

MARK BOX		
PROBLEM	POINTS	
1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
take home	10	
%	100	

NAME: James BondCLASS PIN: 007**INSTRUCTIONS:**

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;**
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
 - (a) Sections 10.1 - 10.6, 10.8 for the inclass problems
 - (b) whole of Ch 10 for in class fill-in-blank and true/false problems
 - (c) Section 10.7. 10.9, 10.10 for the take home part.

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

→ 142. Spring 06, Exam 3 and Fall 08, Exam 2

3. Spring 06, Exam 3

4. Serious Series Problem #2

5. Maple Lab homework # 9.

6. Spring 06, Exam 3

7. HMWK

7. HMWK 10.8 #41

8. HMWK 10.8 #35

9. HMWK 10.8 #12

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. Sequences Let $-\infty < r < \infty$. (Fill-in-the blanks with *exists* or *does not exist*, i.e. *DNE*)

- If $|r| < 1$, then $\lim_{n \rightarrow \infty} r^n$ exists
- If $|r| > 1$, then $\lim_{n \rightarrow \infty} r^n$ DNE
- If $r = 1$, then $\lim_{n \rightarrow \infty} r^n$ exists
- If $r = -1$, then $\lim_{n \rightarrow \infty} r^n$ DNE

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ < 1
- diverges if and only if $|r|$ ≥ 1

1c. p -series where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p > 1
- diverges if and only if p ≤ 1

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\underline{n})$ for each $n \in \mathbb{N}$
- f is a positive function
- f is a continuous function
- f is a decreasing (or nonincreasing) function.

Then $\sum a_n$ converges if and only if $\int_1^{\infty} f(x) dx$ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If 0 $< L <$ ∞ , then $\sum a_n$ converges if and only if $\sum b_n$ converges.

1g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If ρ < 1 then $\sum a_n$ converges.
- If ρ > 1 then $\sum a_n$ diverges.
- If ρ = 1 then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n > a_{n+1} for each $n \in \mathbb{N}$ (decreasing)
- $\lim_{n \rightarrow \infty} a_n =$ 0

then $\sum (-1)^n a_n$ converges

1i. n^{th} -term test for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ diverges.

1j. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ converges
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ converges and $\sum |a_n|$ diverges
- $\sum a_n$ is divergent if and only if $\sum a_n$ diverges

1k. If a power series in $x - x_0$ has radius of convergence R where $0 < R < \infty$, then the power series is:

- absolutely convergent for $x \in (x_0 - R, x_0 + R)$
- divergent for $x < x_0 - R$ and $x_0 - R < x$.

for 1l - 1o

Let $y = f(x)$ be a function with derivatives of all orders in an interval I containing x_0 .

Let $y = p_N(x)$ be the N^{th} -order Taylor polynomial of $y = f(x)$ about x_0 .

Let $y = R_N(x)$ be the N^{th} -order Taylor remainder of $y = f(x)$ about x_0 .

Let $y = p_\infty(x)$ be the Taylor series of $y = f(x)$ about x_0 .

1l. In open form (i.e., with ... and without a \sum -sign)

$$p_N(x) = f^0(x_0) + f^{(1)}(x_0)(x-x_0)^1 + \frac{f^{(2)}(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(N)}(x_0)}{N!}(x-x_0)^N$$

1m. In closed form (i.e., with a \sum -sign and without ...)

$$p_N(x) = \sum_{n=0}^N \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n$$

1n. In closed form (i.e., with a \sum -sign and without ...)

$$p_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n$$

1o. We know that $f(x) = p_N(x) + R_N(x)$. Taylor's BIG Theorem tells us that, for each $x \in I$,

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!} (x-x_0)^{N+1}$$
 for some c between x and x_0 .

1p. Did you write your PIN on the cover page (under your name)? It's worth 5 points. 😊

2. Circle T if the statement is TRUE. Circle F if the statement is FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.
Scoring: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

HMWK
Ch 10
Review
g.c.d

T
T

F

If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n \rightarrow \infty} a_n = L$ and $f: [0, \infty) \rightarrow \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number then $\lim_{x \rightarrow \infty} f(x) = L$. Consider $f(x) = \sin(2\pi x)$

F

If a function $f: [0, \infty) \rightarrow \mathbb{R}$ satisfies that $\lim_{x \rightarrow \infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{n \rightarrow \infty} a_n = L$.

T

F

If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.

F

If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T

F

If $r \neq 1$ and $S_N = \sum_{n=17}^N r^n$, then $S_N = \frac{r^{17} - r^{N+1}}{1-r}$ for each $N > 17$.

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

Thm 10.4.3
 $a_n = \frac{1}{n}, b_n = -\frac{1}{n}$

$$S_N = r^{17} + r^{18} + \dots + r^N$$

$$r S_N = r^{18} + r^{19} + \dots + r^N + r^{N+1}$$

$$(1-r) S_N = r^{17} - r^{N+1}$$

3. For the following **SEQUENCES**:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your **ANSWER IN** the box and show your **WORK BELOW** the box.

3a.

$$\lim_{n \rightarrow \infty} \frac{(4n+1)(5n+2)}{17n^2} = \frac{20}{17}$$

$$\frac{(4n+1)(5n+2)}{17n^2} = \frac{20n^2 + (\text{who cares})n + (\text{some constant})}{17n^2}$$

3b.

$$\lim_{n \rightarrow \infty} (-1)^n \frac{(4n+1)(5n+2)}{17n^2} = \text{DNE}$$

oscillating

3c.

$$\lim_{n \rightarrow \infty} (0.9999999917)^n = 0$$

$$\lim_{n \rightarrow \infty} r^n = 0 \quad \text{when } |r| < 1$$

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

absolutely convergent

conditionally convergent

divergent

• abs. conv? consider $\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$
p-series $p = 1/2 < 1 \Rightarrow$ diverges
So not abs. conv.

• cond. conv? Consider $\sum \frac{(-1)^n}{\sqrt{n}} = \sum (-1)^n \frac{1}{\sqrt{n}}$

A.S.T. with $u_n = \frac{1}{\sqrt{n}}$.

① u_n 's are decreasing since $u_n = \frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}} = u_{n+1}$

② $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$

So by A.S.T., $\sum (-1)^n \frac{1}{\sqrt{n}}$ converges

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$$

absolutely convergent

~~conditionally convergent~~ b/c $\frac{2^n 3^n}{n^n} > 0$.

divergent

Hint: $\frac{2^n 3^n}{n^n} = \left(\frac{2+3}{n}\right)^n$

Root Test

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = \lim_{n \rightarrow \infty} \left[\left(\frac{2+3}{n} \right)^n \right]^{1/n}$$

$$= \lim_{n \rightarrow \infty} \frac{6}{n} = 0 < 1$$

⇓
converges

6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{1}{(\ln n)^4}$$

absolutely convergent

~~conditionally convergent~~ b/c

divergent

Hint: For any $0 < q < \infty$, if n is big enough then $\ln n < n^q$ and so $\frac{1}{(n^q)^4} < \frac{1}{(\ln n)^4}$.

↙

$$\frac{1}{(\ln n)^4} \stackrel{q=\frac{1}{4}}{>} \frac{1}{(n^{1/4})^4} = \frac{1}{n}$$

$\sum \frac{1}{n}$ diverges, p-series, $p=1 \leq 1$.

CT $\Rightarrow \sum \frac{1}{(\ln n)^4}$ diverges

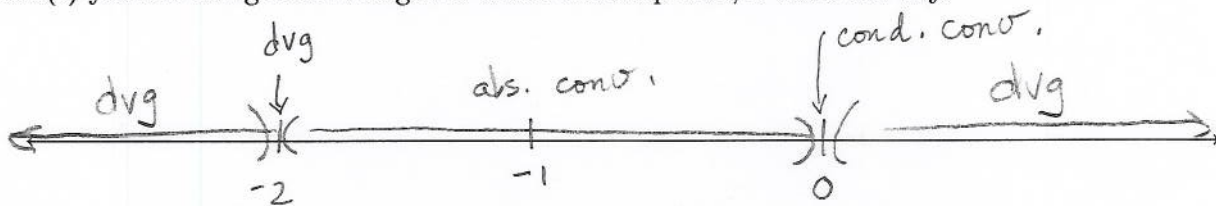
7. Consider the formal power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^n}{n}$$

$(x+1)^n = (x - -1)^n$

The center is $x_0 = \underline{-1}$ and the radius of convergence is $R = \underline{1}$.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



$$\rho = \lim_{n \rightarrow \infty} \left| \frac{(x+1)^{n+1}}{n+1} \cdot \frac{n}{(x+1)^n} \right| = \lim_{n \rightarrow \infty} \frac{n}{n+1} |x+1| = |x+1| \lim_{n \rightarrow \infty} \frac{n}{n+1} = |x+1|$$

or

$$\rho = \lim_{n \rightarrow \infty} \left| \frac{(-1)^n (x+1)^n}{n} \right|^{1/n} = \lim_{n \rightarrow \infty} \frac{|x+1|}{n^{1/n}} = |x+1| \lim_{n \rightarrow \infty} \frac{1}{n^{1/n}} = |x+1|$$

abs conv. when $|x+1| < 1$

Check endpoints

$x = -2$ $\sum (-1)^n \frac{(-2+1)^n}{n} = \sum \frac{(-1)^n (-1)^n}{n} = \sum \frac{1}{n}$ divg
p-series
 $p=1$

$x = 0$ $\sum \frac{(-1)^n (0+1)^n}{n} = \sum \frac{(-1)^n}{n}$ ← converges by AST

$\frac{1}{n} \rightarrow 0$
 $\frac{1}{n} > \frac{1}{n+1}$

So $\sum \frac{(-1)^n}{n}$ is
cond. conv.

8a. Let

$$a_n = \frac{x^{2n+1}}{(2n+1)!}$$

Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

$$\frac{a_{n+1}}{a_n} = \frac{x^2}{(2n+2)(2n+3)}$$

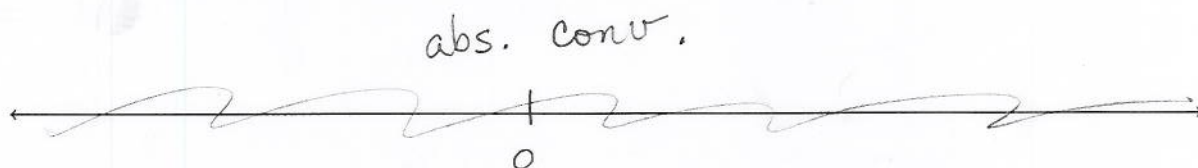
$$\begin{aligned} \frac{a_{n+1}}{a_n} &= \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} = \frac{x^{2n+3}}{(2n+3)!} \frac{(2n+1)!}{x^{2n+1}} \\ &= \frac{x^{2n+3}}{x^{2n+1}} \frac{(2n+1)!}{(2n+3)!} = \frac{x^{2n+1} \cdot x^2}{x^{2n+1}} \cdot \frac{(2n+1)!}{(2n+1)!(2n+2)(2n+3)} \end{aligned}$$

8b. Consider the formal power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

The center is $x_0 = 0$ and the radius of convergence is $R = \infty$.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x^2}{(2n+2)(2n+3)} \right| = \lim_{n \rightarrow \infty} \frac{|x|^2}{(2n+2)(2n+3)}$$

$$= |x|^2 \lim_{n \rightarrow \infty} \frac{1}{(2n+2)(2n+3)} = |x|^2 \cdot 0 < 1$$

↑
always !!