Prof. Girardi	Math 142	Spring 2010	04.19.10	Exam 3 - inclass part

MARK BOX		
PROBLEM	POINTS	
1&2	7	
3a,b,c	20	
4a,b,c,d	33	
5	10	
take-home	30	
%	100	

NAME: ______________________________

PIN: ______________________________

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Stewart, 6^{th} ed., ET): take home part 11.9–11.11 and inclass part 6.1–6.3 .

Problem Inspiration: Mostly homework and old exam problems. See the solution key for details.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

1 & 2. Fill-in-the-blanks/boxes.

- **In 1a and 2a, fill in the blank with:** **perpendicular or parallel.**
- **In 1b, 1c, 1d, 2b, 2c, fill in the blank with a formula involving *some of*:**
  2 , π , radius , radius$_{\text{big}}$, radius$_{\text{little}}$, average radius , height , and/or thickness.

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1. **Disk/Washer Method**

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the disk or washer method.

1a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is parallel to the axis of revolution.

1b. If you use the **disk method**, then the volume of a typical disk is:

$\pi\,(\text{radius})^2\,(\text{height})$.

1c. If you use the **washer method**, then the volume of a typical washer is:

$\pi\,(\text{rad.}_{\text{big}})^2\,(\text{height}) - \pi\,(\text{rad.}_{\text{little}})^2(\text{height}) \overset{\text{or}}{=} \pi\left[(\text{rad}_{\text{big}})^2 - (\text{rad}_{\text{little}})^2\right](\text{height})$

1d. If you partition the x-axis, the $\Delta x =$ height .

2. **Shell Method**

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the shell method.

2a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is perpendicular to the axis of revolution.

2b. If you use the **shell method**, then the volume of a typical shell is:

$2\pi\,(\text{average radius})\,(\text{height})\,(\text{thickness})$.

2c. If you partition the x-axis, the $\Delta x =$ thickness $\overset{\text{or}}{=}$ $\text{radius}_{\text{big}} - \text{radius}_{\text{little}}$.

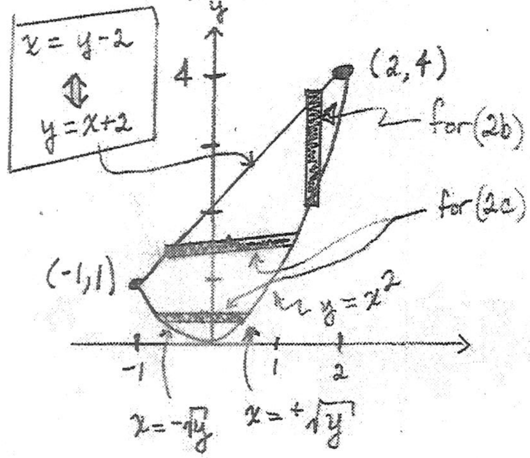
3. Let R be the region enclosed by

$$y = x^2 \qquad \text{and} \qquad y = x + 2\,.$$

Let A be the area of the region R.

3a. The points of intersection of $y = x^2$ and $y = x+2$ are $P = (\underline{-1}, \underline{1})$ and $Q = (\underline{2}, \underline{4})$.
Make a rough sketch of the region R, labeling P and Q.

$$x^2 = x + 2 \iff x^2 - x - 2 = 0 \iff (x-2)(x+1) = 0 \iff x = 2, -1$$



3b. Express the area A as integral(s) with respect to x (so you want dx).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

$$A = \int_{x=-1}^{x=2} \left[(x+2) - (x^2) \right] dx \quad \text{or} \quad \int_{x=-1}^{x=2} (-x^2 + x + 2)\, dx$$

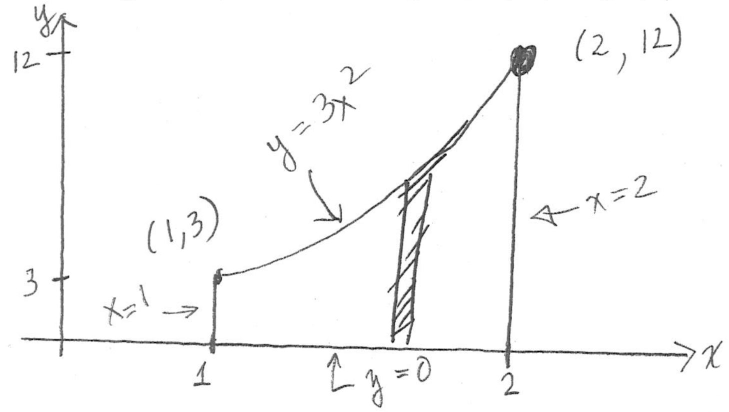
3c. Express the area A as integral(s) with respect to y (so you want dy).
You do NOT have to evaluate the integral(s) nor do lots of algebra.

$$A = \int_{y=0}^{y=1} \left[(+\sqrt{y}) - (-\sqrt{y}) \right] dy + \int_{y=1}^{y=4} \left[(+\sqrt{y}) - (y-2) \right] dy$$

4. Sketched below is the region R that is enclosed by

$$y = 3x^2 \qquad \text{and} \qquad y = 0 \qquad \text{and} \qquad x = 1 \qquad \text{and} \qquad x = 2\,.$$

4a. In the sketch below, draw in a typical rectangle (should it be horizontal or vertical?) that would be used to express the area of R as precisely 1 integral (and not 2 integrals).

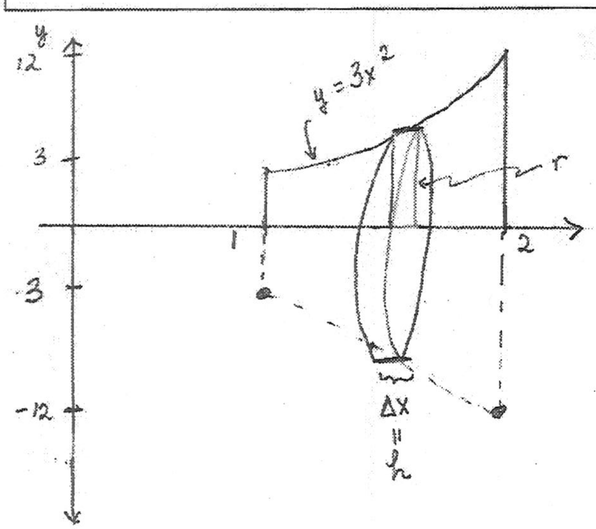


▶. In each of problems 4b, 4c, 4d:

- R will be revolved around some line to create a solid of revolution
- using either the disk, washer, or shell method, express the volume V of the resulting solid of revolution as **one integral** (and NOT as 2 or more integrals).
- In the space provided **below** each problem, make some *good enough sketch* (does not have to be too fancy) to indicate (i.e., help justify) your thinking/reseasoning behind your solution
- you do not have to do lots of algebra to your integrand
- you do not have to integrate your integral.

4b. The volume V of the solid obtained by revolving the region R about the x-axis is

$$V = \int_{x=1}^{x=2} \pi \left(3x^2\right)^2 \, dx \quad \underline{\underline{\text{or}}} \quad 9\pi \int_{x=1}^{x=2} x^4 \, dx$$



Partition x-axis $\Rightarrow$ everything in terms of x

$\Rightarrow$ Disk Method

Volume of typical element

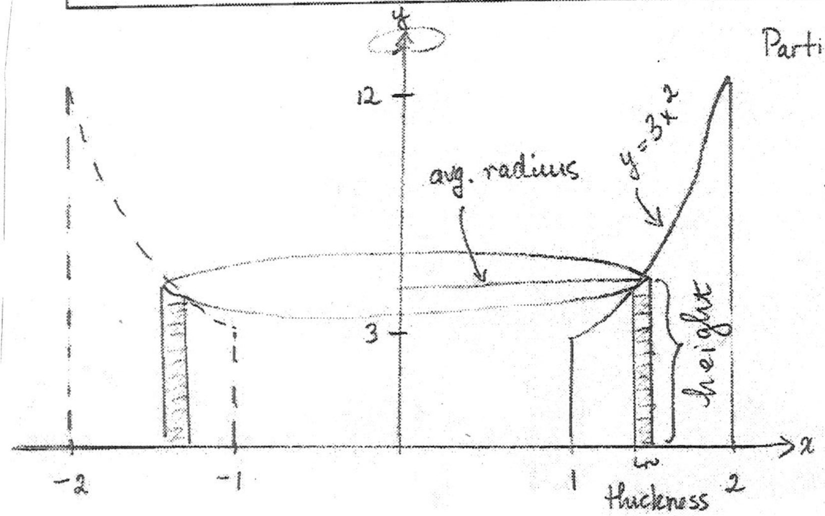
$= (\text{area base})(\text{height})$

$= (\pi r^2) \quad h$

$= \pi \left(3x^2\right)^2 \quad \Delta x$

4c. The volume V of the solid obtained by revolving the region R about the y-axis is

$$V = \int_{x=1}^{x=2} 2\pi x(3x^2)\,dx \overset{\text{or}}{=} 6\pi \int_{x=1}^{x=2} x^3\,dx$$

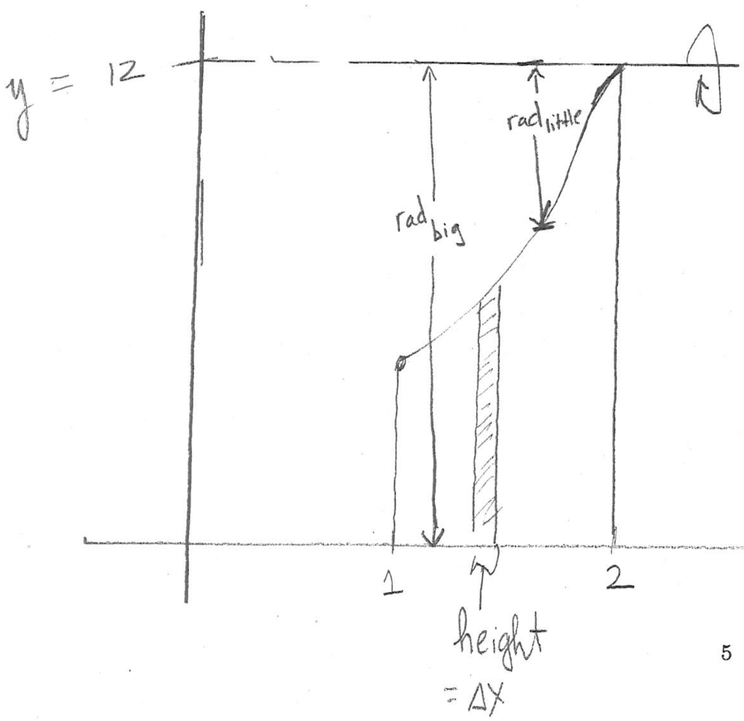


Partition x-axis $\Rightarrow$ everything is in terms of x
$\Rightarrow$ Shell Method

Volume of typical element
$= 2\pi$ (avg. radius)(height)(thickness)
$= 2\pi(x)(3x^2)(\Delta x)$

4d. The volume V of the solid obtained by revolving the region R about the **horizontal** line $y = 12$ is

$$V = \int_{x=1}^{x=2} \pi\left[(12)^2 - (12-3x^2)^2\right] dx$$



partition x-axis $\Rightarrow$ everything in terms of x
$\Rightarrow$ Washer method.

Volume of typical element
= Volume big − Volume little
$= \pi(\text{radius}_{\text{big}})^2(\text{height}) - \pi(\text{radius}_{\text{little}})^2(\text{height})$

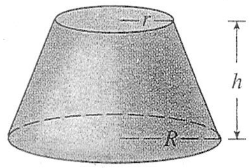
$= \pi\left[\text{rad}_{\text{big}}^2 - \text{rad}_{\text{little}}^2\right]$ height
$= \pi\left[12^2 - \left[12-(3x^2)\right]^2\right]\Delta X$

no hole

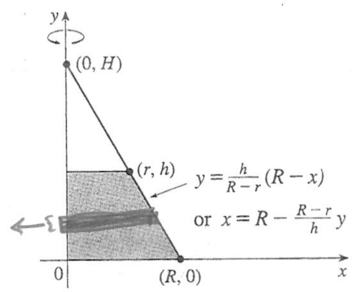
5. Using the disk/~~washer~~ method, express as an integral (do not evaluate) the volume of a frustum of a right circular cone with height h, lower base radius R, and top radius r.

$$V = \pi \int_0^h \left(R - \frac{R-r}{h}\, y\right)^2 dy$$

A frustum of a right circular cone with height h, lower base radius R, and top radius r



height $= \Delta y$



Volume of typical disk (aka tuna can)

$= \pi\,(\text{radius})^2\ \text{height}$

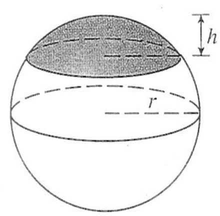
$= \pi \left(R - \frac{R-r}{h}\, y\right)^2 \Delta y$

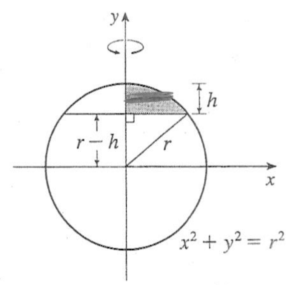
no hole

5. Using the disk/~~washer~~ method, express as an integral (do not evaluate) the volume of a cap of a sphere with radius r and height h.

$$V = \pi \int_{r-h}^{r} \left(\sqrt{r^2 - y^2}\right)^2 dy \overset{\text{or}}{=} \pi \int_{r-h}^{r} (r^2 - y^2)\, dy$$

A cap of a sphere with radius r and height h





$x = +\sqrt{r^2 - y^2}$

Volume of typical disk (aka tuna can)

$= \pi\,(\text{radius})^2\ \text{height}$

$= \pi \left(\sqrt{r^2 - y^2}\right)^2 \Delta y$