Prof. Girardi | Math 142 | Fall 2009 | 11.24.~~08~~ 09 | Exam 3

MARK BOX		
PROBLEM	POINTS	
sign honor code	2	
correct pin	2	
$1-6$	26	
7a	10	
7b	10	
7c	10	
7d	10	
8	10	
9	10	
10	10	
%	100	

NAME (legibly printed): James Bond

class PIN: 007

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  (b) if a line/box is provided, then:
    — show you work BELOW the line/box
    — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 7.1, 7.2, 7.3, 7.4, 7.6, 7.7 .

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

Throughout this exam, you need to only set up the integral expressing the asked for quantity.
You do not have to integrate your integral.
You do not have to do lots of algebra to your integrand.

problem 1–5 : in class quiz & HMWK || Also Fall 08 Exam 3.
problem 6 : HMWK § 7.7 #5

Problem 1 - 6: Fill-in-the blanks/boxes.

- In 1a and 2a, **fill in the blank with: perpendicular or parallel.**
- In 1b, 1c, 1d, 2b, 2c, **fill in the blank with a formula involving** *some of*:
  2 , π , radius , $\text{radius}_{\text{big}}$, $\text{radius}_{\text{little}}$, average radius , height , and/or thickness.

1. **Disk/Washer Method**
   Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the disk or washer method.

1a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is parallel to the axis of revolution.

1b. If you use the **disk method**, then the volume of a typical disk is:

$\pi\,(\text{radius})^2\,(\text{height})$.

1c. If you use the **washer method**, then the volume of a typical washer is:

$\pi\,(\text{rad}_{\text{big}})^2\,(\text{height}) - \pi\,(\text{rad}_{\text{little}})^2\,(\text{height})$ or $\pi\left[(\text{rad}_{\text{big}})^2 - (\text{rad}_{\text{little}})^2\right](\text{height})$

1d. If you partition the z-axis, the $\Delta z =$ height.

2. **Shell Method**
   Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the shell method.

2a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is perpendicular to the axis of revolution.

2b. If you use the **shell method**, then the volume of a typical shell is:

$2\pi\,(\text{average radius})\,(\text{height})\,(\text{thickness})$.

2c. If you partition the z-axis, the $\Delta z =$ thickness or $\text{radius}_{\text{big}} - \text{radius}_{\text{little}}$.

3. **Arc Length**

3a. The arc length L of a smooth curve $y = f(x)$ over the interval $[a, b]$ is defined by the following definite integral.

$$L = \int_{x=a}^{x=b} \sqrt{1+[f'(x)]^2}\,dx \overset{\text{or}}{=} \int_{x=a}^{x=b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

3b. The arc length L of a smooth curve $x = g(y)$ over the interval $[c, d]$ is defined by the following definite integral.

$$L = \int_{y=c}^{y=d} \sqrt{1+[g'(y)]^2}\,dy \overset{\text{or}}{=} \int_{y=c}^{y=d} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy.$$

3c. The arc length L of a curve that is parametrized by

$$x = x(t) \quad , \quad y = y(t) \qquad (a \le t \le b)$$

such that no segment of the curve is traced more than once as t increases from a to b and also $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous functions for $a \le t \le b$, is defined by the following definite integral.

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

4. **Average Value of a Function**

If $y = f(x)$ is continuous on the interval $[a, b]$, the the average value f_{ave} of $y = f(x)$ on $[a, b]$ is defined to be

$$f_{\text{ave}} = \boxed{\frac{\int_{x=a}^{x=b} f(x)\,dx}{b-a}}.$$

5. **Work**

Suppose that Integration-Moose moves in the positive direction along a coordinate line over the interval $[a, b]$ while subjected to a variable force $F(x)$ that is applied in the directins of the motion. The work W preformed by the force on Integration-Moose is defined by the following definite integral.

$$W = \int_{x=a}^{x=b} F(x)\,dx.$$

6. Circle the scenario in which you perform more work:

(1) by rasing a cup of coffee from a table to your mouth

(2) by holding a calculus textbook at shoulder level for 5 minutes.

→ problem source : homework problem Ch 7 Review #6

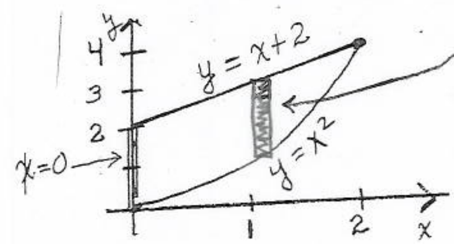
7. Let R be the region in the first quadrant enclosed by

$$y = x^2 \quad \text{and} \quad y = x + 2 \quad \text{and} \quad x = 0\,.$$

In each of problems **7a – 7h**, set up an integral or a sum of integrals that express the desired quantity.

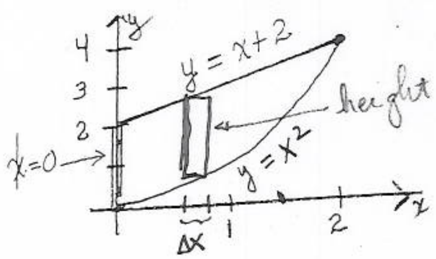
- In the space provided **below** each problem, make some *good enough sketch* (does not have to be too fancy) to indicate (i.e., help justify) your thinking/reseasoning behind your solution
- you do not have to do lots of algebra to your integrand
- you do **not have to integrate/evaulate** your integral.

Extra Credit/Hint In the sketch below, draw in a typical rectangle (should it be horizontal or vertical?) that would be used to express the area of R as precisely 1 integral (and not 2 integrals).



7a. The area A of the region R by integrating with respect to x.

$$A = \int_{x=0}^{x=2} \left[x + 2 - x^2 \right] \, dx$$



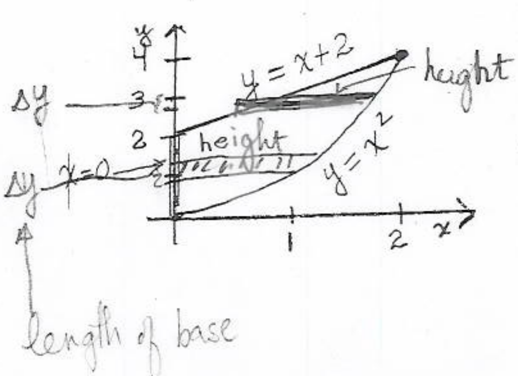
Area Typical rectangle

$= (\text{height})(\text{length of base})$

$= \left[(x+2) - x^2 \right] \Delta x .$

7a. The area A of the region R by integrating with respect to y.

$$A = \int_{y=0}^{y=2} \sqrt{y}\, dy + \int_{y=2}^{y=4} \left[\sqrt{y} - (y-2) \right] dy$$



Area Typical rectangle

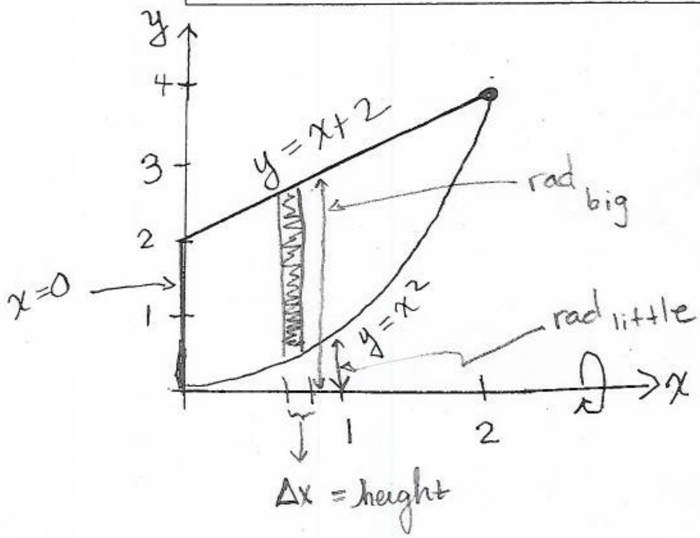
$= (\text{height}) \, (\underbrace{\text{length of base}}_{\Delta y})$

For $0 \le y \le 2$: height $= \sqrt{y} - 0$

For $2 \le y \le 4$: height $= (\sqrt{y}) - (y-2)$

7b. The volume V of the solid obtained by revolving the region R about the x-axis by integrating with respect to x.

$$V = \pi \int_{x=0}^{x=2} \left[(x+2)^2 - (x^2)^2 \right] dx$$



Washer Method

b/c revolving abt x

||

& integrating wrt y

+ there's a hole

Volume of Typical Washer

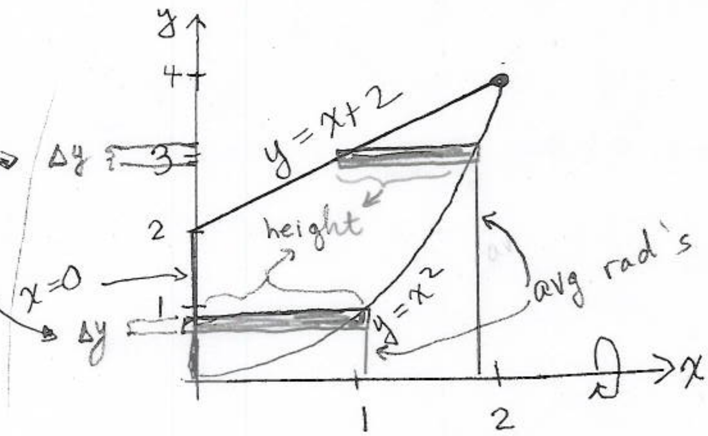
$$= \pi \left[(\text{rad}_{\text{big}})^2 - (\text{rad}_{\text{little}})^2 \right] (\text{height})$$

$$= \pi \left[(x+2)^2 - (x^2)^2 \right] \quad \Delta x$$

7c. The volume V of the solid obtained by revolving the region R about the x-axis by integrating with respect to y.

$$V = 2\pi \int_{y=0}^{y=2} y\sqrt{y}\, dy + 2\pi \int_{y=2}^{y=4} y\left[(\sqrt{y}) - (y-2)\right] dy$$

thickness



Shell Method

b/c revolving abt x # integrating wrt y

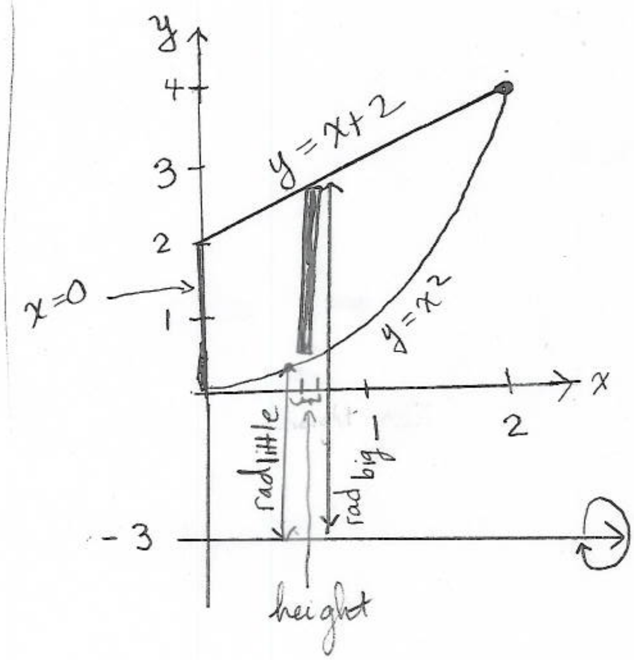
Volume of Typical Shell

$$= 2\pi\,\underbrace{(\text{avg. radius})}_{y}\,(\text{height})\,\underbrace{(\text{thickness})}_{=\,\Delta y}$$

see your last part of 7a.

7d. The volume V of the solid obtained by revolving the region R about the line $y = -3$ by integrating with respect to x.

$$V = \pi \int_{x=0}^{x=2} \left(\left[(x+2) + 3 \right]^2 - \left[x^2 + 3 \right]^2 \right) dx$$



Washer Method.

see 7 b

Volume of Typical Washer

$$= \pi \left[(\text{rad}_{\text{big}})^2 - (\text{rad}_{\text{little}})^2 \right] \text{height}$$

$$= \pi \left[\left[\underbrace{(x+2)+3}_{\text{or } x+5} \right]^2 - \left[x^2 + 3 \right]^2 \right] \Delta x$$

problem source - homework problem § 7.4 # 9

8. Express the arclength of the parameterized curve

$$x(t) = \frac{1}{3}t^3$$
$$y(t) = \frac{1}{2}t^2$$
$$0 \le t \le 1$$

as an integral with respect to t .

arclength = $\displaystyle\int_{t=0}^{t=1} \sqrt{(t^2)^2 + (t^1)^2}\, dt = \int_{t=0}^{t=1} \sqrt{t^4 + t^2}\, dt$

$$\frac{dx}{dt} = \frac{1}{3} \cdot 3t^2 = t^2$$

$$\frac{dy}{dt} = \frac{1}{2} \cdot 2t^1 = t^1$$

$$AL = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_{t=a}^{t=b} \sqrt{(t^2)^2 + (t^1)^2}\, dt$$

Problem Source Fall 08 Exam 3 # 9. Like Homework § 7.7. # 1

9. Express as an integral the work done when a variable force of $F(x) = x^2$ lb in the positive x-direction moves Integration Moose from $x = 7$ to $x = 17$ ft.

work = $\displaystyle\int_{x=7}^{x=17} x^2 \, dx$ **ft-lbs.**

$$\text{Work} = \int_{x=a}^{x=b} F(x)\,dx$$

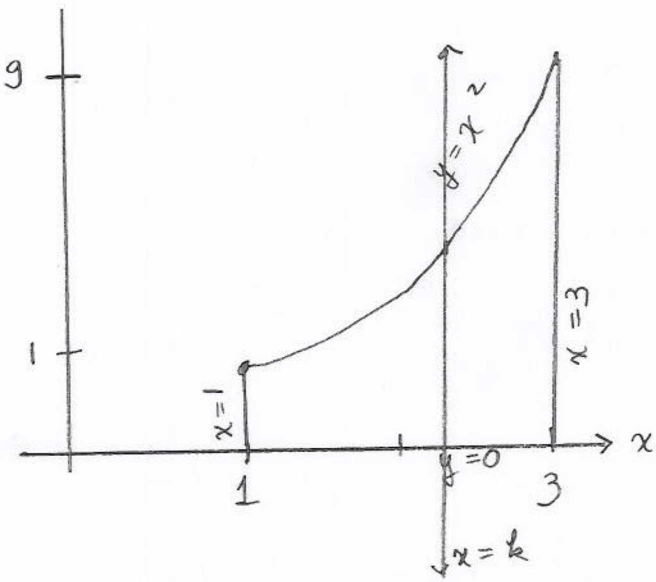
Problem Source § 7.1 # 31 & 33. Also - Fall 2005 Exam 1 # 6
& Fall 2008 Exam 3 # 10

10. Find a vertical line $x = k$ that divides the area enclosed by

$$y = x^2 \quad \text{and} \quad x = 1 \quad \text{and} \quad x = 3 \quad \text{and} \quad y = 0$$

into two equal parts.

ANSWER: the vertical line is $x =$ $\sqrt[3]{14}$.



Want to find k so that

$$\int_1^k x^2\,dx = \int_k^3 x^2\,dx$$

$$\iff \frac{1}{3}x^3 \Big|_{x=1}^{x=k} = \frac{1}{3}x^3 \Big|_{x=k}^{x=3}$$

$$\iff \frac{1}{3}(k^3 - 1) = \frac{1}{3}(3^3 - k^3)$$

$$\iff k^3 - 1 = 27 - k^3$$

$$\iff 2k^3 = 27 + 1$$

$$\iff 2k^3 = 28$$

BTY : $\sqrt[3]{14} \simeq 2.4$

$$\iff k^3 = 14$$

$$\iff k = \sqrt[3]{14}$$