MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
take home	10	
EC for PIN	1	
%	100	

NAME (legibly printed): _____

class PIN: _____

INSTRUCTIONS:
(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**;
      no credit will be given for an answer that *just appears*;
      such explanations help with partial credit
  (b) if a line/box is provided, then:
      — show you work BELOW the line/box
      — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points.
    Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When
    you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
    Ch. 7, 8, 10, § 11.1 - 11.3 .

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain
the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

1.

$$\int x^3 \ln(x)\, dx \quad = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + \text{C}$$

Hint: $D_x \ln x = \frac{1}{x}$ and $\int \frac{dx}{x} = \ln |x| + C$.

So $y = \ln x$ is hard to integrate but easy to differeniate so which method should you try?

2.

$$\int \frac{x^2 + x - 2}{3x^3 - x^2 + 3x - 1} \, dx \quad = \qquad\qquad\qquad\qquad\qquad + \text{ C}$$

Hint: Bigger Bottoms - yes - thanks! Also $3x^3 - x^2 + 3x - 1 = (3x - 1)(x^2 + 1)$.

Don't be scared of nice fractions.

3.

$$\int \sqrt{3 - 2x - x^2} \ dx \ = \hspace{6cm} + \ \text{C}$$

Warning: your answer **cannot** have trig function of an inverve trig function (e.g. $\cos(\arccos x)$) in it - clean up such an expression via a reference triangle. Hint:

$$
\begin{aligned}
3 - 2x - x^2 \ &= \ -\left[\ x^2 + 2x - 3\ \right] \\
&= \ -\left[\ (x+1)^2\ + \boxed{}\ \right] \\
&= \ -(x+1)^2\ - \boxed{}
\end{aligned}
$$

4. Find the limit of the following **SEQUENCE**. Hint: This is a sequence, not a series.

$$\lim_{n \to \infty} \frac{\sqrt{4n^3 + 7n + 5}}{7n^{\frac{3}{2}} + 8} =$$

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \, \frac{1}{\sqrt{n\,(n+1)}}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=8}^{\infty} (-1)^n \; \frac{n!}{(2n-1)!}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

But before you get started .... let

$$a_n = \frac{n!}{(2n-1)!} \; .$$

Then $a_{n+1} =$

Next, simplify $\frac{a_{n+1}}{a_n}$ so that it has NO factorial sign (that is a ! sign) in it.

$\frac{a_{n+1}}{a_n} =$

Ok, now you should be ready to finish off the problem and check the correct box above.

7. Consider the formal power series
$$\sum_{n=1}^{\infty} \frac{(2x-6)^n}{8^n}$$

The center is $x_0 = $ _____ and the radius of convergence is $R = $ _____ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any. Remember your absolute value signs!

$\longleftrightarrow$

●. Let R be the region in the first quadrant enclosed by $y = 2x$ and $y = x^2$.

8. Express the area of R as integral(s) with respect to x.

Area =

9. Using the shell method, express as integral(s) the volume of the solid generated by revolving R about the vertical line $\underline{x = 3}$.

Volume =