MARK BOX		
PROBLEM	POINTS	
1(23)/2(10)/3(7)	40	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
take home	10	
	150	

NAME **(legibly printed):** ______________________

class PIN: ______________________

If you do not make at least a 50% (i.e. at least 20 of the 40 possible points) on problems 1, 2, and 3, then you will recieve a zero on the remainder of the exam. You were warned.

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): 7.1 - 7.5, 7.8, 11.1 - 11.8, 6.1 - 6.3, 10.3, 10.4. .

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

1. Fill in the blanks.

- $\int \frac{du}{u} \; = \; \_\_\_\_\_\_\_\_\_\_ \; |u| + C$
- $\int \cos u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \sin u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \tan u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \cot u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \sec u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \csc u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \sec^2 u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \sec u \tan u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \csc^2 u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- $\int \csc u \cot u \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} \, du \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ + C$
- Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials

  and [degree of f] $\geq$ [degree of g], then one must first do ____________________
- Integration by parts formula: $\int u \, dv \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$
- Trig substitution: (recall that the *integrand* is the function you are integrating)
  if the integrand involves $a^2 + u^2$, then one makes the substitution $u \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$
- Trig substitution:
  if the integrand involves $a^2 - u^2$, then one makes the substitution $u \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$
- Trig substitution:
  if the integrand involves $u^2 - a^2$, then one makes the substitution $u \; = \; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$
- trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$.
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\cos^2(\theta) \; = \; \frac{1}{2} \; (\; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \;)$.
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\sin^2(\theta) \; = \; \frac{1}{2} \; (\; \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \;)$.
- trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween

  tangent (i.e., tan) and secant (i.e., sec) is ______________________________ .

2. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

- $n^{\mathbf{th}}$**-term test** for an arbitrary series $\sum a_n$.

  If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ ____________________ .

- **Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$
  - converges if and only if $|r|$ ____________________
  - diverges if and only if $|r|$ ____________________

- p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$
  - converges if and only if p ____________________
  - diverges if and only if p ____________________

- **Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

  Let $f\colon [1,\infty) \to \mathbb{R}$ be so that
  - $a_n = f($__________$)$ for each $n \in \mathbb{N}$
  - f is a ____ function
  - f is a ____ function
  - f is a ____ function .

  Then $\sum a_n$ converges if and only if ____ converges.

- **Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.
  - If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ____________________, then $\sum a_n$ ____________________.
  - If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ____________________, then $\sum a_n$ ____________________.

- **Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

  Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

  If __________ $< L <$ __________, then $\sum a_n$ converges if and only if ____________________ .

- **Ratio and Root Tests** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

  Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.
  - If ρ __________ then $\sum a_n$ converges.
  - If ρ __________ then $\sum a_n$ diverges.
  - If ρ __________ then the test is inconclusive.

- **Alternating Series Test** for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

  If
  - a_n __________ a_{n+1} for each $n \in \mathbb{N}$
  - $\lim_{n\to\infty} a_n =$ __________

  then $\sum (-1)^n a_n$ ______________________________

- By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).
  - $\sum a_n$ is <u>absolutely convergent</u> if and only if $\sum |a_n|$ ____________________
  - $\sum a_n$ is <u>conditionally convergent</u> if and only if $\sum a_n$ __________ and $\sum |a_n|$ __________
  - $\sum a_n$ is <u>divergent</u> if and only if $\sum a_n$ ____________________
- Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

Does $\sum |a_n|$ converge? Since $|a_n| \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.	if NO $\Rightarrow$	Does $\lim_{n\to\infty} |a_n| = 0$?	if NO $\Rightarrow$	$\sum a_n$ is ________________
if YES $\Downarrow$		if YES $\Downarrow$		
$\sum a_n$ is ______________________________		Is $\sum a_n$ an alternating series?		
		if YES $\Downarrow$		
		Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?		
		if YES $\Downarrow$		
		$\sum a_n$ is ______________________________		

3. Fill-in-the-blanks/boxes, **using words and/or formula involving *some of*:**

perpendicular, parallel, 2 , π , radius , radius$_{\mathbf{big}}$, radius$_{\mathbf{little}}$, average radius , height , thickness.

►. <u>**Disk/Washer Method**</u>

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the disk or washer method.

- You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is ____________________ to the axis of revolution.

- If you use the **disk method**, then the volume of a typical disk is:

  ____ .

- If you use the **washer method**, then the volume of a typical washer is:

  ____ .

- If you partition the z-axis, the $\Delta z =$ ____ .

►. <u>**Shell Method**</u>

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the shell method.

- You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is ____________________ to the axis of revolution.

- If you use the **shell method**, then the volume of a typical shell is:

  ____ .

- If you partition the z-axis, the $\Delta z =$ ____ .

4. $$\boxed{\int \cos^2 x \; dx \;= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \mathrm{C}}$$

Hint: a formula from problem 1 is handy.

5.

$$\int x^{17} \ln(x)\ dx \ = \qquad\qquad\qquad\qquad\qquad + \text{C}$$

Hint: $D_x \ln x = \frac{1}{x}$ and $\int \frac{dx}{x} = \ln|x| + C$.

So $y = \ln x$ is hard to integrate but easy to differeniate so which method should you try?

6.

$$\boxed{\int \sec^5 x \ \tan^5 x \ dx \ = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \text{C}}$$

Hint: formulas from problem 1 are handy.

7.

$$\int \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx \;=\; \qquad\qquad + \text{C}$$

Hint: Do we have (Strictly) Bigger Bottoms ?

8a. Complete the square:

$x^2 - 6x + 13 \ =$

8b.

$$\int \frac{1}{\sqrt{x^2 - 6x + 13}}\, dx \ = \qquad\qquad + \text{C}$$

9. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{(n+2)\,(n+7)}}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

Warning: there is a square root in the denominator ... many of you overlooked $\sqrt{\ }$'s on Exams ... see it?.

10. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=8}^{\infty} (-1)^n \, \frac{(n+1)!}{(2n)!}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

But before you get started .... let

$$a_n = \frac{(n+1)!}{(2n)!} \, .$$

Then $a_{n+1} =$

Next, simplify $\frac{a_{n+1}}{a_n}$ so that it has NO factorial sign (that is a ! sign) in it.

$\frac{a_{n+1}}{a_n} =$

Ok, now you should be ready to finish off the problem and check the correct box above.

11. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+6)^n}{10^n}$$

The center is $x_0 =$ ____________________ and the radius of convergence is $R =$ ____________________ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any. Remember your absolute value signs! Algrebra hint:

$$\left|\frac{(2x+6)^n}{10^n}\right| = \left|\left(\frac{2x+6}{10}\right)^n\right| = \left|\left(\frac{x+3}{5}\right)^n\right| = \left(\frac{|x+3|}{5}\right)^n .$$

Now, do you want to use the ratio or root test?

- Let R be the region in the first quadrant enclosed by $y = 2x$ and $y = x^2$.

12. Express the area of R as integral(s) with respect to x. (so want dx in there)

Area =

13. Using the shell method, express as integral(s) the volume of the solid generated by revolving R about the vertical line $\underline{x = 3}$.

Volume =