Prof. Girardi M	Iath 142 Fa	'all 2008	10.30.08	Exam 2 -	Take Home Part
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PROBLEM	POINTS	
a - j	10	
TOTAL	10	

NAME (legibly printed):	
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INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*; such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) This exam covers (from Calculus by Anton, Bivens, Davis 8th ed.): § 10.7, 10.9, 10.10.

Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature :		
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Taylor/Maclaurin Polynomials and Series

Do parts (a) - (j) for the following problem.

$$f(x) = \frac{1}{1+x}$$
 $x_0 = 4$ $J = (2,6)$.

You might find it easier to do problems (a) - (j) in a different order. Just do what you find easiest.

- On parts (a) (i), use ideas from only Sections 10.7 and 10.9, i.e., use only:
 - the definition of Taylor polynominal
 - the definition of Taylor series
 - the theorem/error-estimate on the N^{th} -Remainder term for Talyor polyominals.

Do **NOT** use a known Taylor Series (i.e., do not use methods from Section 10.10).

- On part (j), the very last part, use a known Taylor Series (as from the handout Commonly Used Taylor Series) and methods from Section 10.10.
- **a.** Find the following. Note the first column are functions of x and the second column are numbers.

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$
$f^{(4)}(x) =$	$f^{(4)}(x_0) =$

b. Find N^{th} -order Taylor polynomial of y = f(x) about x_0 in OPEN form for N = 0, 1, 2, 3, 4.

 $P_0(x) =$

 $P_1(x) =$

 $P_2(x) =$

 $P_3(x) =$

 $P_4(x) =$

c.	Find the Taylor series of $y = f(x)$ about x_0 in OPEN form.		
	$P_{\infty}(x) =$		

d. Find the Taylor series of y = f(x) about x_0 in CLOSED form.

$$P_{\infty}(x) =$$

e. Find the n^{th} Taylor coefficient of y = f(x) about x_0 .

$$c_n =$$

f. Find the interval I of convergence of the Taylor series y = f(x) about x_0 . Recall, the interval of convergence is the set of points for which the series converges, either absolutely or conditionally. (Hint: use the ratio or root test and then check the endpoints.)

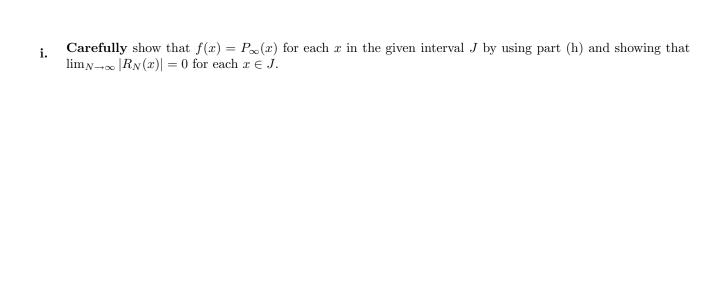
I =

g. Consider the given interval J and fix an $N \in \mathbb{N}$. Find a good upper bound for the maximum of $\left|f^{(N+1)}(c)\right|$ on the interval J. Your answer can have an N in it but it cannot have an: x, x_0, c . (Note that J is a subset of I but Prof. G. might have picked a smaller J than I to make the problem easier.)

$$\max_{c \in J} \left| f^{(N+1)}(c) \right| \le$$

h. Consider the given interval J and fix an $N \in \mathbb{N}$. For each $x \in J$, find a good upper bound for the maximum of $|R_N(x)|$. Your answer can have an N and x in it but it cannot have an: x_0 , c.

 $|R_N(x)| \le$



j. Using a known Taylor Series (as from the handout Commonly Used Taylor Series) and methods from Section 10.10, find a power series expansion (in CLOSED form) for y = f(x) about x_0 . Also, say when this power series expansion is valid by examining when the Commonly Used Taylor Series is valid. Show all your work and work in a logical fashion.

 $f(x) = \sum_{n=0}^{\infty}$ (x - 4)ⁿ which is valid for