

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3 a-d	20	
4a-e	20	
5	10	
take home	30	
%	100	

NAME: Key

PIN: 17

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that just appears;**
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Stewart, 6th ed., ET):
11.8, 6.1-6.3, 10.3-10.4 and the take home was over 11.9-11.11 .


Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : 

2010 Spring Exam 2 #6.

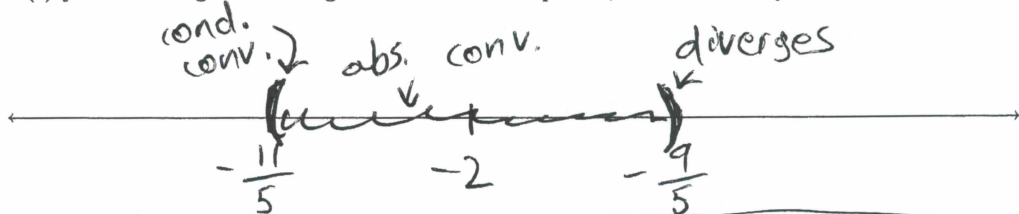
1. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x+10)^n}{n}$$

Hint: $(5x+10)^n = [5(x+2)]^n = 5^n(x+2)^n = 5^n(x-(-2))^n$

The center is $x_0 = -2$ and the radius of convergence is $R = \frac{1}{5}$.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



~~$(5x+10)^n$~~
 ~~$5(x+2)^n$~~
 ~~$5^n(x+2)^n$~~
 ~~$5^n(x-(-2))^n$~~

Endpoint
 $a_n = \frac{(5(-\frac{11}{5})+10)^n}{n}$
 $\frac{(-11+10)^n}{n} = \frac{(-1)^n}{n}$

Endpoint

$a_n = \frac{(5(-\frac{9}{5})+10)^n}{n}$
 $= \frac{1^n}{n} = \frac{1}{n}$

diverges,
harmonic

AST: $\frac{(-1)^n}{n} = a_n$ $\frac{1}{n} = u_n$ u_n diverges, harmonic

~~$u_{n+1} < u_n$~~ $u_{n+1} < u_n$ ✓

$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ ✓

conditionally convergent

Ratio Test

$\lim_{n \rightarrow \infty} \left| \frac{(5x+10)^{n+1}}{n+1} \cdot \frac{n}{(5x+10)^n} \right|$

$|5(x+2)| < 1$
 $|x+2| < \frac{1}{5}$
 $|x-(-2)| < \frac{1}{5}$

$= \lim_{n \rightarrow \infty} |5x+10| \lim_{n \rightarrow \infty} \left| \frac{n}{n+1} \right|$
 $= |5x+10| \cdot 1 < 1$

~~$(5x+10)^{n+1}$~~
 ~~$n+1$~~
 ~~$(5x+10)^n$~~

2010 Spring Exam 2 # 7

2. Consider the formal power series

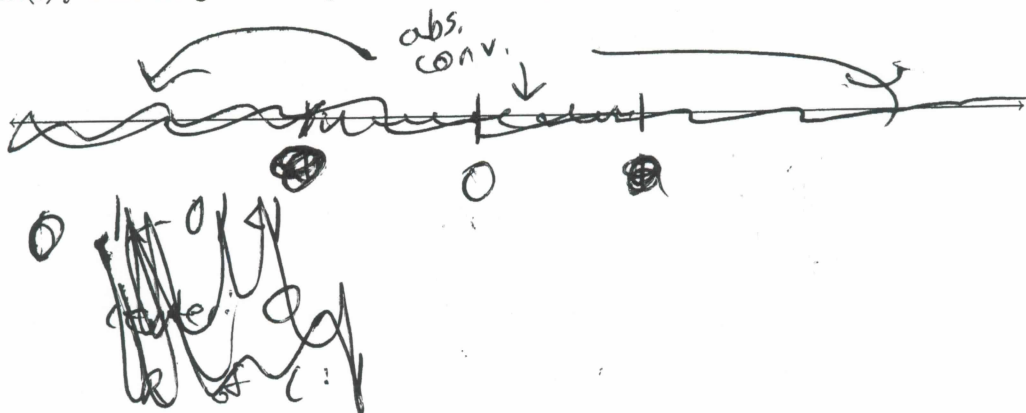
$$\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}$$

Hint 1: $\frac{x^n}{(\ln n)^n} = \left[\frac{x}{\ln n}\right]^n$ so would you rather use the root test or the ratio test?

Hint 2: $\ln(a^r) = r \ln(a)$ but $(\ln(a))^r \neq r \ln(a)$

The center is $x_0 = 0$ and the radius of convergence is $R = \infty$.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



$$a_n = \left(\frac{x}{\ln(n)}\right)^n$$

Root Test:

$$\rho = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{x}{\ln(n)}\right)^n} = \lim_{n \rightarrow \infty} \left|\frac{x}{\ln(n)}\right|$$

$$\rho = |x| \lim_{n \rightarrow \infty} \frac{1}{\ln(n)} \leftarrow \text{because when } n \rightarrow \infty, \ln(n) \text{ is greater than 1}$$

$$\rho = |x| \cdot 0 < 1$$

for all x

Fall 2009, Exam 3 # 7

~~HWK Ch 7 Review #6~~

3. ~~X~~ THIS PROBLEM HAS PARTS: 7a, 7b, 7c, 7d. The region R is the same for all 4 parts.

Let R be the region in the first quadrant enclosed by $y = 2x$ and $y = x + 4$ and $x = 0$.

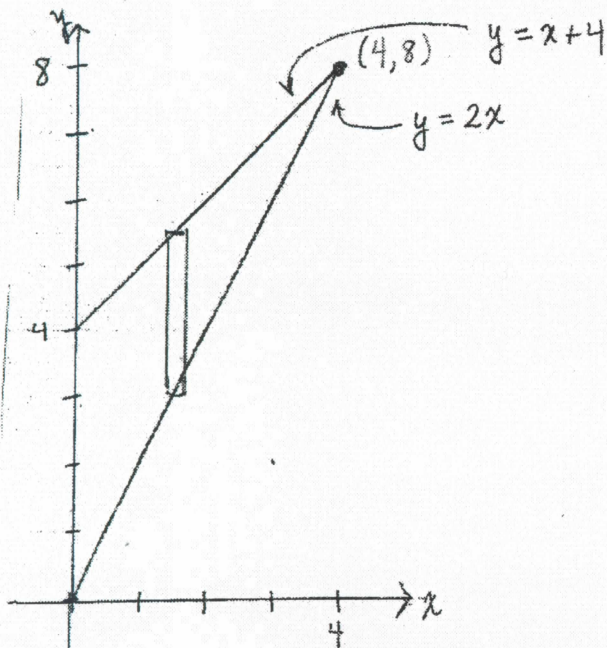
3. 7a. Express the area of R as integral(s) with respect to x .

$$\text{Area} = \int_{x=0}^{x=4} [(x+4) - (2x)] dx$$

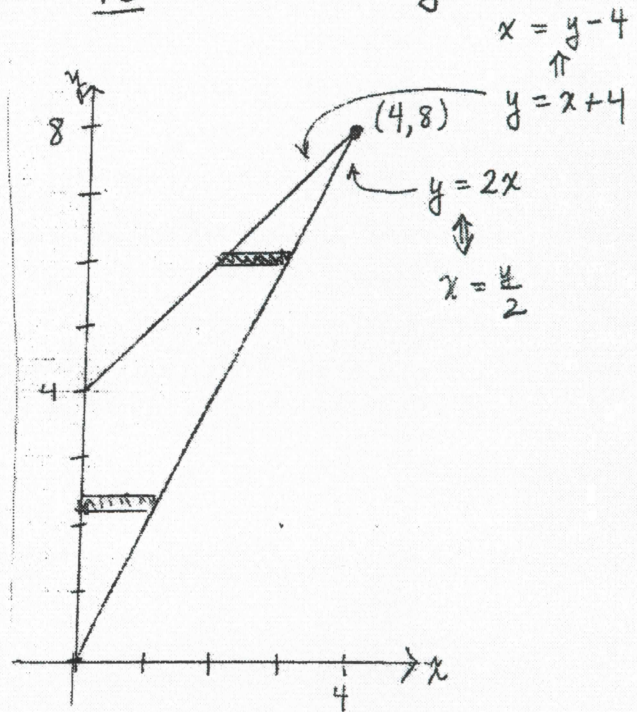
3. 7b. Express the area of R as integral(s) with respect to y .

$$\text{Area} = \int_{y=0}^{y=4} \left[\frac{y}{2} \right] dy + \int_{y=4}^{y=8} \left[\left(\frac{y}{2} \right) - (y-4) \right] dy$$

7a w.r.t. x

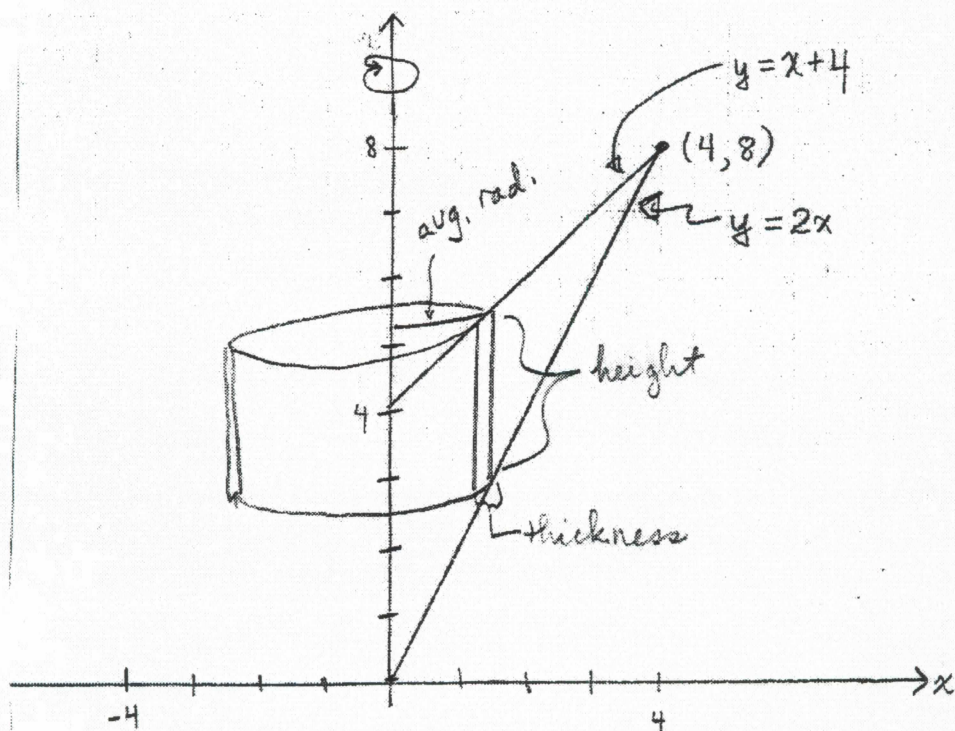


7b w.r.t. y



- 3 c. Using the shell method, express as integral(s) the volume of the solid generated by revolving R about the y -axis.

$$\text{Volume} = \int_{x=0}^{x=4} 2\pi x [(x+4) - (2x)] dx$$



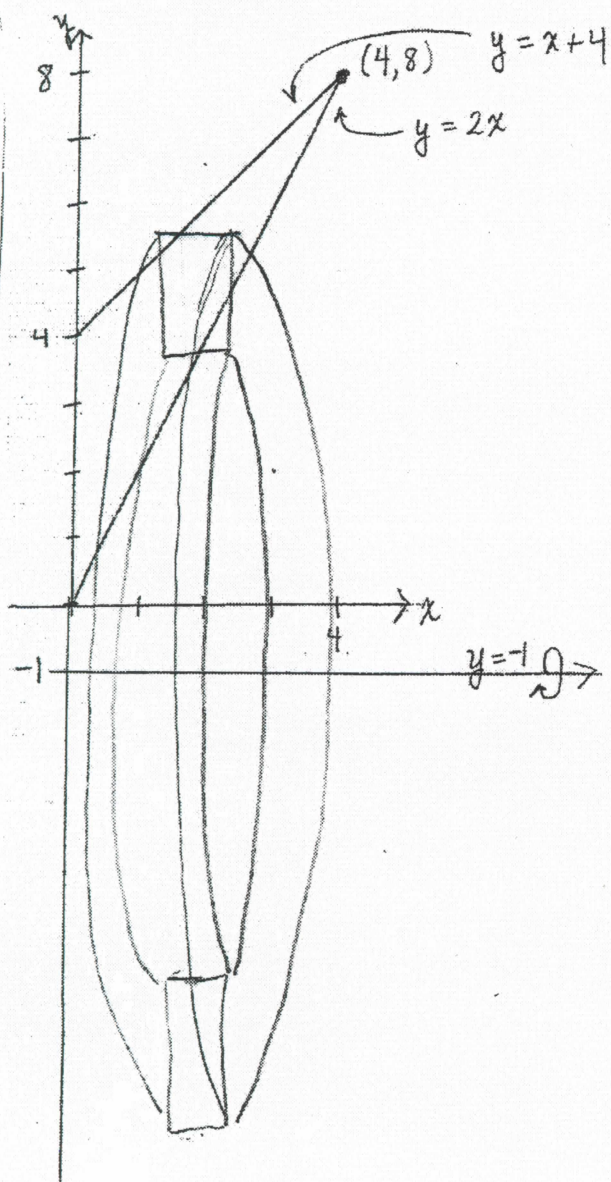
Shell \Rightarrow partition axis \perp to axis of revolution \Rightarrow partition x -axis

$$\begin{aligned} V_{\text{typical shell}} &= (2\pi) (\text{average radius}) (\text{height}) (\text{thickness}) \\ &= (2\pi) (x) [(x+4) - (2x)] \Delta x \end{aligned}$$

→ partition axis \parallel to axis of revolution \Rightarrow partition x -axis
 have a "hole" so washer (not disk)

3. d. Using the disk/washer method, express as integral(s) the volume of the solid generated by revolving R about the line $y = -1$.

$$\text{Volume} = \int_{x=0}^{x=4} \pi \left[(x+5)^2 - (2x+1)^2 \right] dx$$



$$\begin{aligned} \text{Volume typical Washer} \\ &= \text{Volume Big Washer} \\ &\quad - \text{Volume Little Washer} \end{aligned}$$

$$\begin{aligned} &= \pi (\text{Big Radius})^2 (\text{height}) \\ &\quad - \pi (\text{Little radius})^2 (\text{height}) \end{aligned}$$

$$\begin{aligned} &= \pi ((x+4)+1)^2 (\Delta x) \\ &\quad - \pi ((2x)+1)^2 (\Delta x) \end{aligned}$$

Fall 2009 Final Exam - Take home

4 1. Consider the curve in polar coordinate

$$r = 5 - 5 \sin \theta.$$

4 1a. The period of $r = 5 - 5 \sin \theta$ is 2π .

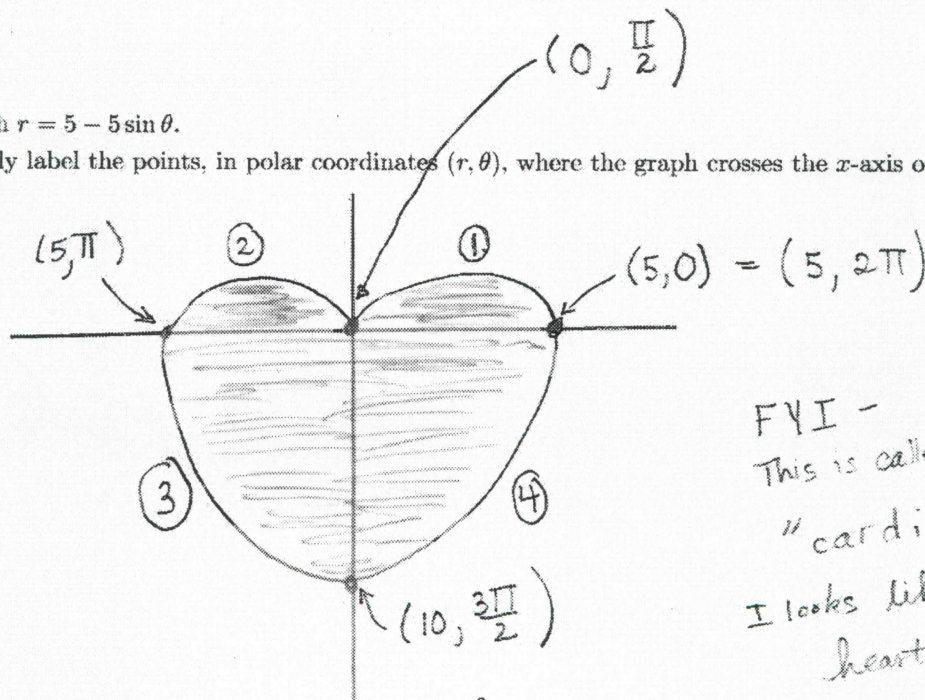
4 1a. $\frac{\text{the period of } r = 5 - 5 \sin \theta}{4} = \frac{\pi}{2}$

4 1c. Make a chart, as we did in class, to help you graph $r = 5 - 5 \sin \theta$.

	θ	$\sin \theta$	$5 \sin \theta$	$-5 \sin \theta$	$r = 5 + -5 \sin \theta$
①	$0 \rightarrow \frac{\pi}{2}$	$0 \rightarrow 1$	$0 \rightarrow 5$	$0 \rightarrow -5$	$5 \rightarrow 0$
②	$\frac{\pi}{2} \rightarrow \pi$	$1 \rightarrow 0$	$5 \rightarrow 0$	$-5 \rightarrow 0$	$0 \rightarrow 5$
③	$\pi \rightarrow \frac{3\pi}{2}$	$0 \rightarrow -1$	$0 \rightarrow -5$	$0 \rightarrow 5$	$5 \rightarrow 10$
④	$\frac{3\pi}{2} \rightarrow 2\pi$	$-1 \rightarrow 0$	$-5 \rightarrow 0$	$5 \rightarrow 0$	$10 \rightarrow 5$

1d. Graph $r = 5 - 5 \sin \theta$.

Clearly label the points, in polar coordinates (r, θ) , where the graph crosses the x -axis or y -axis.



FVI -
This is called a
"cardioid" -
I looks like a
heart.

- 4e 2. Express the area enclosed by $r = 5 - 5 \sin \theta$ as an integral with respect to θ (ok ... with respect to θ means a $d\theta$ in there).
(You do not have to evaluate this integral.)

area = may ways to do this!

$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta$$

There are many answers (due to symmetry)

$$A = \frac{1}{2} \int_0^{2\pi} [5 - 5 \sin \theta]^2 d\theta$$

$$\equiv 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} [5 - 5 \sin \theta]^2 d\theta$$

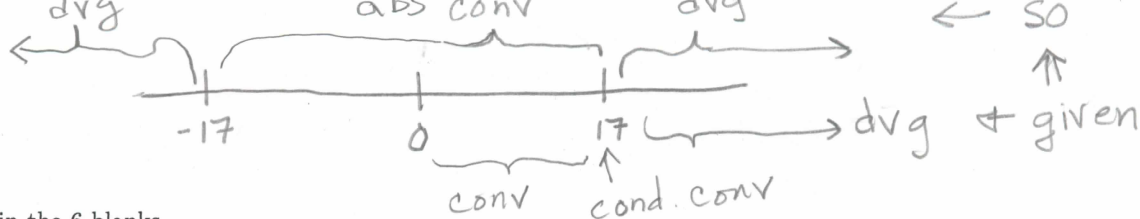
$$\equiv 2 \cdot \frac{1}{2} \int_0^{\pi/2} [5 - 5 \sin \theta]^2 d\theta + 2 \cdot \frac{1}{2} \int_{\frac{3\pi}{2}}^{2\pi} [5 - 5 \sin \theta]^2 d\theta$$

$$\equiv 2 \cdot \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} [5 - 5 \sin \theta]^2 d\theta + 2 \cdot \frac{1}{2} \int_{\pi}^{\frac{3\pi}{2}} [5 - 5 \sin \theta]^2 d\theta$$

$$\equiv 2 \cdot \frac{1}{2} \int_{\pi/2}^{3\pi/2} [5 - 5 \sin \theta]^2 d\theta \quad \text{... goosh, we could go on & on.}$$

$$\text{FYI: } [5 - 5 \sin \theta]^2 = 25 - 50 \sin \theta + 25 \sin^2 \theta = 25 [\sin^2 \theta - 2 \sin \theta + 1]$$

$$L = 25 (1 - \sin \theta)^2$$



5. Fill-in the 6 blanks.

Consider the power series

$$\sum_{n=1}^{\infty} (-1)^n a_n x^n = \sum_{n=1}^{\infty} (-1)^n a_n (x-0)^n$$

where all of the a_n 's are positive. Let's say that you know that

if $0 < x < 17$ then $\sum (-1)^n a_n x^n$ converges

if $x = 17$ then $\sum (-1)^n a_n x^n$ conditionally converges

if $17 < x$ then $\sum (-1)^n a_n x^n$ diverges.

Then this power series has:

center at $x_0 = 0$ and radius of convergence $R = 17$.

Also, what can you say about the following interval? Fill in the blanks below with:

- is absolutely convergent
- is conditionally convergent
- is divergent
- inconclusive (not enough information given to decide in general).

if $-17 < x < 0$ then $\sum (-1)^n a_n x^n$ abs. conv.

if $x < -17$ then $\sum (-1)^n a_n x^n$ divergent

if $x = 0$ then $\sum (-1)^n a_n x^n$ abs conv.

if $x = -17$ then $\sum (-1)^n a_n x^n$ divergent

When $x = -17$, looking at:

$$\sum (-1)^n a_n x^n = \sum (-1)^n a_n (-17)^n = \sum [(-1)(-17)]^n a_n = \sum (17)^n a_n.$$

Know $a_n > 0$ and if $x = 17$ then $\sum (-1)^n a_n x^n$ is cond. conv. so

• $\sum (-1)^n a_n 17^n = \sum (-1)^n (17)^n a_n$ converges

• $\sum |(-1)^n a_n 17^n| = \sum (17)^n a_n$ diverges.

2010 Spring, Exam 2, #8