Prof. Girardi | Math 142 | Fall 2008 | 12.08.08 | Final Exam

| MARK BOX | | |
|---|---|---|
| PROBLEM | POINTS | |
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 5 | |
| 6 | 5 | |
| 7 | 5 | |
| 8 | 5 | |
| 9 | 5 | |
| 10 | 5 | |
| 11 | 5 | |
| 12 | 5 | |
| 13 | 5 | |
| 14 | 5 | |
| 15 | 5 | |
| 16 | 5 | |
| 17 | 5 | |
| take home | 30 | |
| % | 100 | |

**NAME** (legibly printed): Integration Moose

**class PIN:** ______________________

**INSTRUCTIONS**:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
   (b) if a line/box is provided, then:
      — show you work BELOW the line/box
      — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8$^{\text{th}}$ ed.): Ch. 7, 8, 10, § 11.1 - 11.3 .

**Problem Inspiration**: See the answer key.

### Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

From Exam 1 this year.

**1. Fill in the blanks.**

**1a.** $\int \frac{du}{u} =$ ln $|u| + C$

**1b.** If $a$ is a constant and $a > 0$ but $a \neq 1$, then $\int a^u\, du =$ $\frac{a^u}{\ln a}$ $+ C$

**1c.** $\int \cos u\, du =$ $\sin u$ $+ C$

**1d.** $\int \sec^2 u\, du =$ $\tan u$ $+ C$

**1e.** $\int \sec u \tan u\, du =$ $\sec u$ $+ C$

**1f.** $\int \sin u\, du =$ $-\cos u$ $+ C$

**1g.** $\int \csc^2 u\, du =$ $-\cot u$ $+ C$

**1h.** $\int \csc u \cot u\, du =$ $-\csc u$ $+ C$

**1i.** $\int \tan u\, du =$ $-\ln|\cos u| + C \overset{\text{or}}{=} \ln|\sec u| + C$ $\cancel{+C}$

**1j.** $\int \cot u\, du =$ $\ln|\sin u| + C \overset{\text{or}}{=} -\ln|\csc u| + C$ $\cancel{+C}$

**1k.** $\int \sec u\, du =$ $\ln|\sec u + \tan u| + C \overset{\text{or}}{=} -\ln|\sec u - \tan u| + C$ $\cancel{+C}$

**1l.** $\int \csc u\, du =$ $-\ln|\csc u + \cot u| + C \overset{\text{or}}{=} \ln|\csc u - \cot u| + C$ $\cancel{+C}$

**1m.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}}\, du =$ $\arcsin \frac{u}{a}$ $+ C$

**1n.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2}\, du =$ $\frac{1}{a} \arctan \frac{u}{a}$ $+ C$

**1o.** If $a$ is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}}\, du =$ $\frac{1}{a} \operatorname{arcsec} \frac{|u|}{a}$ $+ C$

**1p.** Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where $f$ and $g$ are polyonomials and [degree of $f$] $\geq$ [degree of $g$], then one must first do long division

**1q.** Integration by parts formula: $\int u\, dv =$ $uv - \int v\, du$

**1r.** Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u =$ $a \sin\theta$

**1s.** Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u =$ $a \tan\theta$

**1t.** Trig substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u =$ $a \sec\theta$

**1u.** trig formula ... your answer should involve trig functions of $\theta$, and not of $2\theta$: $\sin(2\theta) =$ $2\sin\theta\cos\theta$.

**1v.** trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$.

**1w.** trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$.

**1x.** trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is $1 + \tan^2\theta = \sec^2\theta$.

**1y.** $\arcsin(-\frac{1}{2}) =$ $\frac{-\pi}{6}$ **RADIANS**. (your answer should be an angle)

From Exam 2 this year.

**2.** Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

**Hint: I do NOT want to see the words absolute nor conditional on this page!**

**2a.** $n^{\text{th}}$**-term test** for an arbitrary series $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ diverges.

**2b. Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ $< 1$
- diverges if and only if $|r|$ $\geq 1$

**2c.** $p$**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if $p$ $> 1$
- diverges if and only if $p$ $\leq 1$

**2d. Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f(n)$ for each $n \in \mathbb{N}$
- $f$ is a positive function
- $f$ is a continuous function
- $f$ is a nonincreasing (or decreasing) function .

Then $\sum a_n$ converges if and only if $\int_1^\infty f(x)\,dx$ converges.

**2e. Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

**2f. Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If $0 < L < \infty$, then $\sum a_n$ converges if and only if $\sum b_n$ converges

**2g. Ratio and Root Tests** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If $\rho$ $< 1$ then $\sum a_n$ converges.
- If $\rho$ $> 1$ then $\sum a_n$ diverges.
- If $\rho$ $= 1$ then the test is inconclusive.

**2h. Alternating Series Test** for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- $a_n$ $>$ $a_{n+1}$ for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n = 0$

then $\sum (-1)^n a_n$ converges

**2i.** By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ converges
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ converges and $\sum |a_n|$ diverges
- $\sum a_n$ is divergent if and only if $\sum a_n$ diverges

**2j.** If a power series in $x - x_0$ has radius of convergence $R$ where $0 < R < \infty$, then the power series is:

- absolutely convergent for $|x - x_0| < R$ i.e. $(x_0 - R, x_0 + R)$
- divergent for $|x - x_0| > R$ i.e. $(-\infty, x_0 - R) \cup (x_0 + R, \infty)$.

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**3. Fill-in the blanks/boxes** . — From Exam 3 this year.

- **In 3a and 3e, fill in the blank with: perpendicular or parallel.**
- **In 3b, 3c, 3d, 3e, 3f, fill in the blank with a formula involving *some of*:**

**2 , $\pi$ , radius , radius$_{\text{big}}$ , radius$_{\text{little}}$ , average radius , height , and/or thickness.**

- **Disk/Washer Method**

Let's say you revolve some region in the $xy$-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the disk or washer method.

**3a.** You should partition the coordinate axis (i.e., the $x$-axis or the $y$-axis) that is parallel to the axis of revolution.

**3b.** If you use the **disk method**, then the volume of a typical disk is:

$\pi(\text{radius})^2(\text{height})$.

**3c.** If you use the **washer method**, then the volume of a typical washer is:

$\pi(\text{rad}_{\text{big}})^2(\text{height}) - \pi(\text{rad}_{\text{little}})^2(\text{height}) \overset{\text{or}}{=} \pi(\text{rad}_{\text{big}}^2 - \text{rad}_{\text{little}}^2)(\text{height})$.

**3d.** If you partition the $z$-axis, the $\Delta z =$ height.

- **Shell Method**

Let's say you revolve some region in the $xy$-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the shell method.

**3e.** You should partition the coordinate axis (i.e., the $x$-axis or the $y$-axis) that is perpendicular to the axis of revolution.

**3f.** If you use the **shell method**, then the volume of a typical shell is:

$2\pi(\text{average radius})(\text{height})(\text{thickness})$.

**3g.** If you partition the $z$-axis, the $\Delta z =$ thickness $\overset{\text{or}}{=}$ $\text{radius}_{\text{big}} - \text{radius}_{\text{little}}$.

- **Arc Length**

**3h.** The arc length $L$ of a smooth curve $y = f(x)$ over the interval $[a, b]$ is defined by the following definite integral.

$$L = \int_{x=a}^{x=b} \sqrt{1+[f'(x)]^2}\, dx \overset{\text{or}}{=} \int_{x=a}^{x=b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx.$$

**3i.** The arc length $L$ of a smooth curve $x = g(y)$ over the interval $[c, d]$ is defined by the following definite integral.

$$L = \int_{y=c}^{y=d} \sqrt{1+[g'(y)]^2}\, dy \overset{\text{or}}{=} \int_{y=c}^{y=d} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy.$$

**3j.** The arc length $L$ of a curve that is parametrized by

$$x = x(t) \quad , \quad y = y(t) \qquad (a \leq t \leq b)$$

such that no segment of the curve is traced more than once as $t$ increases from $a$ to $b$ and also $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous functions for $a \leq t \leq b$, is defined by the following definite integral.

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

- **Average Value of a Function**

If $y = f(x)$ is continuous on the interval $[a, b]$, the the average value $f_{\text{ave}}$ of $y = f(x)$ on $[a, b]$ is defined to be

$$f_{\text{ave}} = \boxed{\frac{\int_a^b f(x)\, dx}{b-a}}.$$

- **Work**

**3k.** Suppose that Integration-Moose moves in the positive direction along a coordinate line over the interval $[a, b]$ while subjected to a variable force $F(x)$ that is applied in the directins of the motion. The work $W$ preformed by the force on Integration-Moose is defined by the following definite integral.

$$W = \int_{x=a}^{x=b} F(x)\, dx.$$

**3l.** Circle the scenario in which you perform more work:

(1) by raising a cup of coffee from a table to your mouth

(2) by holding a calculus textbook at shoulder level for 5 minutes.

Example from Class.

**4.** $\boxed{\int x^2 e^{(x^3)}\,dx = \frac{1}{3} e^{x^3} + \text{C}}$

$$\boxed{\begin{aligned} u &= x^3 \\ du &= 3x^2\,dx \end{aligned}}$$

$$\int x^2 e^{x^3}\,dx = \frac{1}{3}\int e^{x^3}\,3x^2\,dx$$

$$= \frac{1}{3}\int e^u\,du$$

$$= \frac{1}{3} e^u + C$$

$$= \frac{1}{3} e^{x^3} + C$$

100 $\int$'s # 5.

**5.** $\boxed{\int \frac{\tan x}{cos^2 x}\,dx = \frac{\sec^2 x}{2} + C \quad \underline{\underline{or}} \quad \frac{1}{2\cos^2 x} + C}$

Hint: integration buddies .

$$\int \frac{\tan x}{\cos^2 x}\,dx = \int \frac{\sin x}{\cos x}\,\frac{1}{\cos^2 x}\,dx = \int \frac{\sin x\,dx}{\cos^3 x}$$

$$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\,dx \end{array}} \qquad = -\int \frac{1}{u^3}\,du = -\int u^{-3}\,du$$

$$= -\frac{u^{-2}}{-2} + c = \frac{1}{2u^2} = \frac{1}{2\cos^2 x} \quad \underline{\underline{or}} \quad \frac{\sec^2 x}{2}$$

Example from class

**6.** $\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2 e^x + C$

| | |
|---|---|
| $u = x^2$ $\quad dv = e^x \, dx$<br>$du = 2x \, dx$ $\quad v = e^x$ | $\tilde{u} = x$ $\quad d\tilde{v} = e^x \, dx$<br>$d\tilde{u} = dx$ $\quad \tilde{v} = e^x$ |

$$\int x^2 e^x \, dx = x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 \left[ x e^x - \int e^x \, dx \right]$$

$$= x^2 e^x - 2x e^x + 2 \int e^x \, dx$$

Practice Problems #2h.

**7.**

$$\boxed{\int e^{5x}\cos x\,dx = \frac{e^{5x}}{26}\left(\sin x + 5\cos x\right) + C}$$

Hint: bring to the other side idea .

WAY #1

| | | | |
|---|---|---|---|
| $u = e^{5x}$ | $dv = \cos x\,dx$ | $\hat{u} = e^{5x}$ | $dv = \sin x\,dx$ |
| $du = 5e^{5x}\,dx$ | $v = \sin x$ | $d\hat{u} = 5e^{5x}\,dx$ | $v = -\cos x$ |

$$\int e^{5x}\cos x\,dx = e^{5x}\sin x - 5\int e^{5x}\sin x\,dx$$

$$= e^{5x}\sin x - 5\left[-e^{5x}\cos x - -5\int e^{5x}\cos x\,dx\right]$$

$$= e^{5x}\sin x + 5e^{5x}\cos x - 25\int e^{5x}\cos x\,dx$$

$$26\int e^{5x}\cos x\,dx = e^{5x}\sin x + 5e^{5x}\cos x \qquad (+K)$$

WAY #2

| | | | |
|---|---|---|---|
| $u = \cos x$ | $dv = e^{5x}\,dx$ | $u = \sin x$ | $dv = e^{5x}\,dx$ |
| $du = -\sin x\,dx$ | $v = \frac{1}{5}e^{5x}$ | $du = \cos x\,dx$ | $v = \frac{1}{5}e^{5x}$ |

$$\int e^{5x}\cos x\,dx = \frac{1}{5}e^{5x}\cos x - -\frac{1}{5}\int e^{5x}\sin x\,dx$$

$$= \frac{1}{5}e^{5x}\cos x + \frac{1}{5}\left[\frac{1}{5}e^{5x}\sin x - \frac{1}{5}\int e^{5x}\cos x\,dx\right]$$

$$= \frac{1}{5}e^{5x}\cos x + \frac{1}{25}e^{5x}\sin x - \frac{1}{25}\int e^{5x}\cos x\,dx$$

$$\frac{26}{25}\int e^{5x}\cos x\,dx = \frac{1}{5}e^{5x}\cos x + \frac{1}{25}e^{5x}\sin x \qquad +K$$

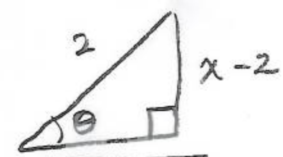
$$26\int e^{5x}\cos x\,dx = 5e^{5x}\cos x + e^{5x}\sin x \qquad +\tilde{K}$$

100 $\int$'s #40.

**8.** $$\boxed{\int \frac{x}{\sqrt{4x-x^2}}\,dx = 2\arcsin\left(\frac{x-2}{2}\right) - \sqrt{4x-x^2} + \mathrm{C}}$$

Hint: $4x - x^2 = 4 - (x-2)^2$

$$\boxed{\begin{aligned} x-2 &= 2\sin\theta \\ x &= 2+2\sin\theta \\ dx &= 2\cos\theta\, d\theta \end{aligned}} \longrightarrow \sin\theta = \frac{x-2}{2}$$




$$\sqrt{2^2-(x-2)^2} = \sqrt{4x-x^2}$$

$$4x-x^2 = 4-(x-2)^2 = 4-(2\sin\theta)^2 = 4-4\sin^2\theta = 4(1-\sin^2\theta) = 4\cos^2\theta$$

$$\int \frac{x\ dx}{\sqrt{4x-x^2}} = \int \frac{(2+2\sin\theta)(2\cos\theta\, d\theta)}{\sqrt{4\cos^2\theta}} = \frac{2\cdot 2}{\sqrt{4}} \int \frac{(1+\sin\theta)(\cos\theta)\, d\theta}{\cos\theta}$$

$$= 2\int (1+\sin\theta)\, d\theta$$

$$= 2\left[\theta - \cos\theta\right] + C$$

$$= 2\theta - 2\cos\theta + C$$

$$\downarrow$$

$$\cos\theta = \frac{\sqrt{4x-x^2}}{2}$$

**9.** $\boxed{\displaystyle\int \frac{3x+5}{x^3-x^2-x+1}\,dx = \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{-4}{x-1} \qquad + \text{C}}$

Hint: $x^3 - x^2 - x + 1 = (x+1)(x-1)^2$

$$\frac{3x+5}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$\Rightarrow \quad 3x+5 = A(x-1)^2 + B(x-1)(x+1) + C(x+1)$$

$$x=1 \rightarrow 8 = 2C \Rightarrow \boxed{C=4}$$

$$x=-1 \rightarrow 2 = 4A \Rightarrow \boxed{A=\tfrac{1}{2}}$$

$$x^2 : \; 0 = A + B \qquad \Rightarrow \boxed{B = -\tfrac{1}{2}}$$

$$x : \; 3 = -2A + C$$

$$x^0 : \; 5 = A - B + C$$

$$\int \frac{3x+5}{x^3-x^2-x+1}\,dx = \int \left[ \frac{1}{2}\,\frac{1}{x+1} + \frac{-1}{2}\,\frac{1}{x-1} + 4(x-1)^{-2} \right] dx$$

$$= \frac{1}{2}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{4(x-1)^{-1}}{-1} + C$$

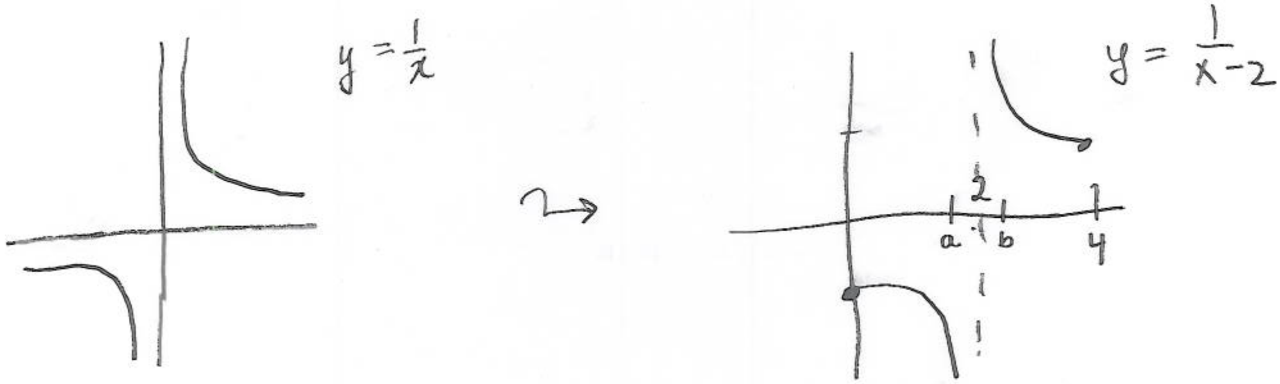
or

$$\frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|$$

Improper Integral

**10.** $\int_0^4 \frac{dx}{x-2} =$ DNE

HINT: first make a rough sketch of the function $f(x) = \frac{1}{x-2}$ on the interval $[0, 4]$.




$$\int_0^4 \frac{dx}{x-2} = \int_0^2 \frac{dx}{x-2} + \int_2^4 \frac{dx}{x-2}$$

$$= \lim_{a \to 2^-} \int_0^a \frac{dx}{x-2} + \lim_{b \to 2^+} \int_b^4 \frac{dx}{x-2}$$

$$= \left[ \lim_{a \to 2^-} \ln|x-2| \Big|_{x=0}^{x=a} \right] + \left[ \lim_{b \to 2^+} \ln|x-2| \Big|_{x=b}^{x=4} \right]$$

$$= \left[ \lim_{a \to 2^-} \ln|a-2| + \ln 2 \right] + \left[ \lim_{b \to 2^+} \ln 2 - \ln|b-2| \right]$$

DNE, $= -\infty$

DNE, $= +\infty$

Practice Problems # 3c.

**11.** Sequences

**11a.** $\dfrac{d}{dx}\left(\dfrac{x+2}{x+5}\right) =$

$$= \frac{(1)(x+5) - (x+2)(1)}{(x+5)^2} = \frac{x+5-x-2}{(x+5)^2} = \frac{3}{(x+5)^2}$$

**11b.** $\lim_{n\to\infty}\left(\frac{n+2}{n+5}\right)^n \;= e^{-3}$

$$y := \left(\frac{x+2}{x+5}\right)^x \xrightarrow{x\to\infty} \cancel{1^\infty} \text{ indeterminate form.}$$

$$\ln y = x \ln\left(\frac{x+2}{x+5}\right)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} x \ln\left(\frac{x+2}{x+5}\right) = \lim_{x\to\infty} \frac{\ln\left(\frac{x+2}{x+5}\right)}{x^{-1}} \quad \boxed{\begin{array}{c}\frac{0}{0}\\ \text{L'H}\\ \text{see 11a.}\end{array}}$$

$$= \lim_{x\to\infty} \frac{D_x \ln\left(\frac{x+2}{x+5}\right)}{D_x\ x^{-1}} = \lim_{x\to\infty} \frac{\frac{x+5}{x+2}\cdot\frac{3}{(x+5)^2}}{-x^{-2}}$$

$$= \lim_{x\to\infty} -3\ \frac{x^2}{(x+2)(x+5)} = -3.$$

Serious Series # 2

**12.** Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$$

- [ ] absolutely convergent
- [x] conditionally convergent
- [ ] divergent

- $\sum_{n=1}^{\infty} \left| (-1)^n \frac{1}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ divg. p-series. $p = \frac{1}{2} < 1$.

- $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ conv. by AST b/c

$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n a_n$ where $a_n = \frac{1}{\sqrt{n}}$

- $a_n$ dec? yes, b/c $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$.
- $\lim_{n \to \infty} a_n = 0$? yes, clear.

Series Series # 14

**13.** Let

$$a_n = \frac{3^n\ n!}{(2n)!}$$

**13a.** Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.
Hint: $(2\ (n+1)\ )! = (2n+2)!$

$$\boxed{\frac{a_{n+1}}{a_n} = \frac{3(n+1)}{(2n+1)(2n+2)} \quad \text{or} \quad \frac{3n+3}{4n^2+6n+2}}$$

$$\frac{a_{n+1}}{a_n} = \frac{3^{n+1}\,(n+1)!}{[2(n+1)]!}\ \frac{(2n)!}{3^n\, n!} = \frac{3^{n+1}}{3^n}\ \frac{(n+1)!}{n!}\ \frac{(2n)!}{(2n+2)!}$$

$$= \frac{3^n \cdot 3^1}{3^n}\ \frac{(n!)\,(n+1)}{(n!)}\ \cdot\ \frac{(2n)!}{(2n!)(2n+1)(2n+2)}$$

**13b.** Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n\ \frac{3^n\ n!}{(2n)!}$$

- [x] absolutely convergent
- [ ] conditionally convergent
- [ ] divergent

• abs. conv? Look at $\sum \left| (-1)^n \frac{3^n n!}{(2n)!} \right| = \sum \frac{3^n n!}{(2n)!}$

Try ratio test.

$$\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} \overset{(13a)}{=} \lim_{n\to\infty} \frac{3n+3}{4n^2+6n+2} = 0 < 1$$

$\Rightarrow$ conv

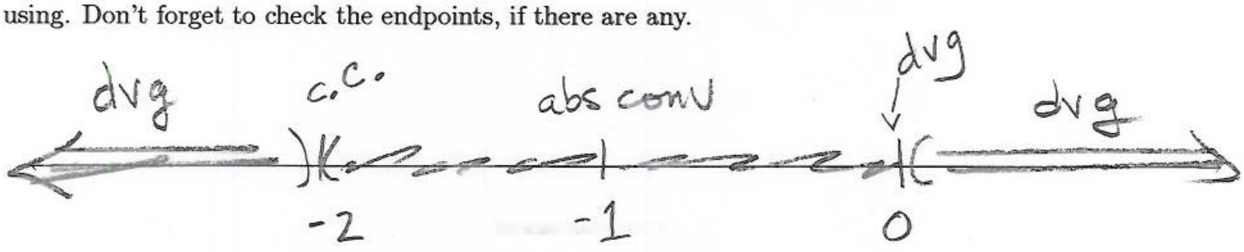
Homework § 10.8 # 41

**14.** Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n}$$

The center is $x_0 =$ $-1$ and the radius of convergence is $R =$ $1$.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.




$$\rho = \lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{n+1} \cdot \frac{n}{(x+1)^n} \right| = \lim_{n\to\infty} |x+1| \frac{n}{n+1} = |x+1| \lim_{n\to\infty} \frac{n}{n+1}$$

$$= |x+1| < 1$$

<u>Check endpts</u>

$\underline{x=0}$ $\quad \sum \frac{(x+1)^n}{n} = \sum \frac{1}{n}$. dvg, p-series, $p = 1 \le 1$

$\underline{x=-2}$ $\quad \sum \frac{(x+1)^n}{n} = \underbrace{\sum \frac{(-1)^n}{n}}$.

c.c.

Conv. by AST b/c
- $\{\frac{1}{n}\}$ dec.
- $\lim_{n\to\infty} \frac{1}{n} = 0$

Fall 06 Final Exam # 18.

~~18~~ 15. Using the fact that

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n \quad \text{when} \quad |r| < 1\,, \qquad (*)$$

find a power series expansion of

$$\frac{x}{4+100x^2}$$

and state when it is valid. Simplify your answer so that your power series has the form $\sum_{n=0}^{\infty} c_n x^{\text{some power}}$ for some constants $c_n$.

$$\boxed{\frac{x}{4+100x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n (25)^n}{4} x^{2n+1} \qquad \text{valid when } |x| < \frac{1}{5}}$$

$$\frac{x}{4+100x^2} = x\left[\frac{1}{4+100x^2}\right] = \frac{x}{4}\left[\frac{1}{1+25x^2}\right] = \frac{x}{4}\left[\frac{1}{1-(-25x^2)}\right]$$

$$= \frac{x}{4}\sum_{n=0}^{\infty}(-25x^2)^n = \sum_{n=0}^{\infty}\frac{x^1}{4}(-1)^n(25)^n x^{2n}$$

$$\boxed{r = -25x^2}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n(25)^n}{4}x^{2n+1}$$

valid $\iff |r| < 1 \iff |-25x^2| < 1$

$\iff 25|x|^2 < 1$

$\iff |x|^2 < \frac{1}{25}$

$\iff |x| < \frac{1}{5}$

Fall 08, Exam 3, #7

- Let $R$ be the region in the first quadrant enclosed by $y = 2x$ and $y = x^2$.

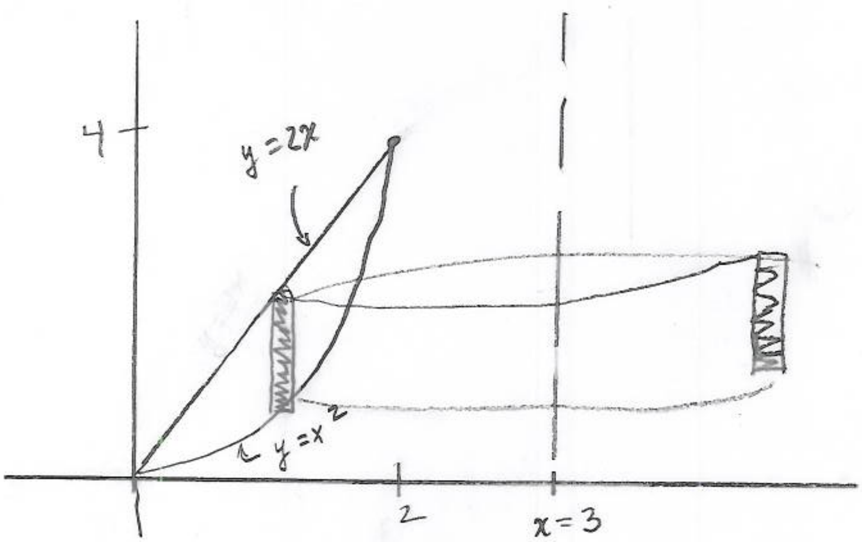
$2x = x^2$
$0 = x^2 - 2x = x(x-2)$
$x = 0 \rightarrow (0,0)$
$x = 2 \rightarrow (2,4)$

**16.** Express the area of $R$ as integral(s) with respect to $x$.

Area = $\displaystyle\int_{x=0}^{x=2} \left[ (2x) - (x^2) \right] dx$

**17.** Using the shell method, express as integral(s) the volume of the solid generated by revolving $R$ about the line $x = 3$.

Volume = $\displaystyle\int_{x=0}^{x=2} 2\pi (3-x)(2x-x^2)\, dx$




$V_{\text{typical shell}} = 2\pi\, (\text{avg. rad.})\, (\text{height})\, (\text{thickness})$

$\downarrow$

$\Delta x$