NAME: ________________________________ **PIN**:________

Prof. Girardi	Math 142	Fall 2011	12.09.11	Final Exam

INSTRUCTIONS:

(1) This test consists of 25 multiple choice problems, each worth 4 points. The exam is copied two sided. You should turn in just this top piece of paper, with your answers indicated on the back of this page. You can take home the rest of the exam.
(2) You may **not** use a calculator, books, personal notes.
(3) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(4) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET):
§: 7.1 - 7.5. 7.8, 11.1 - 11.9, 6.1-6.3, 10.3-10.4 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : ________________________________

Mark your solution with an X.

Your Solution Table					
PROBLEM					
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

1. Evaluate the integral

$$\int_0^1 \frac{x}{x^2+9}\,dx$$

a. $\frac{1}{2}\left(\ln 10 - \ln 9\right)$
b. $\frac{1}{2}\left(\ln 1 - \ln 0\right)$
c. $\left(\ln 10 - \ln 9\right)$
d. $\left(\ln 1 - \ln 0\right)$
e. None of these

2. Evaluate the integral

$$\int_0^4 \frac{x}{x+9}\,dx\ .$$

Hint: do you have (strictly) bigger bottoms?
a. $4 - 9\ln(13) + 9\ln(9)$
b. $13 - 9\ln(4) + \ln(3)$
c. $(1/(9\ln(13)) - \ln(3)$
d. $4 - 13\ln(9) + 3\ln(18)$
e. None of these

3. Evaluate $\int_0^1 \frac{1}{x^p}\,dx$ for different valuses of p. Which of the following integrals is equal to 1.25?
a. $\int_0^1 \frac{1}{x^{0.2}}\,dx$
b. $\int_0^1 \frac{1}{x^{0.5}}\,dx$
c. $\int_0^1 \frac{1}{x^{0.7}}\,dx$
d. $\int_0^1 \frac{1}{x^{2}}\,dx$
e. $\int_0^1 \frac{1}{x^{2.5}}\,dx$

4. Evaulate the integral

$$\int_0^4 \left(x^2+1\right)e^{-x}\,dx\ .$$

a. $-27e^4 + 3$
b. $-27e^{-4} - 1$
c. $27e^{-4} + 3$
d $-27e^{-4} + 3$
e. none of these

5. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x\,dx\ .$$

a. $-\frac{2}{15}$
b. $\frac{2}{15}$
c. $-\frac{4}{7}$
d. $\frac{4}{7}$
e. none of these

6. Evaluate the integral

$$\int \frac{dx}{(x^2+2x+2)^2} \; .$$

Hint: complete the square: $x^2 + 2x + 2 = (x \pm \; ? \;)^2 \pm \; ?$.

a. $\frac{1}{2}\left(\arctan(x+1) + \frac{x+1}{x^2+2x+2}\right) + C$

b. $\frac{1}{2}\left(\tan(x+1) + \frac{1}{x^2+2x+2}\right) + C$

c. $\frac{1}{2}\left(\tan(x+1) + \frac{x+1}{x^2+2x+2}\right) + C$

d. $\frac{1}{2}\left(\arctan(x+1) + \frac{1}{x^2+2x+2}\right) + C$

e. none of these

7. Evaluate the integral

$$\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx \; .$$

Hint: Do we have (strictly) bigger bottoms?

Recall: $(\ln a) + (\ln b) = \ln(ab)$ and $(\ln a) - (\ln b) = \ln(\frac{a}{b})$

a. $\ln(6) + \frac{7}{6}$

b. $\ln(6) + \frac{-1}{6}$

c. $\ln(\frac{2}{3}) + \frac{7}{6}$

d. $\ln(\frac{2}{3}) + \frac{-1}{6}$

e. none of these

8. Evaluate the integral

$$\int_1^e x \ln x \, dx \; .$$

a. $e^2 - 1$

b. $\frac{e^2}{4} + \frac{1}{2}$

c. $\frac{e^2}{4} + \frac{1}{4}$

d. $\frac{e^2}{4} - \frac{1}{4}$

e. none of these

9. Evaluate the integral

$$\int_1^\infty \frac{dx}{(2x+1)^3} \; .$$

a. $\frac{1}{18}$

b. $\frac{1}{36}$

c. diverges to infinity

d. does not exist but also does not diverges to infinity

e. none of these

10. Evaluate the integral

$$\int_0^e \frac{dx}{x-2} \; .$$

a. $\ln|e-2| - \ln 2$

b. $\ln 2 - \ln|e-2|$

c. diverges to infinity

d. does not exist but also does not diverges to infinity

e. none of these

11. Compute

$$\lim_{n\to\infty} \frac{17n^3+4n^2-5}{19n^5+3n^4-8n^3+n^2-8}\ .$$

a. $\frac{17}{19}$

b. 0

c. diverges to infinity

d. does not exist but also does not diverge to infinity

e. none of these.

12. Compute

$$\lim_{n\to\infty} \frac{\sqrt{9n^4+1}}{17n^2+n+3}\ .$$

a. $\frac{9}{17}$

b. $\frac{3}{17}$

c. 0

d. diverges

e. none of these.

13. Compute

$$\lim_{n\to\infty} \left(\frac{-1}{2}\right)^n\ .$$

a. 0

b. 1

c. diverges to infinity

d. does not exist but does not diverges to infinity

e. none of these.

14. Consider the following two series.

$$\text{Series A is} \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

$$\text{Series B is} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n}\ .$$

a. both series converge absolutely

b. both series diverge

c. series A converges conditionally and series B diverges

d. series A diverges and series B converges conditionally

e. none of these

15. Consider the formal series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)(n+7)}}\ .$$

a. is absolutely convergent, as can be shown by the limit comparison test (LCT) with $b_n = \frac{1}{n^2}$

b. is conditionally convergent as can by shown by using only the AST and not other tests.

c. converges conditionally as can be shown by using the LCT with $b_n = \frac{1}{n}$ as well as the AST .

d. diverges

e. none of these

16. Consider the formal seris $\sum_{n=1}^{\infty} a_n$ where

$$a_n = (-1)^n \frac{(n+1)!}{(2n)!}$$

and let

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| .$$

a. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = \frac{1}{2}$.
b. $\sum_{n=1}^{\infty} a_n$ converges absolutely because $\rho = 0$.
c. $\rho = 1$ so the Ratio Test fails for $\sum_{n=1}^{\infty} a_n$
d. $\sum_{n=1}^{\infty} a_n$ diverges
e. none of these

17. Find the sum of the series

$$\sum_{n=10}^{\infty} \frac{3^{n+1}}{4^n} .$$

a. $12\left(\frac{3}{4}\right)^{10}$
b. $4\left(\frac{3}{4}\right)^{10}$
c. $12\left(\frac{3}{4}\right)^{11}$
d. $4\left(\frac{3}{4}\right)^{11}$
e. none of these

18. What is the LARGEST interval for which the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n}$$

is absolutely convergent?

a. $(1, 5)$
b. $(-4, -2)$
c. $(-5, -1)$
d. $[-5, -1]$
e. none of these

19. Suppose that the radius of convergence of a power series $\sum_{n=0}^{\infty} c_n x^n$ is 16. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^{2n}$?

a. 256
b. 4
c. 1
d. 16
e. none of these

20. In class we learned that, for each $x \in \mathbb{R}$,

$$\cos x \;=\; \sum_{n=0}^{\infty}(-1)^n \,\frac{x^{2n}}{(2n)!}\ .$$

Use this to find a Taylor expansion about the center $x_0 = 0$ (i.e., Maclaurin series) for

$$f(x) = x\cos(4x)\ .$$

a. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{n!}$
b. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n+1}}{(2n)!}$
c. $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{2n} x^{2n}}{(2n)!}$
d. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 4^{2n} x^{2n+1}}{(2n!}$
e. none of these

21. Let R be the region bounded by

$$y = x$$
$$y = x^2$$

between $x = 0$ and $x = 1$. Express, as an integral, the volume of rotation about the x axis. (Hint: Washer).

a. $\pi \int_{x=0}^{x=1} \left[x^2 - x\right]^2 \, dx$
b. $\pi \int_{x=0}^{x=1} \left[x - x^2\right]^2 \, dx$
c. $\pi \int_{x=0}^{x=1} \left[x^2 - (x^2)^2\right] \, dx$
d. $\pi \int_{x=0}^{x=1} \left[(x^2)^2 - x^2\right] \, dx$
e. none of these

22. Let R be the same region as in the previous problem and again, rotate it around the x-axis. Using the shell method, exress the volume of rotation as an integral.

a. $2\pi \int_{x=0}^{x=1} \left(y - \sqrt{y}\right) \, dy$
b. $2\pi \int_{x=0}^{x=1} \left(\sqrt{y} - y\right) \, dy$
c. $2\pi \int_{x=0}^{x=1} (y) \left(y - \sqrt{y}\right) \, dy$
d. $2\pi \int_{x=0}^{x=1} (y) \left(\sqrt{y} - y\right) \, dy$
e. none of these

23. Now let R be the region in the first quadrant bounded by the lines $y = 2x$ and $y = x + 4$ and the y-axis. Now rotate this region about the line $y = -1$. Using the disk/washer method, express as an integral this volume.

a. $\pi \int_{x=0}^{x=4} \left[(x+5)^2 - (2x+1)^2\right] \, dx$
b. $\pi \int_{x=0}^{x=4} \left[(x+5) - (2x+1)\right]^2 \, dx$
c. $\pi \int_{x=0}^{x=4} \left[(x+4)^2 - (2x)^2\right] \, dx$
d. $\pi \int_{x=0}^{x=4} \left[(x+4) - (2x)\right]^2 \, dx$
e. none of these

24. Express the polar equation

$$r = 2\sin\theta$$

in Cartesion equations.

a. $x^2 + (y-2)^2 = 2$

b. $x^2 + (y-1)^2 = 1$

c. $(x-1)^2 + y^2 = 1$

dc. $(x-2)^2 + y^2 = 2$

e. none of these

25. Express the area enclosed by $r = 5 - 5\sin\theta$ as on integral.

a. $\frac{1}{2}\int_0^{2\pi}[5 - 5\sin\theta]^2 \, d\theta$

b. $\int_0^{2\pi}[5 - 5\sin\theta]^2 \, d\theta$

c. $\frac{1}{2}\int_0^{2\pi}[5 - 5\sin\theta] \, d\theta$

d. $\frac{1}{2}\int_0^{2\pi}[5^2 - 5^2\sin^2\theta] \, d\theta$

e. none of these