

MARK BOX		
PROBLEM	POINTS	
1	25	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Extra Credit	5	
%	100	

NAME: Key

class PIN: _____

(* Extra Credit: 5 point for knowing yor PIN number.

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning;**
no credit will be given for an answer that just appears;
 such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
 Check that your copy of the exam has all of the problems.
- (3) You may **not** use an electronic device, a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
 Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.8. .

Problem Inspiration: If I told you here, you would know what method to use. So see the solution key, which will be available from the course homepage after the exam.

Hints:

- (1) **You can check your answers to the indefinite integrals by differentiating.**
- (2) **For more partial credit, box your $u - du$ substitutions.**

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

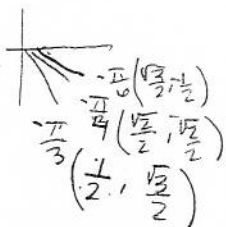
Signature : _____

You were warned on the first day of the semester.

1. Fill in the blanks (each worth 1 point).

- $\int \frac{du}{u} = \underline{\ln |u| + C}$
- If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u du = \underline{\frac{a^u}{\ln a}} + C$
- $\int \cos u du = \underline{\sin u} + C$
- $\int \sin u du = \underline{-\cos u} + C$
- $\int \tan u du = \underline{\ln |\sec u|}$ or $\underline{-\ln |\cos u|} + C$
- $\int \cot u du = \underline{\ln |\sin u|}$ or $\underline{-\ln |\csc u|} + C$
- $\int \sec u du = \underline{\ln |\sec u + \tan u|}$ or $\underline{-\ln |\sec u - \tan u|} + C$
- $\int \csc u du = \underline{\ln |\csc u - \cot u|}$ or $\underline{-\ln |\csc u + \cot u|} + C$
- $\int \sec^2 u du = \underline{\tan u} + C$
- $\int \sec u \tan u du = \underline{\sec u} + C$
- $\int \csc^2 u du = \underline{-\cot u} + C$
- $\int \csc u \cot u du = \underline{-\csc u} + C$
- If a is a constant and $a > 0$ then $\int \frac{1}{a^2+u^2} du = \underline{\frac{1}{a} \tan^{-1} \frac{u}{a}} + C$
- If a is a constant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} du = \underline{\sin^{-1} \frac{u}{a}} + C$
- If a is a constant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} du = \underline{\frac{1}{a} \sec^{-1} \frac{|u|}{a}} + C$
- Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polynomials and $[\text{degree of } f] \geq [\text{degree of } g]$, then one must first do long division
- Integration by parts formula: $\int u dv = \underline{uv - \int v du}$
- Trig substitution: (recall that the *integrand* is the function you are integrating) if the integrand involves a^2+u^2 , then one makes the substitution $u = \underline{a \tan \theta}$
- Trig substitution: if the integrand involves a^2-u^2 , then one makes the substitution $u = \underline{a \sin \theta}$
- Trig substitution: if the integrand involves u^2-a^2 , then one makes the substitution $u = \underline{a \sec \theta}$
- trig formula ... your answer should involve trig functions of θ , and not of 2θ : $\sin(2\theta) = \underline{2 \sin \theta \cos \theta}$
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\cos^2(\theta) = \frac{1}{2} (\underline{1 + \cos 2\theta})$.
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\sin^2(\theta) = \frac{1}{2} (\underline{1 - \cos 2\theta})$.
- trig formula ... since $\cos^2 \theta + \sin^2 \theta = 1$, we know that the corresponding relationship between tangent (i.e., \tan) and secant (i.e., \sec) is $1 + \tan^2 \theta = \sec^2 \theta$
- $\arcsin(-\frac{1}{2}) = \underline{-\frac{\pi}{6}}$ RADIANS. (your answer should be an angle)

S + C



a "warm-up" problem

2.

$$\int e^{17x} dx = \frac{1}{17} e^{17x} + C$$

$$\left| \begin{array}{l} u = 17x \\ du = 17 dx \end{array} \right| = \frac{1}{17} \int e^u du$$

$$= \left(\frac{1}{17}\right) e^u + C$$

$$= \left| \frac{1}{17} e^{17x} + C \right|$$

$$\left| \begin{array}{l} \text{check:} \\ e^{17x} (17) \left(\frac{1}{17}\right) = e^{17x} \end{array} \right|$$

or

$$u = e^{17x}$$

$$du = 17 e^{17x} dx$$

$$\int e^{17x} dx = \frac{1}{17} \int du = \frac{1}{17} u + C = \frac{1}{17} e^{17x}$$

Homework § 8.2 # 1

3.

$$\int x e^x dx = x e^x - e^x \quad \text{or} \quad (x-1) e^x + c$$

$$\begin{array}{l} u = x \quad dv = e^x dx \\ du = dx \quad v = e^x \end{array}$$

$$= x e^x - \int e^x dx$$

$$= \boxed{x e^x - e^x + c}$$

$$\text{check} = \boxed{x e^x + e^x - e^x = x e^x \checkmark}$$

Homework § 8.2 # 33

4.

$$\int \ln(x+2) dx = (x+2) \ln(x+2) - x + C$$

$$\begin{array}{l} \text{let } u = \ln(x+2) \quad dv = dx \\ du = \frac{1}{x+2} dx \quad v = x \end{array}$$

$$x \ln(x+2) - \int \frac{x}{x+2} dx$$

$$x \ln(x+2) - \int \frac{x+2-2}{x+2} dx$$

like adding 0
but helps make
easier to integrate

$$x \ln(x+2) - \int \frac{x+2}{x+2} - \frac{2}{x+2} dx$$

$$x \ln(x+2) - \int \left(1 - \frac{2}{x+2}\right) dx$$

$$x \ln(x+2) - x + 2 \int \frac{1}{x+2} dx$$

$$x \ln(x+2) - x + 2 \ln(x+2) + C$$

$$(x+2) \ln(x+2) - x + C$$

→ OR

$$\frac{x}{x+2} = 1 + \frac{-2}{x+2}$$

$$\begin{array}{r} x+2 \overline{) x} \\ \underline{-(x+2)} \\ -2 \end{array}$$

or

$$\begin{array}{l} u = \ln(x+2) \quad dv = dx \\ du = \frac{dx}{x+2} \quad v = x+2 \end{array}$$

$$\int \ln(x+2) dx = (x+2) \ln(x+2) - \int \frac{x+2}{x+2} dx$$

$$= (x+2) \ln(x+2) - \int 1 dx$$

$$= (x+2) \ln(x+2) - x + C$$

100 Integrals - # 35

5.

$$\int \sec^3 x \tan^3 x \, dx = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

$$\left. \begin{array}{l} \text{let } s = \sec x \\ ds = \sec x \tan x \, dx \end{array} \right\}$$

$$\left. \begin{array}{l} \text{let } t = \tan x \\ dt = \sec^2 x \, dx \end{array} \right\}$$

↑ doesn't work.

$$\int \sec^2 x \tan^2 x [\sec x \tan x \, dx]$$

$$\int \sec^2 x (\sec^2 x - 1) [\sec x \tan x \, dx]$$

$$\int (\sec^4 x - \sec^2 x) [\sec x \tan x \, dx]$$

$$\int (s^4 - s^2) ds$$

$$\frac{1}{5} s^5 - \frac{1}{3} s^3 + C$$

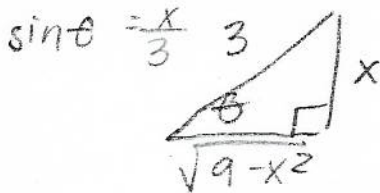
$$\frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

100 Integrals # 26

6.

$$\int \frac{x^2}{\sqrt{9-x^2}} dx = \frac{9}{2} \sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C$$

$$\begin{aligned} \text{let } x &= 3\sin\theta \\ dx &= 3\cos\theta d\theta \end{aligned}$$



$$\theta = \sin^{-1}\left(\frac{x}{3}\right)$$

$\frac{9}{3}$

$$\int \frac{9\sin^2\theta \cdot 3\cos\theta d\theta}{\sqrt{9-9\sin^2\theta}}$$

$$9 \int \frac{\sin^2\theta \cos\theta d\theta}{\sqrt{1-\sin^2\theta}}$$

$$9 \int \frac{\sin^2\theta \cos\theta d\theta}{\cancel{\cos\theta}}$$

$$9 \int \sin^2\theta d\theta$$

$$9 \int \frac{1-\cos 2\theta}{2} d\theta$$

$$\frac{9}{2} \int (1-\cos 2\theta) d\theta$$

$$\frac{9}{2}\theta - \frac{9}{2} \int \cos 2\theta d\theta$$

$$\frac{9}{2}\theta - \frac{9}{2} \cdot \frac{1}{2} \sin 2\theta + C$$

$$\frac{9}{2}\theta - \frac{9}{4} \sin 2\theta + C$$

$$\frac{9}{2}\theta - \frac{9}{2} \sin\theta \cos\theta + C$$

$$\frac{9}{2} \sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2} \left[\frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3} \right] + C$$

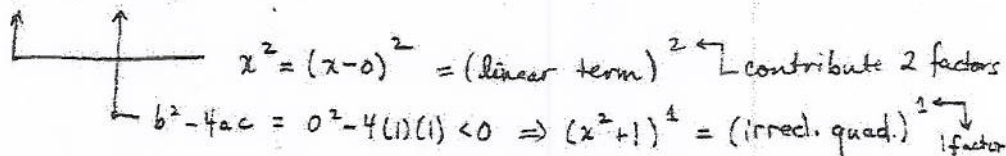
$$\frac{9}{2} \sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2} x\sqrt{9-x^2} + C$$

An example from class.

$$7. \int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} dx = 2 \ln|x| + \frac{1}{x} + \frac{3}{2} \ln|x^2+1| - 2 \tan^{-1} x + K$$

• bigger bottoms? **Yes**/No [degree num] = 3 < 4 = [degree den].

• $x^4 + x^2 = x^2(x^2+1)$



$$\frac{5x^3 - 3x^2 + 2x - 1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} = \frac{Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2}{x^2(x^2+1)}$$

→ should always be the same ←

⇒ $5x^3 - 3x^2 + 2x - 1 = Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2$ $x=0 \Rightarrow -1 = B$

Equate coefficients (short way) ← do on board for them

$$\begin{array}{l} x^3 : 5 = A + C \\ x^2 : -3 = B + D \\ x : 2 = A \\ \text{constant} : -1 = B \end{array} \Rightarrow \begin{array}{l} C = 3 \\ D = -2 \end{array}$$

so

$$\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} = \int \left[\frac{2}{x} + \frac{-1}{x^2} + \frac{3x-2}{x^2+1} \right] dx$$

$u = x^2 + 1$
 $du = 2x dx$

$$= 2 \int \frac{dx}{x} - 1 \int x^{-2} dx + 3 \int \frac{1}{2} (x^2+1)^{-1} \frac{(2x dx)}{du} - 2 \int \frac{dx}{x^2+1}$$

$$= 2 \ln|x| - \frac{x^{-1}}{-1} + \frac{3}{2} \ln|x^2+1| - 2 \tan^{-1} x + K$$

Homework § 8.8 # 57

8. LaPlace Transform (from a homework problem)

A **transform** is a formula that converts, or *transforms*, one function into another function.

Consider a function of t , denoted by $y = f(t)$. The **LaPlace Transform** of this function $y = f(t)$ is a (new) function, namely the function

$$y = \mathcal{L}\{f(t)\}(s),$$

which is a function of s . The formula for the LaPlace Transform of $y = f(t)$ is

$$\mathcal{L}\{f(t)\}(s) = \int_{t=0}^{t=\infty} e^{-st} f(t) dt. \quad (8)$$

where, in the integral in (8) above, s is treated as a constant.

The LaPlace Transform of the function

$$f(t) = e^{2t}$$

is the function

$$\mathcal{L}\{f(t)\}(s) = \frac{1}{s-2} \quad \text{for } s > 2$$

Hint: thus, if $f(t) = e^{2t}$, then by equation (8),

$$\mathcal{L}\{f(t)\}(s) = \int_{t=0}^{t=\infty} e^{-st} f(t) dt = \int_{t=0}^{t=\infty} e^{-st} e^{2t} dt$$

$$\int_{t=0}^{t=\infty} e^{-st} e^{2t} dt = \int_{t=0}^{t=\infty} e^{(2-s)t} dt = \lim_{b \rightarrow \infty} \int_{t=0}^{t=b} e^{(2-s)t} dt$$

$$\boxed{u = (2-s)t} \quad \begin{matrix} du = (2-s)dt \\ \hookrightarrow \lim_{b \rightarrow \infty} \frac{e^{(2-s)t}}{(2-s)} \Big|_{t=0}^{t=b} \end{matrix}$$

$$= \lim_{b \rightarrow \infty} \left[\frac{e^{(2-s)b}}{(2-s)} - \frac{e^0}{(2-s)} \right] = \left[\lim_{b \rightarrow \infty} \frac{e^{(2-s)b}}{2-s} + \lim_{b \rightarrow \infty} -\frac{1}{(2-s)} \right]$$

$$= \left[\frac{1}{2-s} \lim_{b \rightarrow \infty} e^{(2-s)b} \right] + \frac{-1}{2-s}$$

$$= \frac{1}{2-s} \cdot 0 + \frac{-1}{2-s}$$

$$= \frac{1}{s-2}$$

given $s > 2$
 so $0 > 2-s$
 so//
 if $b \rightarrow \infty$
 then $(2-s)b \rightarrow -\infty$
 and $e^{(2-s)b} \rightarrow 0$