

MARK BOX		
PROBLEM	POINTS	
1	5	
2	5	
TOTAL	10	

NAME (legibly printed): James Bond

class PIN: 007

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that just appears;
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): § 11.1, 11.2, 11.3 .

Problem Inspiration: just like the homework.

This take home part of the final is due at the beginning of our in class final on
December 8 at 2pm.

You may use your notes, book, and calculator. However, you may not discuss this
examine with anyone other than yourself!

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : James Bond

1. Consider the curve in polar coordinate

$$r = 5 - 5 \sin \theta .$$

1a. The period of $r = 5 - 5 \sin \theta$ is 2π .

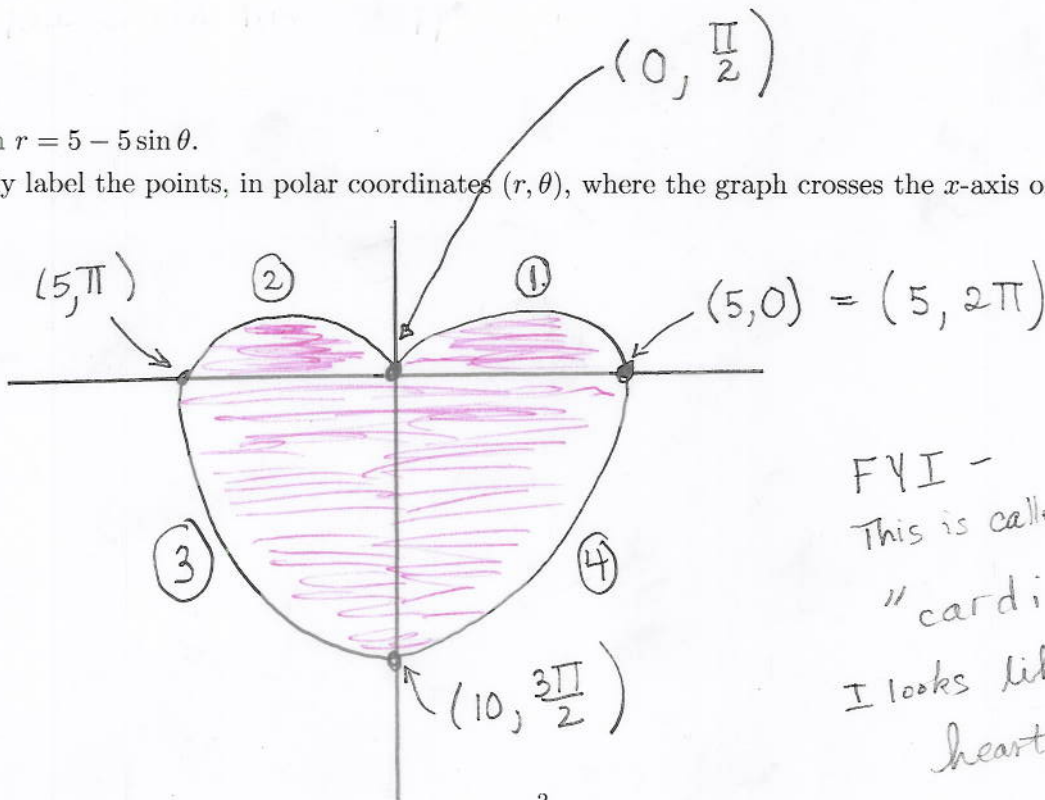
1a. $\frac{\text{the period of } r = 5 - 5 \sin \theta}{4} = \frac{\pi}{2}$

1c. Make a chart, as we did in class, to help you graph $r = 5 - 5 \sin \theta$.

	θ	$\sin \theta$	$5 \sin \theta$	$-5 \sin \theta$	$r = 5 + -5 \sin \theta$
①	$0 \rightarrow \frac{\pi}{2}$	$0 \rightarrow 1$	$0 \rightarrow 5$	$0 \rightarrow -5$	$5 \rightarrow 0$
②	$\frac{\pi}{2} \rightarrow \pi$	$1 \rightarrow 0$	$5 \rightarrow 0$	$-5 \rightarrow 0$	$0 \rightarrow 5$
③	$\pi \rightarrow \frac{3\pi}{2}$	$0 \rightarrow -1$	$0 \rightarrow -5$	$0 \rightarrow 5$	$5 \rightarrow 10$
④	$\frac{3\pi}{2} \rightarrow 2\pi$	$-1 \rightarrow 0$	$-5 \rightarrow 0$	$5 \rightarrow 0$	$10 \rightarrow 5$

1d. Graph $r = 5 - 5 \sin \theta$.

Clearly label the points, in polar coordinates (r, θ) , where the graph crosses the x -axis or y -axis.



FYI -
This is called a
"cardioid" -
It looks like a
heart.

2. Express the area enclosed by $r = 5 - 5 \sin \theta$ as an integral with respect to θ
 (ok ... with respect to θ means a $d\theta$ in there).
 (You do not have to evaluate this integral.)

area = many ways to do this!

$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta$$

There are many answers (due to symmetry)

$$A = \frac{1}{2} \int_0^{2\pi} [5 - 5 \sin \theta]^2 d\theta$$

$$\text{or } 2 \cdot \frac{1}{2} \int_{-\pi/2}^{\pi/2} [5 - 5 \sin \theta]^2 d\theta$$

$$\text{or } 2 \cdot \frac{1}{2} \int_0^{\pi/2} [5 - 5 \sin \theta]^2 d\theta + 2 \cdot \frac{1}{2} \int_{\frac{3\pi}{2}}^{2\pi} [5 - 5 \sin \theta]^2 d\theta$$

$$\text{or } 2 \cdot \frac{1}{2} \int_{\frac{\pi}{2}}^{\pi} [5 - 5 \sin \theta]^2 d\theta + 2 \cdot \frac{1}{2} \int_{\pi}^{\frac{3\pi}{2}} [5 - 5 \sin \theta]^2 d\theta$$

$$\text{or } 2 \cdot \frac{1}{2} \int_{\pi/2}^{3\pi/2} [5 - 5 \sin \theta]^2 d\theta \quad \dots \text{ goosh, we could go on \& on.}$$

$$\text{FYI: } [5 - 5 \sin \theta]^2 = 25 - 50 \sin \theta + 25 \sin^2 \theta = 25 [\sin^2 \theta - 2 \sin \theta + 1]$$

$$L = 25 (1 - \sin \theta)^2$$