

MARK BOX		
PROBLEM	POINTS	
a - j	30	
TOTAL	30	

NAME: \_\_\_\_\_

class PIN: \_\_\_\_\_

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**INSTRUCTIONS:**

- (1) To receive credit you must:
    - (a) **work in a logical fashion, show all your work, indicate your reasoning;**  
no credit will be given for an answer that *just appears*;  
such explanations help with partial credit
    - (b) if a line/box is provided, then:
      - show you work **BELOW** the line/box
      - put your answer on/in the line/box
    - (c) if no such line/box is provided, then box your answer
  - (2) The MARK BOX indicates the problems along with their points.  
Check that your copy of the exam has all of the problems.
  - (3) This exam covers (from *Calculus* by Stewart 6<sup>th</sup> ed,ET): § 11.9, 11.10, 11.11 .
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**Problem Inspiration:** just like the homework.

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**Honor Code Statement**

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : \_\_\_\_\_

**Due Friday April 16 by 1pm.**

**Either hand me your paper in class or  
slip your paper under my office (LC 309C) door.**

## Taylor/Maclaurin Polynomials and Series

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Do parts (a) - (j) for the following problem.

$$f(x) = (4 - x)^{-2} \quad x_0 = 1 \quad J = (0, 2) .$$

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You might find it easier to do problems (a) - (j) in a different order. Just do what you find easiest.

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- On parts (a) - (i), use ideas from only Sections 11.10 and 11.11, i.e., use only:
  - the definition of Taylor polynomial
  - the definition of Taylor series
  - the theorem/error-estimate on the  $N^{\text{th}}$ -Remainder term for Taylor polynomials.Do **NOT** use a known Taylor Series (i.e., do not use methods from Section 11.9).
- On part (j), the very last part, use a known Taylor Series (as from the handout **Commonly Used Taylor Series**) and methods from Section 11.9.

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- a. Find the following. Note the first column are functions of  $x$  and the second column are numbers.

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$
$f^{(4)}(x) =$	$f^{(4)}(x_0) =$

- b. Find  $N^{\text{th}}$ -order Taylor polynomial of  $y = f(x)$  about  $x_0$  in OPEN form for  $N = 0, 1, 2, 3, 4$ .

$P_0(x) =$
$P_1(x) =$
$P_2(x) =$
$P_3(x) =$
$P_4(x) =$

- c. Find the Taylor series of  $y = f(x)$  about  $x_0$  in OPEN form.

$$P_{\infty}(x) =$$

- d. Find the Taylor series of  $y = f(x)$  about  $x_0$  in CLOSED form.

$$P_{\infty}(x) =$$

- e. Find the  $n^{\text{th}}$  Taylor coefficient of  $y = f(x)$  about  $x_0$ .

$$c_n =$$

- f. Find the interval  $I$  of convergence of the Taylor series  $y = f(x)$  about  $x_0$ . Recall, the interval of convergence is the set of points for which the series converges, either absolutely or conditionally. (Hint: use the ratio or root test and then check the endpoints.)

$$I =$$

- g. Consider the given interval  $J$  and fix an  $N \in \mathbb{N}$ . Find a good upper bound for the maximum of  $|f^{(N+1)}(c)|$  on the interval  $J$ . Your answer can have an  $N$  in it but it cannot have an:  $x, x_0, c$ . (Note that  $J$  is a subset of  $I$  but Prof. G. might have picked a smaller  $J$  than  $I$  to make the problem easier.)

$$\max_{c \in J} |f^{(N+1)}(c)| \leq$$

- h. Consider the given interval  $J$  and fix an  $N \in \mathbb{N}$ . For each  $x \in J$ , find a good upper bound for the maximum of  $|R_N(x)|$ . Your answer can have an  $N$  and  $x$  in it but it cannot have an:  $x_0, c$ .

$$|R_N(x)| \leq$$

- i. **Carefully** show that  $f(x) = P_\infty(x)$  for each  $x$  in the given interval  $J$  by using part (h) and showing that  $\lim_{N \rightarrow \infty} |R_N(x)| = 0$  for each  $x \in J$ .

- j. Using a known Taylor Series (as from the handout **Commonly Used Taylor Series**) and methods from Section 11.9, find a power series expansion (in **CLOSED** form) for  $y = f(x)$  about  $x_0$ . Also, say when this power series expansion is valid by examining when the **Commonly Used Taylor Series** is valid. **Show all your work and work in a logical fashion.**

$$f(x) = \sum_{n=0}^{\infty} \boxed{\phantom{0000000000}} (x-1)^n \quad \text{which is valid for } \boxed{\phantom{0000000000}} .$$

Warning: the above sum starts with  $n = 0$ . Hints:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} = (1-r)^{-1} \quad \text{valid when } |r| < 1 \tag{1}$$

$$\frac{1}{4-x} = \frac{1}{3-(x-1)} = \frac{1}{3} \left[ \frac{1}{1-\left(\frac{x-1}{3}\right)} \right] \tag{2}$$

$$D_x \frac{1}{4-x} = D_x (4-x)^{-1} = (4-x)^{-2} \tag{3}$$