MARK BOX		
PROBLEM	POINTS	
1	5	
2	5	
3	5	
TOTAL	15	

NAME (legibly printed): Key

class PIN: ____________

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): § 11.1, 11.2, 11.3 .

Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.
I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : ____________

1. Consider the point, in **polar coordinates**,

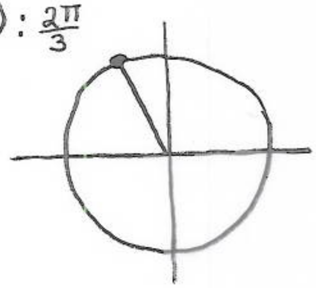
$$P = (r\,,\,\theta) = \left(4, \frac{2\pi}{3}\right) .$$

In **cartesian coordinates**, the point P is given by

$$P = (x, y) = (\underline{\ -2\ }\,,\ \underline{\ 2\sqrt{3}\ }) .$$

Below graph, and CLEARLY label, the following points.

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) : \frac{2\pi}{3}$



$$P = \left(4, \frac{2\pi}{3}\right)$$

$$Q = \left(-4, \frac{2\pi}{3}\right)$$

$$R = \left(4, -\frac{2\pi}{3}\right)$$

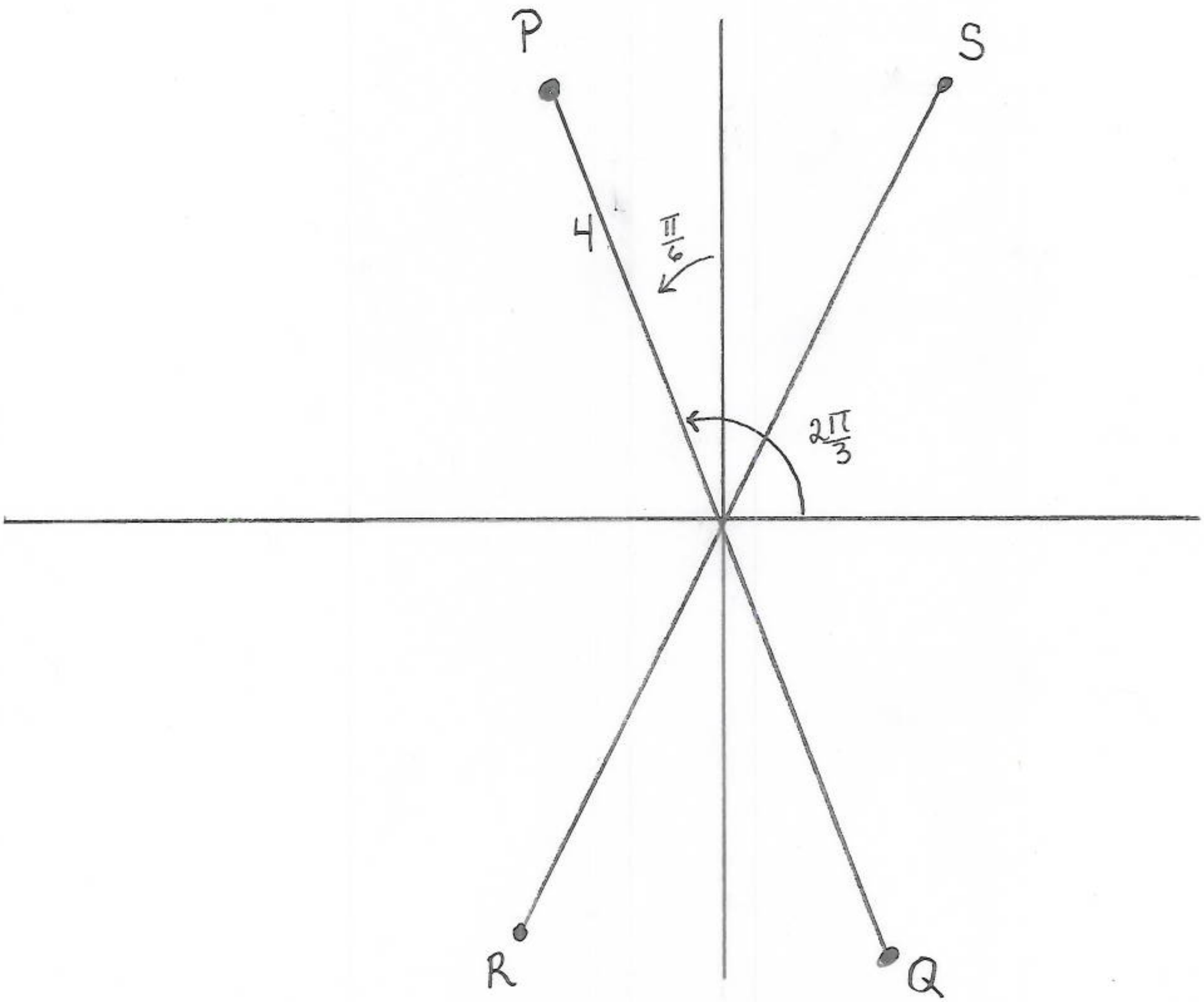
$$S = \left(-4, -\frac{2\pi}{3}\right) .$$

$x = r\cos\theta = 4\cos\frac{2\pi}{3}$

$= 4\left(-\frac{1}{2}\right) = -2$

$y = r\sin\theta = 4\sin\frac{2\pi}{3}$

$= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$



2. Consider the curve in polar coordinate

$\boxed{\S 11.1 \quad \# 42}$

$$r^2 = 9\sin(2\theta) \ .$$

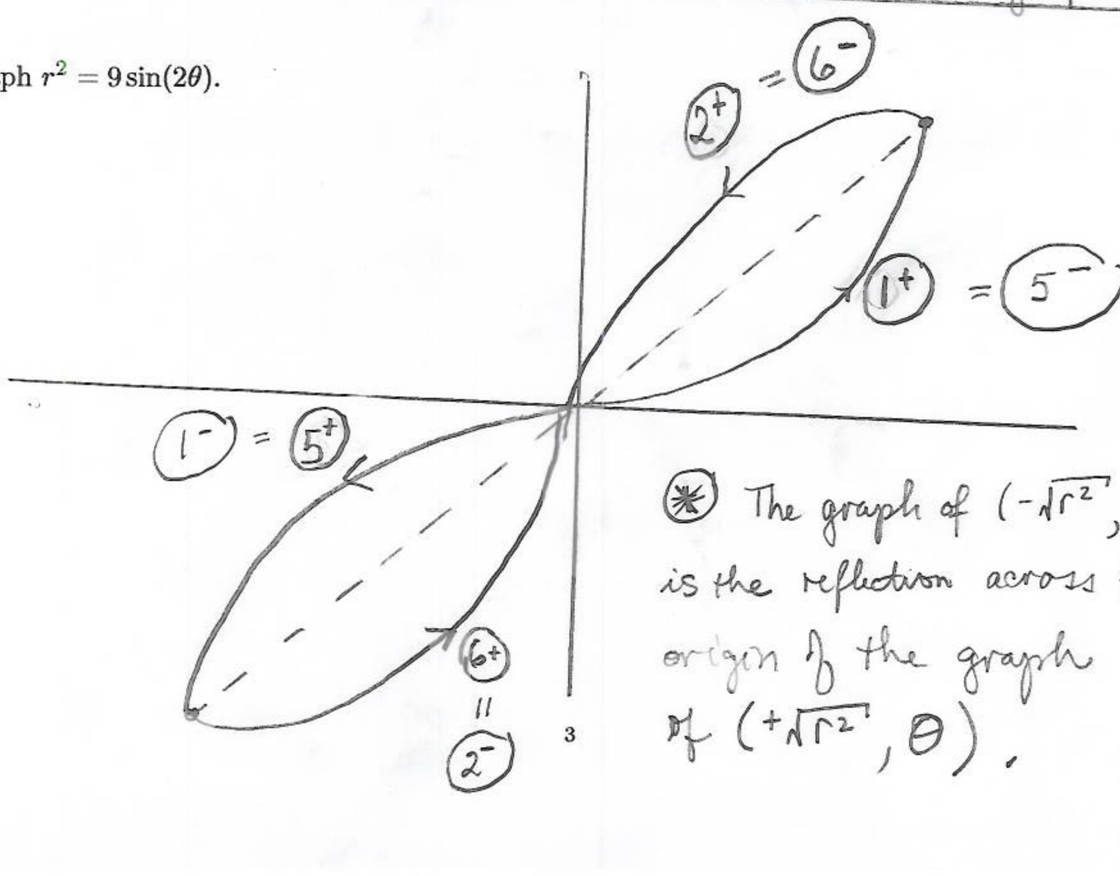
2a. The period of $r^2 = 9\sin(2\theta)$ is $\underline{\frac{2\pi}{2} = \pi}$.

2a. $\frac{\text{the period of } r^2 = 9\sin(2\theta)}{4} = \underline{\frac{\pi}{4}}$

2c. Make a chart, as we did in class, to help you graph $r^2 = 9\sin(2\theta)$.

	θ	2θ	$r^2 = 9\sin 2\theta$	$r = +\sqrt{9\sin 2\theta}$	$r = -\sqrt{9\sin 2\theta}$
1	$0 \to \frac{\pi}{4}$	$0 \to \frac{\pi}{2}$	$0 \to 9$	$0 \to 3$	$0 \to -3$
2	$\frac{\pi}{4} \to \frac{\pi}{2}$	$\frac{\pi}{2} \to \pi$	$9 \to 0$	$3 \to 0$	$-3 \to 0$
3	$\frac{\pi}{2} \to \frac{3\pi}{4}$	$\pi \to \frac{3\pi}{2}$	$0 \to -9$	no graph	no graph
4	$\frac{3\pi}{4} \to \pi$	$\frac{3\pi}{2} \to 2\pi$	$-9 \to 0$	no graph	no graph
5	$\pi \to \frac{5\pi}{4}$	2π ...	$0 \to 9$	$0 \to 3$	$0 \to -3$
6	$\frac{5\pi}{4} \to \frac{3\pi}{2}$	just go around unit circle again	$9 \to 0$	$3 \to 0$	$-3 \to 0$
7	$\frac{3\pi}{2} \to \frac{7\pi}{4}$		$0 \to -9$	no graph	no graph
8	$\frac{7\pi}{4} \to 2\pi$		$-9 \to 0$	no graph	no graph

2d. Graph $r^2 = 9\sin(2\theta)$.



✳ The graph of $(-\sqrt{r^2}, \theta)$ is the reflection across the origin of the graph of $(+\sqrt{r^2}, \theta)$.

3. Express the area enclosed by $r^2 = 9\sin(2\theta)$ as an integral with respect to θ (ok ... with respect to θ means a $d\theta$ in there).
(You do not have to evaluate this integral.)

area =

$$\text{Area} = \frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} [f(\theta)]^2 \, d\theta$$

$$= \frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} 9\sin(2\theta)\, d\theta$$

... many answers from here, eg:

$$\text{Area} = 2 \cdot \frac{1}{2}\int_{\theta=0}^{\theta=\frac{\pi}{2}} 9 \sin 2\theta \, d\theta$$

$$\overset{\text{or}}{=} 4 \cdot \frac{1}{2}\int_{\theta=0}^{\theta=\frac{\pi}{4}} 9 \sin 2\theta \, d\theta$$