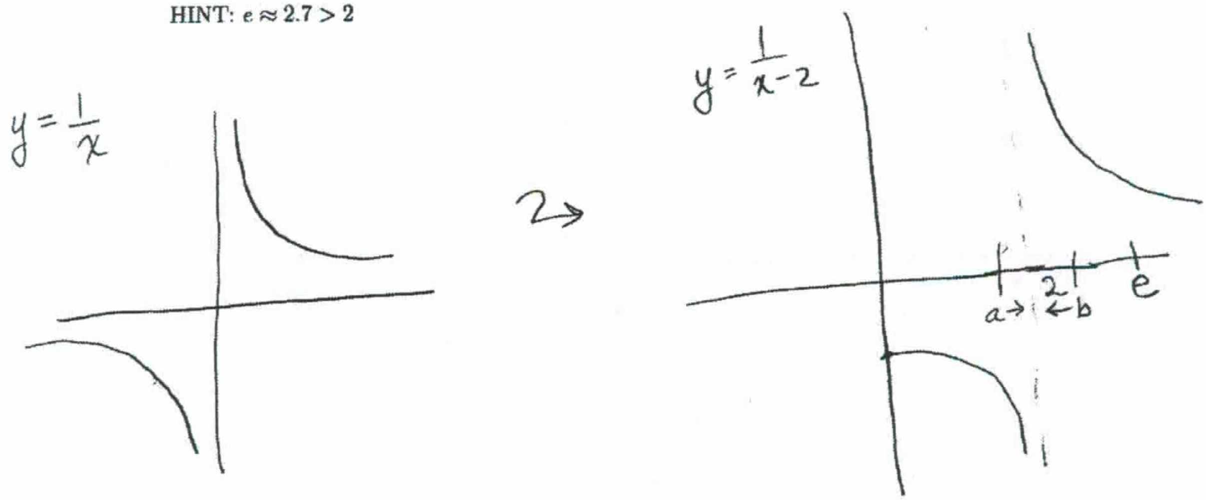
$$= -\frac{1}{4}\lim_{b\to\infty}\left[\frac{1}{(2b+1)^2} - \frac{1}{3^2}\right]$$

$$= -\frac{1}{4}\left[0 - \frac{1}{3^2}\right]$$

$$= \frac{1}{4}\cdot\frac{1}{9}$$

$$\boxed{\int_0^e \frac{dx}{x-2} = \text{DNE}}$$

HINT: $e \approx 2.7 > 2$



$$\int_0^e \frac{dx}{x-2} = \lim_{a\to 2^+} \int_0^a \frac{dx}{x-2} + \lim_{b\to 2^-} \int_b^e \frac{dx}{x-2}$$

$$= \lim_{a\to 2^+} \ln|x-2| \Big|_{x=0}^{x=a} + \lim_{b\to 2^-} \ln|x-2| \Big|_{x=b}^{x=e}$$

$$= \lim_{a\to 2^+} \left[\ln|a-2| - \ln 2 \right] + \lim_{b\to 2^-} \left[\ln|e-2| - \ln|b-2| \right]$$

$$= \underbrace{\left[\lim_{a\to 2^+} \ln|a-2| \right]}_{=-\infty} - \ln 2 + \ln|e-2| - \underbrace{\lim_{b\to 2^-} \ln|b-2|}_{=-\infty}$$

[redacted]. Compute

$$\lim_{n\to\infty} \frac{17n^3+4n^2-5}{19n^5+3n^4-8n^3+n^2-8}.$$

divide through by n (to the highest power) $= n^5$

$$\lim_{n\to\infty} \frac{17n^3+4n^2-5}{19n^5+3n^4-8n^3+n^2-8}$$

$$= \lim_{n\to\infty} \frac{\frac{17}{n^2}+\frac{4}{n^3}-\frac{5}{n^5}}{19+\frac{3}{n}-\frac{8}{n^2}+\frac{1}{n^3}-\frac{8}{n^5}}$$

$$= \frac{0+0+0}{19+0-0+0-0} = \frac{0}{19} = 0.$$

$$\lim_{n\to\infty} \frac{\sqrt{9n^4+1}}{17n^2+n+3}$$

$$= \lim_{n\to\infty} \frac{\dfrac{\sqrt{9n^4+1}}{\sqrt{n^4}}}{\dfrac{17n^2+n+3}{n^2}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{9+\frac{1}{n^4}}}{17+\frac{1}{n}+\frac{3}{n^2}}$$

$$= \frac{\sqrt{9}}{17}$$

$$= \frac{3}{17}$$

13. $\lim_{n \to \infty} \left(-\frac{1}{2}\right)^n = 0$

Geometric Series

$r = -\frac{1}{2}$

$|r| = \left|-\frac{1}{2}\right| < 1$

so converges to zero.

$\sum \frac{1}{n}$ divgeres. $\left(\begin{array}{l} \text{harmonic series} \quad = \text{or} = \\ \text{p-series} \quad p = 1 \leq 1 \end{array} \right)$

=

Now consider

$\sum \frac{(-1)^n}{n}$

Let $u_n = \frac{1}{n}$.

- u_n is decreasing
- $\lim_{n \to \infty} u_n = 0$

So by AST, $\sum \frac{(-1)^n}{n}$ conv.

=

So $\sum \frac{1}{n}$ divg. and $\sum \frac{(-1)^n}{n}$ conv. cond.

Problem Inspiration : 09 Fall final #5

Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}}$$

- ☐ ~~absolutely convergent~~ LCT w/ $b_n = \frac{1}{n}$
- ☒ conditionally convergent then use AST
- ☐ divergent

Warning: there is a square root in the denominator ... many of you overlooked $\sqrt{\ }$'s on Exams ... see it?.

Abs. Conv? Consider $\sum \left| (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}} \right| = \sum \frac{1}{\sqrt{(n+2)(n+7)}}$

Thinking Land $a_n = \frac{1}{\sqrt{(n+2)(n+7)}} \overset{n \text{ big}}{\approx} \frac{1}{\sqrt{n \cdot n}} = \frac{1}{n} = b_n$

LCT. $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{\sqrt{(n+2)(n+7)}} = 1$

more details $= \lim_{n\to\infty} \sqrt{\left[\frac{n^2}{(n+2)(n+7)}\right]} = \sqrt{1} = 1 = L$, $0 < L < \infty$

so $\sum b_n$ & $\sum a_n$ do the same thing. $\sum b_n$ divg (harmonic series)

so $\sum \left| (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}} \right|$ dvg so not abs. conv.

Cond. Conv? Let's use AST w/ $0 \le u_n = \frac{1}{\sqrt{(n+2)(n+7)}}$

(1) u_n dec., ie $u_n > u_{n+1}$? yes clear.

(2) $\lim_{n\to\infty} \frac{1}{\sqrt{(n+2)(n+7)}} = 0$ ☺

so, by AST, $\sum (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}}$ conv.

Problem Inspiration : Spring 2010, Ex 2, # 5

10. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=8}^{\infty} (-1)^n \frac{(n+1)!}{(2n)!}$$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

But before you get started .... let

$$a_n = \frac{(n+1)!}{(2n)!}.$$

Then $a_{n+1} = \dfrac{((n+1)+1)!}{(2(n+1))!} = \dfrac{(n+2)!}{(2n+2)!}$

Next, simplify $\frac{a_{n+1}}{a_n}$ so that it has NO factorial sign (that is a ! sign) in it.

$\dfrac{a_{n+1}}{a_n} = \dfrac{n+2}{(2n+1)(2n+2)}$ or $\dfrac{n+2}{4n^2+6n+2}$

Ok, now you should be ready to finish off the problem and check the correct box above.

$$\frac{a_{n+1}}{a_n} = \frac{(n+2)!}{(2n+2)!} \cdot \frac{(2n)!}{(n+1)!} = \frac{(n+2)!}{(n+1)!} \frac{(2n)!}{(2n+2)!} = \frac{(n+1)!\,(n+2)}{(n+1)!} \cdot \frac{(2n)!}{(2n)!\,(2n+1)(2n+2)}$$

Abs. Conv? Consider $\sum \frac{(n+1)!}{(2n)!}$ & use Ratio Test

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \overset{\text{above}}{=} \lim_{n\to\infty} \frac{n+2}{(2n+1)(2n+2)} = 0 < 1$$

$\Downarrow$ Ratio Test

conv.

$$\sum_{n=10}^{\infty} \frac{3^{n+1}}{4^n} = \sum_{n=10}^{\infty} 3 \cdot \left(\frac{3}{4}\right)^n$$

Let $S_N = \sum_{n=10}^{N} 3 \cdot \left(\frac{3}{4}\right)^n$.

$$1 \quad S_N = 3\left[\left(\tfrac{3}{4}\right)^{10} + \left(\tfrac{3}{4}\right)^{11} + \dots + \left(\tfrac{3}{4}\right)^{N}\right]$$

$$\tfrac{3}{4} \quad S_N = 3\left[\qquad \left(\tfrac{3}{4}\right)^{11} + \dots + \left(\tfrac{3}{4}\right)^{N} + \left(\tfrac{3}{4}\right)^{N+1}\right]$$

$$\left(1-\tfrac{3}{4}\right) S_N = 3\left[\left(\tfrac{3}{4}\right)^{10} - \left(\tfrac{3}{4}\right)^{N+1}\right]$$

$$\tfrac{1}{4} \quad S_N = 3\left[\left(\tfrac{3}{4}\right)^{10} - \left(\tfrac{3}{4}\right)^{N+1}\right]$$

$$S_N = 12\left[\left(\tfrac{3}{4}\right)^{10} - \left(\tfrac{3}{4}\right)^{N+1}\right]$$

$$\lim_{N\to\infty} S_N = 12 \cdot \left(\frac{3}{4}\right)^{10}$$

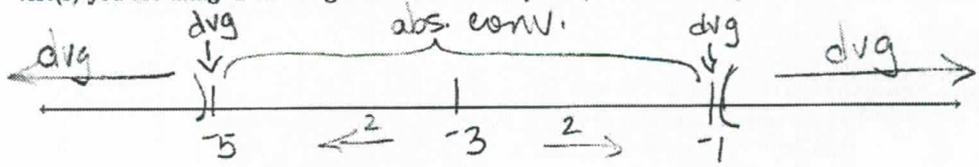
Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n}$$

Hint: $(2x+6)^n = [2(x+3)]^n = 2^n(x+3)^n = 2^n(x-(-3))^n$

The center is $x_0 =$ -3 and the radius of convergence is $R =$ 2.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



Ratio Test $\quad \rho = \lim_{n\to\infty} \left| \frac{(2x+6)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{(2x+6)^n} \right| = \lim_{n\to\infty} \frac{|2x+6|}{4} = \frac{|2x+6|}{4}$

$\updownarrow$ or

Root Test. $\quad \rho = \lim_{n\to\infty} \left| \frac{(2x+6)^n}{4^n} \right|^{1/n} = \lim_{n\to\infty} \frac{|2x+6|}{4} = \frac{|2x+6|}{4}$

$\rho < 1 \iff |2x+6| < 4 \iff 2|x+3| < 4 \iff |x+3| < 2 \iff |x-(-3)| < 2$

endpts. $\quad -3+2 = -1$ and $-3-2 = -5$

Check endpts

$x = -1$: $\sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n} = \sum_{n=1}^{\infty} \frac{4^n}{4^n} = \sum_{n=1}^{\infty} 1 = 1+1+1+1+\dots = \infty$ dvg

$x = -5$ $\quad \sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n} = \sum_{n=1}^{\infty} \frac{(-4)^n}{4^n} = \sum_{n=1}^{\infty} \frac{(-1\cdot 4)^n}{4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{4^n}$

$= \sum_{n=1}^{\infty} (-1)^n = -1+1-1+1-1+1-1+\dots$

osc btw -1 & 0 $\Rightarrow$ dvg

Suppose that the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ is 16. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

a. 256

b. 4

c. 1

d. 16

e. none of these

$\sum c_n x^{2n} = \sum c_n (x^2)^n$ converges when:

$|x^2| < 16$

$|x|^2 < 16$

$|x| < 4$

20. In class we learned that, for each $x \in \mathbb{R}$,

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} .$$

Use this to find a Taylor expansion about the center $x_0 = 0$ (i.e., Maclaurin series) for

$$f(x) = x\cos(4x) .$$

a. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{n!}$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

d. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 4^{2n} x^{2n+1}}{(2n!}$

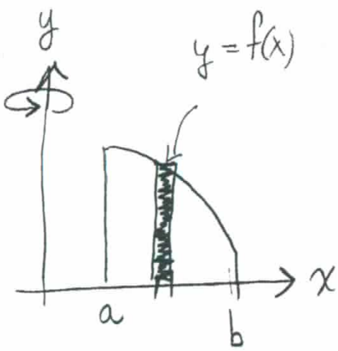
e. none of these

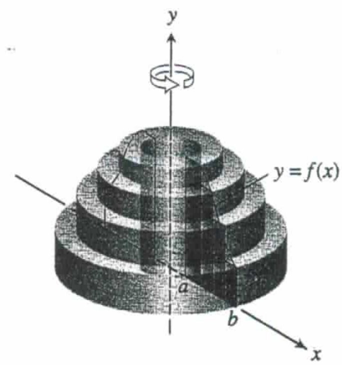
Since $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, then

$$\cos(4x) = \sum_{n=0}^{\infty} (-1)^n \frac{4^{2n} x^{2n}}{(2n)!} .$$

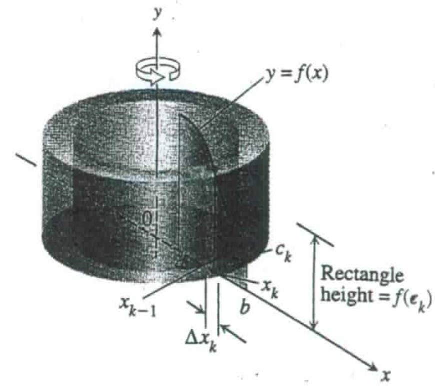
$\therefore$ $F(x) = x\cos(4x) = x \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$$





5.27 A solid of revolution approximated by cylindrical shells.



5.28 The shell swept out by the kth rectangle.

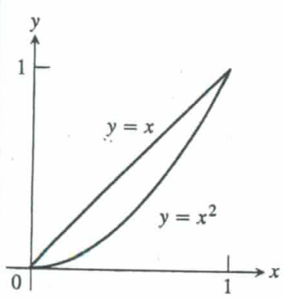
Volume of typical shell = $\Delta V_k = 2\pi \times$ average shell radius $\times$ shell height $\times$ thickness,

Table 5.1 Washers vs. shells

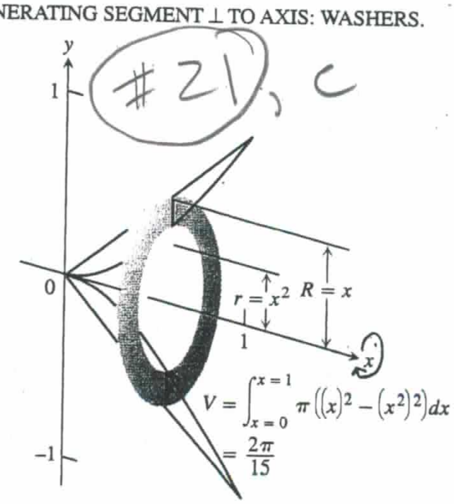
partition (|| -to) axis of revolution → GENERATING SEGMENT ⊥ TO AXIS: WASHERS.

parition ⊥-to axis of revolution → GENERATING SEGMENT || TO AXIS: SHELLS.

The region bounded by
$y = x,\ y = x^2$
or
$x = y,\ x = \sqrt{y}$

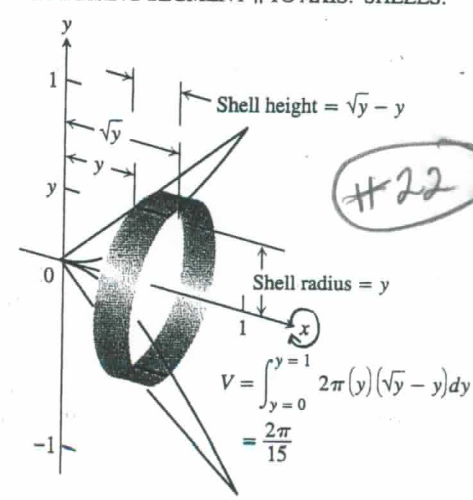


#21, c

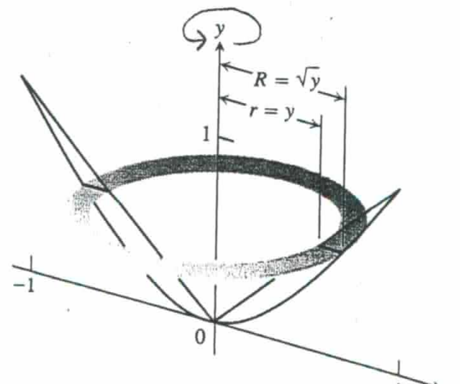


$$V = \int_{x=0}^{x=1} \pi\left((x)^2 - (x^2)^2\right)dx = \frac{2\pi}{15}$$

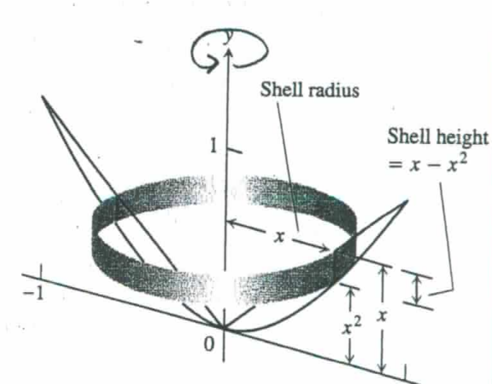
#22, d



$$V = \int_{y=0}^{y=1} 2\pi(y)(\sqrt{y} - y)dy = \frac{2\pi}{15}$$



$$V = \int_{y=0}^{y=1} \pi\left((\sqrt{y})^2 - (y)^2\right)dy = \frac{\pi}{6}$$



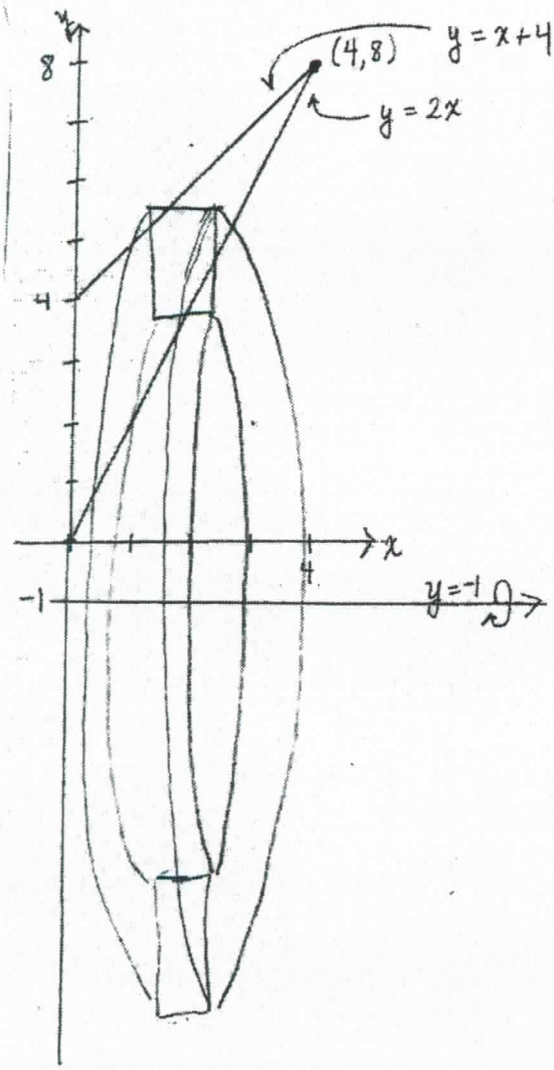
$$V = \int_{x=0}^{x=1} 2\pi(x)(x - x^2)dx = \frac{\pi}{6}$$

6.21

→ partition axis ∥ to axis of revolution ⇒ partition x-axis
have a "hole" so washer (not disk)

3 **7d.** Using the disk/washer method, express as integral(s) the volume of the solid generated by revolving R about the line $y = -1$.

Volume = $\displaystyle\int_{x=0}^{x=4} \pi \left[(x+5)^2 - (2x+1)^2 \right] dx$



Volume typical Washer

= Volume Big Washer
− Volume Little Washer

= π (Big Radius)^2 (height)
− π (little radius)^2 (height)

= $\pi\ ((x+4)+1)^2\ (\Delta x)$
− $\pi\ ((2x)+1)^2\ (\Delta x)$

~~Ex. 4~~ Convert the Cartesian equation $xy=1$ into a polar equations.

$$xy = 1 \iff (r\cos\theta)(r\sin\theta) = 1$$
$$\iff r^2 \cos\theta \sin\theta = 1$$
$$\iff \frac{r^2}{2}(2\cos\theta\sin\theta) = 1$$
$$\iff \frac{r^2}{2}(\sin 2\theta) = 1$$
$$\iff \boxed{r^2 \sin 2\theta = 2}$$

← ok, but can you do better (to help with graphing) try to write it using only one trig. function.

24

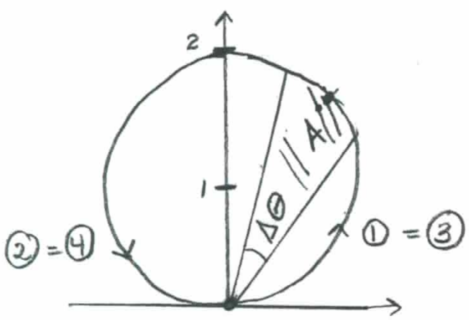
~~Ex. 5.~~ Convert the polar equation $\boxed{r = 2\sin\theta}$ into a Cartesion equation.

$$r = 2\sin\theta \iff r^2 = 2r\sin\theta$$
$$\iff x^2 + y^2 = 2y$$

← ok, but can you do better? what is it?

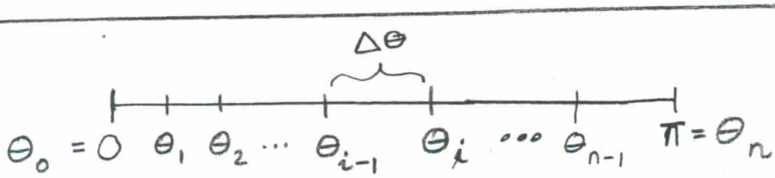
$$\iff x^2 + y^2 - 2y + 1 = 1$$
$$\iff x^2 + (y-1)^2 = 1.$$

← circle with $\begin{cases} \text{radius} = 1 \\ \text{center} = (0,1) \end{cases}$



Let's look closer

θ	$\sin\theta$	$r = 2\sin\theta$
$0 \xrightarrow{①} \frac{\pi}{2}$	$0 \to 1$	$0 \to 2$
$\frac{\pi}{2} \xrightarrow{②} \pi$	$1 \to 0$	$2 \to 0$
$\pi \xrightarrow{③} \frac{3\pi}{2}$	$0 \to -1$	$0 \to -2$
$\frac{3\pi}{2} \xrightarrow{④} 2\pi$	$-1 \to 0$	$-2 \to 0$



Let's talk area

$$\text{Area of typical sector} = \frac{1}{2} r^2 \theta = \frac{1}{2}(2\sin\theta)^2 \Delta\theta = 2\sin^2\theta\,\Delta\theta$$

$$\text{Area of circle} = \int_{\theta=0}^{\theta=\pi} (2\sin^2\theta)\,d\theta \overset{\text{trig}}{\underset{\text{ID}}{=}} \int_{\theta=0}^{\theta=\pi} (1-\cos 2\theta)\,d\theta \overset{\text{Calc.}}{=} \pi.$$

25

~~4e~~

~~2~~. Express the area enclosed by $r = 5 - 5\sin\theta$ as an integral with respect to θ
(ok ... with respect to θ means a $d\theta$ in there).
(You do not have to evaluate this integral.)

area = may ways to do this!

$$A = \frac{1}{2}\int_{\alpha}^{\beta} [f(\theta)]^2 \, d\theta$$

There are many answers (due to symmetry)

$$A = \frac{1}{2}\int_0^{2\pi} [5 - 5\sin\theta]^2 \, d\theta$$

$$\text{or } 2 \cdot \frac{1}{2}\int_{-\pi/2}^{\pi/2} [5 - 5\sin\theta]^2 \, d\theta$$

$$\text{or } 2 \cdot \frac{1}{2}\int_0^{\pi/2} [5 - 5\sin\theta]^2 \, d\theta + 2\,\frac{1}{2}\int_{\frac{3\pi}{2}}^{2\pi} [5 - 5\sin\theta]^2 \, d\theta$$

$$\text{or } 2 \cdot \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} [5 - 5\sin\theta]^2 \, d\theta + 2 \cdot \frac{1}{2}\int_{\pi}^{\frac{3\pi}{2}} [5 - 5\sin\theta]^2 \, d\theta$$

$$\text{or } 2 \cdot \frac{1}{2}\int_{\pi/2}^{3\pi/2} [5 - 5\sin\theta]^2 \, d\theta$$ ... gosh, we could go on & on.

FYI: $[5 - 5\sin\theta]^2 = 25 - 50\sin^1\theta + 25\sin^2\theta = 25[\sin^2\theta - 2\sin\theta + 1]$

3

$\quad = 25(1 - \sin\theta)^2$