Prof. Girardi	Math 142	Spring 2010	02.12.10	Exam 1 - take-home

MARK BOX		
PROBLEM	POINTS	
1	1	
2	1	
3	1	
4	1	
5	1	
6	1	
7	4	
%	10	

NAME: Sol'n Key

CLASS PIN: 17

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* (ET) Stewart 6^{th} ed.):
  (a) Sections 7.1 – 7.5, 7.8 for the inclass problems
  (b) Section 11.1 for the take home part.

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

1 − 3 are ~~poly~~ poly so divide thru by $n^{(\text{highest power})}$

☺. For the following **SEQUENCES**:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

$\div n^2$

1. $\boxed{\lim_{n\to\infty} \frac{(2n+1)(5n+2)}{17n^2} = \frac{10}{17}}$

$$\lim_{n\to\infty} \frac{(2n+1)(5n+2)}{17n^2} = \lim_{n\to\infty} \frac{10n^2+9n+2}{17n^2} =$$

$$\lim_{n\to\infty} \frac{10 + \frac{\text{who cares}}{n} + \frac{\text{who cares}}{n^2}}{17} = \frac{10}{17}$$

$\div n^2$

2. $\boxed{\lim_{n\to\infty} \frac{5n+2}{17n^2} = 0}$

$$\lim_{n\to\infty} \frac{5n+2}{17n^2} = \lim_{n\to\infty} \frac{\frac{5}{n}+\frac{2}{n^2}}{17} = \frac{0+0}{17} = 0$$

$\div n^2$

3. $\boxed{\lim_{n\to\infty} \frac{5n^2+2}{17n} = +\infty}$

$$\lim_{n\to\infty} \frac{5n^2+2}{17n} = \lim_{n\to\infty} \frac{5+\frac{2}{n^2}}{\frac{17}{n}} = \cancel{\frac{5+0}{0}}$$

#4 & 5 : see Theorem 11.1.9 - textbook pg 681 or my §11.1 handout

4.

$$\lim_{n\to\infty} (0.9999999917)^n = 0$$

$r = 0.999999917.$

$|r| < 1$

5.

$$\lim_{n\to\infty} (1.0000000000000017)^n = \text{DNE, divg, } \infty, \ldots$$

$r = 1.00\text{—}017$

$r > 1$

6. A sequence $\{a_n\}$ has the **limit** L , written as

$$\lim_{n\to\infty} a_n = L \ ,$$

if

for every $\varepsilon > 0$ there is a corresponding $N \in \mathbb{N}$ such that if $n > N$ then $|a_n - L| < \varepsilon$.

(Finish filling in the box with the proper Definition 2 (not Def. 1) on page 677. I started you out)

7. Prove that

$$\lim_{n\to\infty}\left(17 - \frac{1}{n^2}\right) = 17$$

by using the definition of limit in the previous problem. An outline of the proof is provided, you just need to fill in the blanks.

Proof: Fix $\varepsilon > 0$.

Pick a natural number $N \in \mathbb{N}$ so big that $\frac{1}{N^2} < \varepsilon$ ... or... $\frac{1}{N} < \sqrt{\varepsilon}$,

which we can do by Archimedes Principle.

Fix $n > N$.

Then $|a_n - L| =$ ______ .

$$|a_n - L| = \left|\left(17 - \frac{1}{n^2}\right) - 17\right|$$

$$= \left|-\frac{1}{n^2}\right|$$

$$= \frac{1}{n^2}$$

$n > \frac{1}{N}$

$$< \frac{1}{N^2}$$

by design

$$< \varepsilon$$