Prof. Girardi	Math 142	Fall 2011	09.22.11	Exam 1

MARK BOX		
PROBLEM	POINTS	
1	25	
2	5	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
take home	10	
%	100	

NAME: Key

class PIN: 17

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use an electronic device, a calculator, books, personal notes.

(4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) If you do not make at least 12.5 out of 25 points on Problem 1, then your score for the entire exam will be whatever you made on Problem 1.

(6) This exam covers (from *Calculus* (ET) by Stewart 6^{th} ed.): Sections 7.1 – 7.5, 7.8, 11.1 .

Hints:

(1) **You can check your answers to the indefinite integrals by differentiating.**

(2) **For more partial credit, box your $u - du$ substitutions.**

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Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

You were warned about this problem.

1. Fill in the blanks (each worth 1 point).

1a. $\int \frac{du}{u} = $ ln $|u| + C$

1b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u\, du = \frac{a^u}{\ln a} + C$

1c. $\int \cos u\, du = \sin u + C$

1d. $\int \sec^2 u\, du = \tan u + C$

1e. $\int \sec u \tan u\, du = \sec u + C$

1f. $\int \sin u\, du = -\cos u + C$

1g. $\int \csc^2 u\, du = -\cot u + C$

1h. $\int \csc u \cot u\, du = -\csc u + C$

1i. $\int \tan u\, du = -\ln|\cos u| + C$ or $\ln|\sec u| + C$ $\cancel{+C}$

1j. $\int \cot u\, du = \ln|\sin u| + C$ or $-\ln|\csc u| + C$ $\cancel{+C}$

1k. $\int \sec u\, du = \ln|\sec u + \tan u| + C$ or $-\ln|\sec u - \tan u| + C$ $\cancel{+C}$

1l. $\int \csc u\, du = -\ln|\csc u + \cot u| + C$ or $\ln|\csc u - \cot u| + C$ $\cancel{+C}$

1m. If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}}\, du = \sin^{-1}(u/a) + C$

1n. If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2}\, du = \frac{1}{a}\tan^{-1}(u/a) + C$

1o. If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}}\, du = \frac{1}{a}\sec^{-1}(|u|/a) + C$

1p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do long division

1q. Integration by parts formula: $\int u\, dv = uv - \int v\, du$

1r. Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u = a\sin\theta$

1s. Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u = a\tan\theta$

1t. Trig substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u = a\sec\theta$

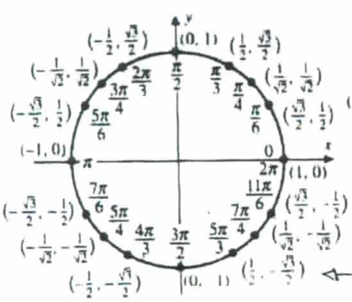
1u. trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) = 2\sin\theta\cos\theta$.

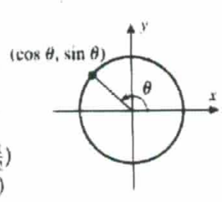
1v. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$.

1w. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$.

1x. trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is $1 + \tan^2\theta = \sec^2\theta$.

1y. $\arctan(-\sqrt{3}) = -\pi/3$ **RADIANS**. (your answer should be an angle)





$\frac{5\pi}{3} = -\frac{\pi}{3}$

1

cot
a
n

$$\tan -\frac{\pi}{3} = \frac{\sin -\frac{\pi}{3}}{\cos -\frac{\pi}{3}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

A "warm up" problem.

2. $\boxed{\int x\ (5x^2+3)^{17}\ dx = \frac{1}{180}(5x^2+3)^{18} + C}$

$$\boxed{\begin{aligned} u &= 5x^2+3 \\ du &= 10x\,dx \end{aligned}}$$

$$\int x(5x^2+3)^{17}\,dx = \frac{1}{10}\int (5x^2+3)^{17}\,(10x\,dx)$$

$$= \frac{1}{10}\int u^{17}\,du$$

$$= \frac{1}{10}\,\frac{u^{18}}{18} + c$$

$$= \frac{u^{18}}{180} + c$$

$$= \frac{1}{180}(5x^2+3)^{18} + c$$

Similar to an example from class.

3. $\int \sin^2 x \cos^3 x \, dx = \dfrac{\sin^3 x}{3} - \dfrac{\sin^5 x}{5} + C$

$\boxed{\cancel{\begin{array}{l} s = \cos x \\ ds = -\sin x \, dx \end{array}}}$

will not work

$\boxed{\begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array}}$

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x \cos^2 x \boxed{\cos x \, dx}$$

$$= \int \sin^2 x \, (1 - \sin^2 x) \boxed{\cos x \, dx}$$

$$= \int t^2 (1 - t^2) \, dt$$

$$= \int (t^2 - t^4) \, dt$$

$$= \frac{t^3}{3} - \frac{t^5}{5} + C$$

$$= \frac{(\sin x)^3}{3} - \frac{(\sin x)^5}{5} + C$$

$\int uv' = uv - \int u'v$

WAY #1

4. $\int e^{5x}\cos(2x)\,dx = \left(\frac{5}{29}\right)e^{5x}\cos(2x) + \left(\frac{2}{29}\right)e^{5x}\sin(2x) + C$

Hint: bring to the other side idea.

$u = \cos(2x) \rightarrow v = \frac{1}{5}e^{5x}$
$du = -2\sin(2x)\,dx \quad dv = e^{5x}\,dx$

or as in class

$u = \cos(2x) \quad dv = e^{5x}\,dx$
$du = -2\sin(2x)\,dx \quad v = \frac{1}{5}e^{5x}$

$$\int e^{5x}\cos(2x)\,dx = \frac{1}{5}e^{5x}\cos(2x) + \frac{2}{5}\int e^{5x}\sin(2x)\,dx$$

$u = \sin(2x) \rightarrow v = \frac{1}{5}e^{5x}$
$du = 2\cos(2x)\,dx \quad dv = e^{5x}\,dx$

or as in class

$u = \sin(2x) \quad dv = e^{5x}\,dx$
$du = 2\cos(2x)\,dx \quad v = \frac{1}{5}e^{5x}$

$$\int e^{5x}\cos(2x)\,dx = \frac{1}{5}e^{5x}\cos(2x) + \frac{2}{5}\left[\frac{1}{5}e^{5x}\sin(2x) - \frac{2}{5}\int e^{5x}\cos(2x)\,dx\right]$$

$$\int e^{5x}\cos(2x)\,dx = \frac{1}{5}e^{5x}\cos(2x) + \frac{2}{25}e^{5x}\sin(2x) - \frac{4}{25}\int e^{5x}\cos(2x)\,dx$$

$$+\frac{4}{25}\int e^{5x}\cos(2x)\,dx \qquad\qquad +\frac{4}{25}\int e^{5x}\cos(2x)\,dx$$

$$\left(\frac{29}{25}\int e^{5x}\cos(2x)\,dx = \frac{1}{5}e^{5x}\cos(2x) + \frac{2}{25}e^{5x}\sin(2x)\right)\cdot\frac{25}{29}$$

multiply both sides by $\frac{25}{29}$

$$\int e^{5x}\cos(2x)\,dx = \frac{5}{29}e^{5x}\cos(2x) + \frac{2}{29}e^{5x}\sin(2x) + C$$

$$1 + \frac{4}{25} = \frac{25+4}{25} = \frac{29}{25}$$

Similar to an example from class.

WAY #2

4. $\boxed{\int e^{5x}\cos(2x)\,dx = \frac{2}{29}e^{5x}\sin(2x) + \frac{5}{29}e^{5x}\cos(2x) \qquad + C}$

Hint: bring to the other side idea.

$$\int e^{5x}\cos(2x)\,dx =$$

$u = e^{5x}$ $\quad dv = \cos(2x)\,dx$

$du = 5e^{5x}\,dx$ $\quad v = \frac{1}{2}\sin(2x)$

$uv - \int v\,du$

$$= \frac{1}{2}e^{5x}\sin(2x) - \frac{5}{2}\int e^{5x}\sin(2x)\,dx =$$

$u = e^{5x}$ $\quad dv = \sin(2x)\,dx$

$du = 5e^{5x}\,dx$ $\quad v = -\frac{1}{2}\cos(2x)$

$$= \frac{1}{2}e^{5x}\sin(2x) - \frac{5}{2}\left(-\frac{1}{2}e^{5x}\cos(2x) - \left(-\frac{5}{2}\int e^{5x}\cos(2x)\,dx\right)\right)$$

$$= \frac{1}{2}e^{5x}\sin(2x) - \frac{5}{2}\left(-\frac{1}{2}e^{5x}\cos(2x) + \frac{5}{2}\int e^{5x}\cos(2x)\,dx\right)$$

[So]

$$\int e^{5x}\cos(2x)\,dx = \frac{1}{2}e^{5x}\sin(2x) + \frac{5}{4}e^{5x}\cos(2x) - \frac{25}{4}\int e^{5x}\cos(2x)\,dx$$

$$\frac{29}{4}\int e^{5x}\cos(2x)\,dx = \frac{1}{2}e^{5x}\sin(2x) + \frac{5}{4}e^{5x}\cos(2x)$$

$$\int e^{5x}\cos(2x)\,dx = \frac{4}{29}\left(\frac{1}{2}e^{5x}\sin(2x) + \frac{5}{4}e^{5x}\cos(2x)\right)$$

$$= \frac{4}{58}e^{5x}\sin(2x) + \frac{20}{116}e^{5x}\cos(2x)$$

$$= \frac{2}{29}e^{5x}\sin(2x) + \frac{5}{29}e^{5x}\cos(2x) + C$$

easier if skip middle step & get from 1st to 3rd step by cancelling fractions .... no need to multiply out & then cancel.

Similar to homework problem §7.1 # 23

5. $\int x^{\frac{1}{3}} \ln x \, dx = \boxed{\dfrac{3(\ln(x))x^{4/3}}{4} - \dfrac{9x^{4/3}}{16} + C}$

$\boxed{\begin{array}{ll} u = \ln x & v = \frac{3}{4}x^{4/3} \\ du = \frac{1}{x}\,dx & dv = x^{1/3}\,dx \end{array}}$

or as in class

$\boxed{\begin{array}{ll} u = \ln x & dv = x^{1/3}\,dx \\ du = \frac{1}{x}\,dx & V = \frac{3}{4}x^{4/3} \end{array}}$

$$= \frac{3((\ln x)x^{4/3})}{4} - \int \frac{3x^{4/3}}{4x}\,dx$$

$$= \frac{3\ln(x)\,x^{4/3}}{4} - \frac{3}{4}\int x^{1/3}\,dx$$

$$= \frac{3(\ln(x)\,x^{4/3})}{4} - \frac{3}{4}\left(\frac{3}{4}x^{4/3}\right) + C$$

$$= \frac{3(\ln(x)\,x^{4/3})}{4} - \frac{9x^{4/3}}{16} + C$$

$$\boxed{\begin{aligned} &= \frac{3\ln(x)\,x^{4/3}}{4} - \frac{9x^{4/3}}{16} + C \\ &\overset{\text{or}}{=} \frac{3x^{4/3}\ln x}{4} - \frac{9x^{4/3}}{16} + C \\ &\overset{\text{or}}{=} \frac{3x^{4/3}}{16}\left[4\ln x - 3\right] + C \end{aligned}}$$

6a. Complete the square by filling in each of the two lines with a (positive or negative) number.

$$x^2 - 6x + 13 = (x + \underline{\ -3\ })^2 + \underline{\ 4\ }.$$

$$\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$$

$$x^2 - 6x + 9 + 13 - 9 = (x-3)^2 + 4$$

6b. $$\boxed{\int \frac{1}{\sqrt{x^2 - 6x + 13}}\,dx = \ln\left|\frac{\sqrt{(x-3)^2+4}}{2} + \frac{x-3}{2}\right| + C \qquad + \text{C}}$$

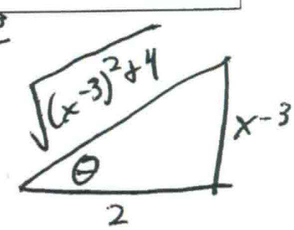
$$= \int \frac{1}{\sqrt{(x-3)^2 + 4}}\,dx$$

$u = a\tan\theta$

$x - 3 = 2\tan\theta$

$$\boxed{\begin{aligned} x &= 2\tan\theta + 3 \\ dx &= 2\sec^2\theta\,d\theta \end{aligned}}$$

$\tan\theta = \frac{x-3}{2}$



$$\sqrt{((2\tan\theta+3)-3)^2 + 4} = \sqrt{(2\tan\theta)^2 + 4} = \sqrt{4\tan^2\theta + 4} = \sqrt{4(\tan^2\theta + 1)}$$

$$= 2\sqrt{\tan^2\theta + 1} = 2\sqrt{\sec^2\theta} = 2\sec\theta$$

choshacao

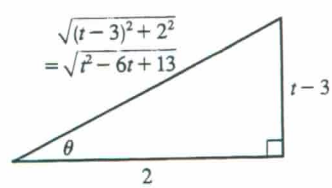
$$= \int \frac{[\cancel{2}\sec^2\theta\,d\theta]}{\cancel{2}\,\cancel{\sec}\,\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{(x-3)^2+4}}{2} + \left(\frac{x-3}{2}\right)\right| + C$$

From §7.3

24. $t^2 - 6t + 13 = (t^2 - 6t + 9) + 4 = (t-3)^2 + 2^2$.

Let $t - 3 = 2\tan\theta$, so $dt = 2\sec^2\theta\,d\theta$. Then



$$\int \frac{dt}{\sqrt{t^2 - 6t + 13}} = \int \frac{1}{\sqrt{(2\tan\theta)^2 + 2^2}}\,2\sec^2\theta\,d\theta$$

$$= \int \frac{2\sec^2\theta}{2\sec\theta}\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C_1 \qquad \text{[by Formula 7.2.1]}$$

$$= \ln\left|\frac{\sqrt{t^2 - 6t + 13}}{2} + \frac{t-3}{2}\right| + C_1$$

$$= \ln\left|\sqrt{t^2 - 6t + 13} + t - 3\right| + C \quad \text{where } C = C_1 - \ln 2$$

7. $\displaystyle\int_1^\infty \frac{1}{(3x+1)^4}\,dx = \frac{1}{576}$

Warning: write your solution in proper form.

$$\int_1^\infty \frac{1}{(3x+1)^4}\,dx$$

$$\lim_{c\to\infty} \int_1^c \frac{1}{(3x+1)^4}\,dx = \lim_{c\to\infty} -\frac{1}{9(3x+1)^3}\Big|_1^c$$

$$= 0 - \left(-\frac{1}{9(3+1)^3}\right)$$

$$= 0 - \left(-\frac{1}{9\cdot 64}\right)$$

$$= 0 + \frac{1}{576}$$

$$\boxed{= \frac{1}{576}}$$

$$\int \frac{1}{(3x+1)^4}\,dx \qquad u = 3x+1 \quad du = 3\,dx$$

$$\frac{1}{3}\int \frac{1}{u^4}\,du$$

$$\frac{1}{3}\int u^{-4}\,du$$

$$= \frac{1}{3}\,\frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{9(3x+1)^3} + C$$

8. Part 8a should help with part 8b.

8a.

$\int e^{(x^2)}\ (2x)\ dx = \quad e^{x^2} \qquad + C$

Way #1 $\boxed{u = x^2 \Rightarrow du = 2x\ dx.}$ $\int e^{x^2} 2x\,dx = \int e^u\,du = e^u + C = e^{x^2} + C$

/

Way #2 $\boxed{t = e^{x^2} \Rightarrow dt = e^{x^2}(2x)\,dx}$ $\int e^{x^2}(2x)dx = \int dt = t = e^{x^2} + C$

8b. The functions $y = e^{x^2}$ and $y = x^2\, e^{x^2}$ do not have elementary antiderivatives.
But the function $y = (2x^2 + 1)\, e^{x^2}$ does have an elementary antiderivative.
Evaulate $\int\ (2x^2 + 1)\ e^{x^2}\ dx$.

$\int (2x^2 + 1)\, e^{x^2}\ dx = \quad x\,e^{x^2} \qquad + C$

Exercise from textbook: § 7.5 # 81

81. The function $y = 2xe^{x^2}$ *does* have an elementary antiderivative, so we'll use this fact to help evaluate the integral.

$$\int(2x^2+1)e^{x^2}\,dx = \int 2x^2e^{x^2}\,dx + \int e^{x^2}\,dx$$

$$= \int x\left(2xe^{x^2}\right)dx + \int e^{x^2}\,dx$$

$$\left[\begin{array}{ll} u = x, & dv = 2xe^{x^2}\,dx, \\ du = dx & v = e^{x^2} \end{array}\right]$$

Now use part 8a to see that parts helps with 1^{st} integra

$$= \left[xe^{x^2} - \int e^{x^2}\,dx\right] + \int e^{x^2}\,dx$$

$$= xe^{x^2} + C$$

$=$

To check $D_x\,(xe^{x^2}) = x\left(e^{x^2}(2x)\right) + (1)\left(e^{x^2}\right)$

$= e^{x^2}\left(2x^2 + 1\right)$ ✓

MARK BOX		
PROBLEM	POINTS	
1	1	
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4	1	
5	1	
6	1	
7	4	
%	10	

NAME: Sol'n Key

CLASS PIN: 17

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 (a) Sections 7.1 – 7.5, 7.8 for the inclass problems
 (b) Section 11.1 for the take home part.

Due Friday Sept. 23 at the beginning of class in LC 102.

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1-3 are $\frac{\text{poly}}{\text{poly}}$ so divide thru by n (highest power)

☺. For the following **SEQUENCES** in 1–5:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

$\div n^3$

1. $\lim_{n\to\infty} \dfrac{5n^2 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = 0$

$$\lim_{n\to\infty} \frac{5n^2 + 4\sqrt{n}}{6n^3+7n^2+1} = \lim_{n\to\infty} \frac{\frac{5n^2}{n^3} + \frac{4n^{1/2}}{n^3}}{\frac{6n^3}{n^3} + \frac{7n^2}{n^3} + \frac{1}{n^3}} =$$

$$\lim_{n\to\infty} \frac{\frac{5}{n} + \frac{4}{n^{5/2}}}{6 + \frac{7}{n} + \frac{1}{n^3}} = \frac{0+0}{6+0+0} = 0$$

$\div n^8$

2. $\lim_{n\to\infty} \dfrac{5n^8 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \infty$ or DNE

$$\lim_{n\to\infty} \frac{5n^8 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \lim_{n\to\infty} \frac{\frac{5n^8}{n^8} + \frac{4n^{1/2}}{n^8}}{\frac{6n^3}{n^8} + \frac{7n^2}{n^8} + \frac{1}{n^8}} =$$

$$\lim_{n\to\infty} \frac{5 + \frac{4}{n^{15/2}}}{\frac{6}{n^5} + \frac{7}{n^6} \frac{1}{n^8}} = \frac{5+\cancel{0}}{0\cancel{+0+0}} = \cancel{\frac{5}{0}} = \cancel{+\infty} = \infty$$

$\div n^3$

3. $\lim_{n\to\infty} \dfrac{5n^3 + 4\sqrt{n}}{6n^3 + 7n^2 + 1} = \dfrac{5}{6}$

$$\lim_{n\to\infty} \frac{5n^3 + 4\sqrt{n}}{6n^3 + 7n^2+1} = \lim_{n\to\infty} \frac{\frac{5n^3}{n^3} + \frac{4n^{1/2}}{n^3}}{\frac{6n^3}{n^3} + \frac{7n^2}{n^3} + \frac{1}{n^3}} =$$

$$\lim_{n\to\infty} \frac{5 + \frac{4}{n^{5/2}}}{6 + \frac{7}{n} + \frac{1}{n^3}} = \frac{5+0}{6+0+0} = \frac{5}{6}$$

#4 & 5.

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & |r|<1 \\ 1 & r=1 \\ \infty & r>1 \\ \text{DNE (osc)} & r<-1 \end{cases}$$

4.

$\lim_{n\to\infty} (-0.917991799917)^n = 0$

$r = -0.917991799917$

$|r| < 1$

5.

$\lim_{n\to\infty} (-1.0000000000000017)^n =$ DNE (osc.)

$r = -1.00 \cdots 017$

$r < -1$

6. A sequence $\{a_n\}$ has the **limit** L , written as

$$\lim_{n\to\infty} a_n = L\ ,$$

if

for every $\varepsilon > 0$ there is $N \in \mathbb{N}$ such that if $n > N$ then $|a_n - L| < \varepsilon$

(Finish filling in the box with the proper Definition 2 (not Def. 1) on page 677. I started you out)

7. Prove that

$$\lim_{n\to\infty}\left(8 + \frac{(-1)^n}{n^3}\right) = 8$$

by using the definition of limit in the previous problem. An outline of the proof is provided, you just need to fill in the blanks.

Proof: Fix $\varepsilon > 0$.

Pick a natural number $N \in \mathbb{N}$ so big that $\frac{1}{N^3} < \varepsilon$ or $\frac{1}{\varepsilon} < N^3$ or,

which we can do by Archimedes Principle.

Fix $n > N$.

Then $|a_n - L| = \left|\left(8 + \frac{(-1)^n}{n^3}\right) - 8\right|$.

$$= \left|\frac{(-1)^n}{n^3}\right|$$

$$= \frac{1}{n^3}$$

$$< \frac{1}{N^3} < \varepsilon .$$

there are several possibilities here

$\frac{1}{N^3} < \varepsilon$ or $\frac{1}{N} < \varepsilon^{1/3}$ or

$\frac{1}{\varepsilon} < N^3$ or $\frac{1}{\varepsilon^{1/3}} < N$