If you do not make at least a 50% on this exam's *first few problems*, then your **total** exam score will be only as many points was you managed to get on the *first few problems*. Here are some typical *first few problems*.

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1. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

Hint: You should NOT write the words absolute nor conditional on Problem 1!

1a. Sequences (Afterall, this is needed for Geometric Series!)

Let $-\infty < r < \infty$. (Fill-in-the blanks with *exists* or *does not exist, i.e. DNE*)

- If $|r| < 1$, then $\lim_{n\to\infty} r^n$ ________________________
- If $|r| > 1$, then $\lim_{n\to\infty} r^n$ ________________________
- If $r = 1$, then $\lim_{n\to\infty} r^n$ ________________________
- If $r = -1$, then $\lim_{n\to\infty} r^n$ ________________________

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ ________________
- diverges if and only if $|r|$ ________________

1c. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p ________________
- diverges if and only if p ________________

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f\colon [1,\infty) \to \mathbb{R}$ be so that

- $a_n = f($________$)$ for each $n \in \mathbb{N}$
- f is a ________________________________ function
- f is a ________________________________ function
- f is a ________________________________ function .

Then $\sum a_n$ converges if and only if ________________________________ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ________________, then $\sum a_n$ ________________.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If ________ $< L <$ ________, then $\sum a_n$ converges if and only if ________________ .

1g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If ρ ________ then $\sum a_n$ converges.
- If ρ ________ then $\sum a_n$ diverges.
- If ρ ________ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum(-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.
If

- a_n __________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ __________

then $\sum(-1)^n a_n$ ______________________________

1i. n^{th}**-term test** for an arbitrary series $\sum a_n$.
If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ ____________________ .

1j. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ ____________________
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ __________ and $\sum |a_n|$ __________
- $\sum a_n$ is divergent if and only if $\sum a_n$ ____________________

1k. If a power series in $x - x_0$ has radius of convergence R where $0 < R < \infty$, then the power series is:

- absolutely convergent for ____
- divergent for ____

1l. Consider a **function** $y = f(x)$ where $f\colon [1,\infty) \to \mathbb{R}$.
Next consider the corresponding **sequence** $\{a_n\}_{n=1}^{\infty}$ where $a_n \stackrel{\text{def.}}{=} f(n)$.

- If the limit of the **function** $y = f(x)$ as $x \to \infty$ is L,

  then the limit of the corresponding **sequence** $\{a_n\}_{n=1}^{\infty}$ as $n \to \infty$ is ____________________.
- If $\lim_{n\to\infty} a_n = L$, is it necessarily true that $\lim_{x\to\infty} f(x) = L$? Circle: **Yes** or **No**

2. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

Does $\sum |a_n|$ converge?
Since $|a_n| \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.

if NO $\Rightarrow$ Does $\lim_{n\to\infty} |a_n| = 0$?

if NO $\Rightarrow$ $\sum a_n$ is ________________

(from the first box) if YES $\Downarrow$

$\sum a_n$ is ______________________________

(from "Does $\lim_{n\to\infty} |a_n| = 0$?") if YES $\Downarrow$

Is $\sum a_n$ an alternating series?

if YES $\Downarrow$

Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?

if YES $\Downarrow$

$\sum a_n$ is ______________________________

3. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is <u>always</u> true and circle F if the statement is NOT <u>always</u> true.
Scoring: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

T F If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

T F If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

T F If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n\to\infty} a_n = L$ and

$f\colon [0,\infty) \to \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n,

then $\lim_{x\to\infty} f(x) = L$.

T F If a function $f\colon [0,\infty) \to \mathbb{R}$ satisfies that $\lim_{x\to\infty} f(x) = L$ and

$\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n,

then $\lim_{n\to\infty} a_n = L$.

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum (a_n + b_n)$ converges.

T F If $\sum (a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $r \neq 1$ and $S_N = \displaystyle\sum_{n=17}^{N} r^n$, then $S_N = \dfrac{r^{17} - r^{N+1}}{1-r}$ for each $N > 17$.

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

4. Geometric Series. Let, for $N \geq 102$,

$$s_N = \sum_{n=102}^{N} 3\frac{2^n}{7^{n+1}} .$$

4a. Do some algebra to write s_N as $\sum_{n=102}^{N} c\, r^n$ for an appropriate constant c and ratio r.

$$s_N = \sum_{n=102}^{N}$$

4b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation $\sum$ sign nor some dots $\ldots$.).

$$s_N =$$

4c. Does $\sum_{n=102}^{\infty} 3\frac{2^n}{7^{n+1}}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=102}^{\infty} 3\frac{2^n}{7^{n+1}}$$