MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
take home	10	
EC for PIN	1	
%	100	

NAME (legibly printed): Integration Moose

class PIN: 17

Thanks for being such a delightful class!
Good Luck in Maths 241 & 242.

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): Ch. 7, 8, 10, § 11.1 - 11.3 .

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

§ 8.2 Lesson 4 - Ex 4a from class

1. $$\int x^3 \ln(x)\, dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C \overset{\text{or}}{=} \frac{x^4}{16}\left[(4\ln x) - 1\right] + C$$

Hint: $D_x \ln x = \frac{1}{x}$ and $\int \frac{dx}{x} = \ln|x| + C$.

So $y = \ln x$ is hard to integrate but easy to differeniate so which method should you try?

→ Parts w/ u = the function that's easy to integrat

$$\boxed{\begin{array}{ll} u = \ln x & dv = x^3\, dx \\ du = \frac{1}{x}\, dx & \leftarrow v = \frac{x^4}{4} \end{array}}$$

$$\int x^3 \ln x\, dx = \frac{x^4 \ln x}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\, dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3\, dx$$

$$= \frac{x^4 \ln x}{4} - \frac{1}{4}\,\frac{x^4}{4} + C$$

§ 8.5 Example 3 page 540

2. $$\boxed{\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx = -\frac{7}{15}\ln|3x-1| + \frac{2}{5}\ln|x^2+1| + \frac{3}{5}\tan^{-1}x + C}$$

Hint: Bigger Bottoms - yes - thanks! Also $3x^3 - x^2 + 3x - 1 = (3x-1)(x^2+1)$.

Don't be scared of nice fractions.

PFD

$$\frac{x^2+x-2}{(3x-1)(x^2+1)} = \frac{A}{3x-1} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(3x-1)}{(3x-1)(x^2+1)}$$

$$\Rightarrow \boxed{x^2+x-2 = A(x^2+1) + (Bx+C)(3x-1)}$$

$3x-1=0 \iff x=\frac{1}{3}$. so plug $\frac{1}{3}$ into ↑ & get

$$\frac{1}{9}+\frac{1}{3}-2 = A\left(\frac{1}{9}+1\right) \xrightarrow{(x)\,9} 1+3-18 = A(1+9)$$

$$\Rightarrow -14 = 10A \Rightarrow A = \frac{-14}{10} = \boxed{-\frac{7}{5} = A}$$

equate coeff.:

x^2: $1 = A + 3B$

x^1: $1 = -B + 3C \Rightarrow B = 3C - 1 = 3\cdot\frac{3}{5} - 1 = \frac{9-5}{5} = \boxed{\frac{4}{5} = B}$

x^0: $-2 = A - C \Rightarrow C = A + 2 = -\frac{7}{5} + \frac{10}{5} = \boxed{\frac{3}{5} = C}$

$$\int \frac{x^2+x-2}{3x^3-x^2+3x-1}\,dx =$$

$$= -\frac{7}{5}\int\frac{dx}{3x-1} + \frac{4}{5}\int\frac{x\,dx}{x^2+1} + \frac{3}{5}\int\frac{dx}{x^2+1}$$

$u = 3x-1$, $du = 3dx$ $\qquad$ $u = x^2+1$, $du = 2dx$

$$= -\frac{7}{5}\frac{1}{3}\int\frac{3dx}{3x-1} + \frac{4}{5}\frac{1}{2}\int\frac{2x\,dx}{x^2+1} + \frac{3}{5}\int\frac{dx}{x^2+1}$$

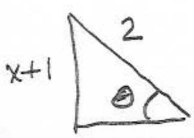
= ↑ up in box

§8.4 Exercise # 39

3. $\boxed{\int \sqrt{3-2x-x^2}\,dx = 2\sin^{-1}\left(\frac{x+1}{2}\right) + \frac{1}{2}\sqrt{3-2x-x^2}\,(x+1) + C}$

Warning: your answer **cannot** have trig function of an inverve trig function (e.g. cos(arccos x)) in it - clean up such an expression via a reference triangle. Hint:

$\sin\theta = \frac{x+1}{2}$



$\sqrt{4-(x+1)^2} = \sqrt{3-2x-x^2}$

$$3-2x-x^2 = -\left[\,x^2+2x-3\,\right]$$

$$= -\left[(x+1)^2 + \boxed{-4}\right]$$

$$= -(x+1)^2 - \boxed{-4} = 4-(x+1)^2$$

$a=2 \qquad u = x+1$

$a^2-u^2 \Rightarrow u = a\sin\theta$

$x+1 = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$$\sqrt{3-2x-x^2} = \sqrt{4-(x+1)^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta}$$

$$= \sqrt{4(1-\sin^2\theta)} = \sqrt{4\cos^2\theta} = 2\cos\theta$$

$$\int\sqrt{3-2x-x^2}\,dx = \int(2\cos\theta)(2\cos\theta\,d\theta) = 4\int\cos^2\theta\,d\theta$$

$$= 4\int\frac{1+\cos 2\theta}{2}\,d\theta = 2\int(1+\cos 2\theta)\,d\theta$$

$$= 2\int d\theta + \int(\cos 2\theta)(2\,d\theta) = 2\theta + \sin 2\theta + C$$

$$= 2\theta + 2\cos\theta\sin\theta + C$$

$$= 2\sin^{-1}\left(\frac{x+1}{2}\right) + 2\left(\frac{\sqrt{3-2x-x^2}}{2}\right)\left(\frac{x+1}{2}\right) + C$$

F06 Final # 12b.

4. Find the limit of the following **SEQUENCE**. Hint: This is a sequence, not a series.

$$\lim_{n\to\infty} \frac{\sqrt{4n^3+7n+5}}{7n^{\frac{3}{2}}+8} = \frac{2}{7}$$

num. behaves like $\sqrt{n^3} = n^{3/2}$ @ ∞.

den. behaves like $n^{3/2}$ @ ∞.

So divide thru by $n^{3/2}$

$$\lim_{n\to\infty} \frac{\sqrt{4n^3+7n+5}}{7n^{3/2}+8} = \lim_{n\to\infty} \frac{\dfrac{\sqrt{4n^3+7n+5}}{\sqrt{n^3}}}{\dfrac{7n^{3/2}+8}{n^{3/2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{\dfrac{4n^3+7n+5}{n^3}}}{7+\dfrac{8}{n^{3/2}}} = \lim_{n\to\infty} \frac{\sqrt{4+\dfrac{7}{n^2}+\dfrac{5}{n^3}}}{7+\dfrac{8}{n^{3/2}}}$$

$$= \frac{\sqrt{4+0+0}}{7+0} = \frac{2}{7}$$

§ 10.6 Exercise 24.

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n\,(n+1)}}$$

- ☐ ~~absolutely convergent~~
- ☒ conditionally convergent
- ☐ divergent

Abs. Conv. Consider $\sum \left| (-1)^n \frac{1}{\sqrt{n(n+1)}} \right| = \sum \frac{1}{\sqrt{n(n+1)}}$

Thinkin' Land $\quad \frac{1}{\sqrt{n(n+1)}} \overset{n \text{ big}}{\approx} \frac{1}{\sqrt{n \cdot n}} = \frac{1}{\sqrt{n^2}} = \boxed{\frac{1}{n} = b_n}$

CT $\quad a_n = \frac{1}{\sqrt{n(n+1)}} \geq \frac{1}{\sqrt{(n+1)(n+1)}} = \frac{1}{n+1} \leftarrow \sum \frac{1}{n+1} = \infty$ or LCT

$L = \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{\sqrt{n(n+1)}} = \lim_{n\to\infty} \frac{\sqrt{n^2}}{\sqrt{n(n+1)}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+n}} =$

$= \sqrt{1} = 1$. & $0 < L < \infty$ so $\sum a_n$ & $\sum b_n$

do the same think. $\sum b_n$ dvg (harmonic series / p-series $p = 1 \leq 1$).

So $\sum \frac{1}{\sqrt{n(n+1)}}$ also dvg.

Cond. Conv? AST w/ $u_n = \frac{1}{\sqrt{n(n+1)}}$

- $\lim_{n\to\infty} u_n = 0$ ✓ clear
- u_n are decreasing ✓ $u_n = \frac{1}{\sqrt{n(n+1)}} \geq \frac{1}{\sqrt{(n+1)(n+2)}} = u_{n+1}$ ✓

So $\sum (-1)^n \frac{1}{\sqrt{n(n+1)}}$ Conv. by AST.

$2n+2-1$ § 10.6 Exercise 29
$2n+1$ and Spring 06 Final

6 ~~11~~. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=8}^{\infty} (-1)^n \frac{n!}{(2n-1)!}$$

- [x] absolutely convergent
- [ ] ~~conditionally convergent~~
- [ ] ~~divergent~~

But before you get started .... let

$$a_n = \frac{n!}{(2n-1)!}.$$

Then $\boxed{a_{n+1} = \frac{(n+1)!}{(2n+1)!}}$

Next, simplify $\frac{a_{n+1}}{a_n}$ so that it has NO factorial sign (that is a ! sign) in it.

$\boxed{\frac{a_{n+1}}{a_n} = \frac{n+1}{(2n+1)(2n)}}$

Ok, now you should be ready to finish off the problem and check the correct box above.

abs conv?

$a_n = \frac{n!}{(2n-1)!}$ $\quad a_{n+1} = \frac{(n+1)!}{(2(n+1)-1)!}$ $\quad a_{n+1} = \frac{(n+1)!}{(2n+2-1)!}$ $\quad a_{n+1} = \frac{(n+1)!}{(2n+1)!}$

ratio Test:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{(2n+1)!} \cdot \frac{(2n-1)!}{n!}$$

$$\lim_{n\to\infty} \frac{(n+1)\cancel{(n)!}}{(2n+1)(2n)\cancel{(2n-1)!}} \cdot \frac{\cancel{(2n-1)!}}{\cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{(2n+1)(2n)}$$

$$\lim_{n\to\infty} \frac{n+1}{4n^2+2n} = \frac{\infty}{\infty} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{8n+2} = \frac{1}{\infty} = 0$$

L = 1 inc
L < 1 con
L > 1 div

∴ The given series ~~series~~ is abs convergent b/c $\sum |a_n|$ converges by the ratio test

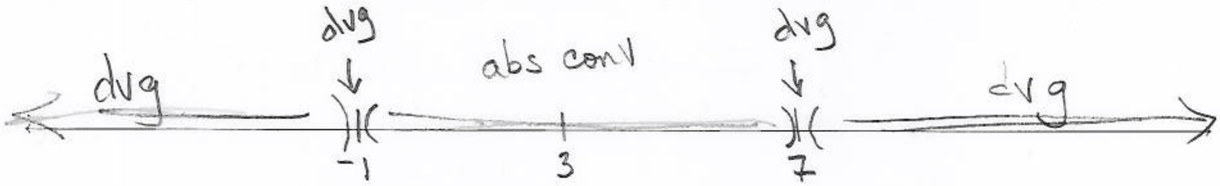
I made it up.

7. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x-6)^n}{8^n} = \sum \frac{2^n (x-3)^n}{8^n}$$

The center is $x_0 =$ 3 and the radius of convergence is $R =$ 4.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any. Remember your absolute value signs!



Root Test $\lim\limits_{n\to\infty} \left|\frac{(2x-6)^n}{8^n}\right|^{1/n} = \lim\limits_{n\to\infty} \frac{|2x-6|^{n\cdot\frac{1}{n}}}{8^{n\cdot 1/n}} = \lim\limits_{n\to\infty} \frac{|2x-6|}{8}$

$= \frac{|2x-6|}{8} = \frac{2|x-3|}{8} = \frac{|x-3|}{4} < 1 \iff |x-3| < 4$

or

Ratio Test $\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\frac{(2x-6)^{n+1}}{8^{n+1}} \cdot \frac{8^n}{(2x-6)^n}\right|$

$= \lim\limits_{n\to\infty} \frac{|2x-6|}{8} = \frac{2x-6}{8} = \frac{2|x-3|}{8} = \frac{|x-3|}{4} < 1$

Check endpts.

$x = 7 : \sum \frac{(2x-6)^n}{8^n} = \sum \frac{(14-6)^n}{8^n} = \sum \frac{8^n}{8^n} = \sum 1 = \underset{\text{dvg.}}{\infty}$

$x = -1 \quad \sum \frac{(2x-6)^n}{8^n} = \sum \frac{(-8)^n}{8^n} = \sum (-1)^n$ dvg (osc).

Fall 08, Exam 3, #7

- Let R be the region in the first quadrant enclosed by $y = 2x$ and $y = x^2$.

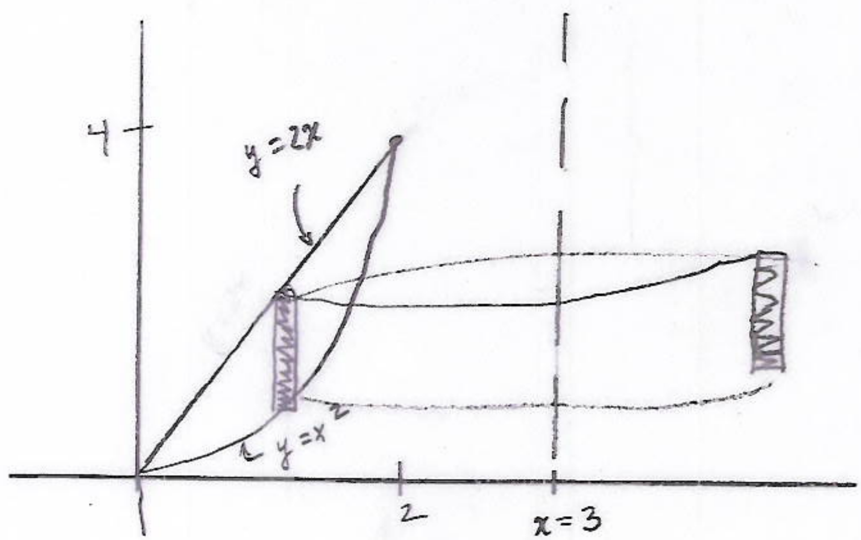
$2x = x^2$
$0 = x^2 - 2x = x(x-2)$
$x = 0 \rightarrow (0,0)$
$x = 2 \rightarrow (2,4)$

8 ~~16~~. Express the area of R as integral(s) with respect to x.

Area = $\displaystyle\int_{x=0}^{x=2} \left[(2x) - (x^2) \right] dx$

9 ~~17~~. Using the shell method, express as integral(s) the volume of the solid generated by revolving R about the line $x = 3$.

Volume = $\displaystyle\int_{x=0}^{x=2} 2\pi (3-x)(2x-x^2)\, dx$



$V_{\text{typical shell}} = 2\pi\ (\text{avg. rad.})\ (\text{height})\ (\text{thickness})$

$\downarrow$

Δx