

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3 a-d	20	
4a-e	20	
5	10	
take home	30	
%	100	

NAME: \_\_\_\_\_

PIN: \_\_\_\_\_

**INSTRUCTIONS:**

- (1) To receive credit you must:
  - (a) **work in a logical fashion, show all your work, indicate your reasoning;**  
no credit will be given for an answer that *just appears*;  
such explanations help with partial credit
  - (b) if a line/box is provided, then:
    - show your work BELOW the line/box
    - put your answer on/in the line/box
  - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.  
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Stewart, 6<sup>th</sup> ed., ET):  
11.8, 6.1-6.3, 10.3-10.4 and the take home was over 11.9-11.11 .

**Honor Code Statement**

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : \_\_\_\_\_

1. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x + 10)^n}{n}$$

Hint:  $(5x + 10)^n = [5(x + 2)]^n = 5^n (x + 2)^n = 5^n (x - (-2))^n$

The center is  $x_0 =$  \_\_\_\_\_ and the radius of convergence is  $R =$  \_\_\_\_\_ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



2. Consider the formal power series

$$\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n} .$$

Hint 1:  $\frac{x^n}{(\ln n)^n} = \left[\frac{x}{\ln n}\right]^n$  so would you rather use the root test or the ratio test?

Hint 2:  $\ln(a^r) = r \ln(a)$  but  $(\ln(a))^r \neq r \ln(a)$

The center is  $x_0 =$  \_\_\_\_\_ and the radius of convergence is  $R =$  \_\_\_\_\_ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



**3. THIS PROBLEM HAS PARTS: a, b,c, and d. The region  $R$  is the same for all 4 parts.**

Let  $R$  be the region in the first quadrant enclosed by  $y = 2x$  and  $y = x + 4$  and  $x = 0$ .

Recall  $x = 0$  is the  $y$ -axis. What is the point of intersection of the two other lines?

**3a.** Express the area of  $R$  as integral(s) with respect to  $x$ .

Area =
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**3b.** Express the area of  $R$  as integral(s) with respect to  $y$ .

Area =
--------

**3c.** Using the shell method, express as integral(s) the volume of the solid generated by revolving  $R$  about the  $y$ -axis.

Volume =

**3d.** Using the disk/washer method, express as integral(s) the volume of the solid generated by revolving  $R$  about the line  $y = -1$ .

Volume =

4. Consider the curve in polar coordinate

$$r = 5 - 5 \sin \theta .$$

4a. The period of  $r = 5 - 5 \sin \theta$  is \_\_\_\_\_.

4b.  $\frac{\text{the period of } r = 5 - 5 \sin \theta}{4} =$  \_\_\_\_\_

4c. Make a chart, as we did in class, to help you graph  $r = 5 - 5 \sin \theta$ .

4d. Graph  $r = 5 - 5 \sin \theta$ .

Clearly label the points, in polar coordinates  $(r, \theta)$ , where the graph crosses the  $x$ -axis or  $y$ -axis.

- 4e. Express the area enclosed by  $r = 5 - 5 \sin \theta$  as an integral with respect to  $\theta$   
(ok ... with respect to  $\theta$  means a  $d\theta$  in there).  
(You do not have to evaluate this integral.)

area =



5. Fill-in the 6 blanks.

Consider the power series

$$\sum_{n=1}^{\infty} (-1)^n a_n x^n$$

where all of the  $a_n$ 's are positive. Let's say that you know that

if  $0 < x < 17$  then  $\sum (-1)^n a_n x^n$  converges

if  $x = 17$  then  $\sum (-1)^n a_n x^n$  conditionally converges

if  $17 < x$  then  $\sum (-1)^n a_n x^n$  diverges .

Then this power series has:

center at  $x_0 =$  \_\_\_\_\_ and radius of convergence  $R =$  \_\_\_\_\_ .

Also, what can you say about the following interval? Fill in the blanks below with:

- is absolutely convergent
- is conditionally convergent
- is divergent
- inconclusive (not enough information given to decide in general).

if  $-17 < x < 0$  then  $\sum (-1)^n a_n x^n$  \_\_\_\_\_

if  $x < -17$  then  $\sum (-1)^n a_n x^n$  \_\_\_\_\_

if  $x = 0$  then  $\sum (-1)^n a_n x^n$  \_\_\_\_\_

if  $x = -17$  then  $\sum (-1)^n a_n x^n$  \_\_\_\_\_