MARK BOX		
PROBLEM	POINTS	
1 a–j	30	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
take home	10	
%	100	

NAME **(legibly printed):** ________________________

class PIN: ________________________

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 10.1 - 10.6, 10.8 for the inclass and Section 10.7. 10.9, 10.10 for the take home. .

Problem Inspiration: See the answer key.

1. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. n^{th}**-term test** for an arbitrary series $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ ___________________ .

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ ___________________
- diverges if and only if $|r|$ ___________________

1c. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p ___________________
- diverges if and only if p ___________________

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f\colon [1, \infty) \to \mathbb{R}$ be so that

- $a_n = f($_________$)$ for each $n \in \mathbb{N}$
- f is a ______________________________________ function
- f is a ______________________________________ function
- f is a ______________________________________ function .

Then $\sum a_n$ converges if and only if ______________________________________ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ___________________, then $\sum a_n$ ___________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ ___________________, then $\sum a_n$ ___________________.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If _________ $< L <$ _________, then $\sum a_n$ converges if and only if ___________________ .

1g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If ρ _________ then $\sum a_n$ converges.
- If ρ _________ then $\sum a_n$ diverges.
- If ρ _________ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum(-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ _________

then $\sum(-1)^n a_n$ ____________________________

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ __________________
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ _________ and $\sum |a_n|$ _________
- $\sum a_n$ is divergent if and only if $\sum a_n$ __________________

1j. If a power series in $x - x_0$ has radius of convergence R where $0 < R < \infty$, then the power series is:

- absolutely convergent for ____________________________________
- divergent for ____

2. Circle T if the statement is TRUE. Circle F if the statement if FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.
Scoreing: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

T F If a function $f\colon [0,\infty) \to \mathbb{R}$ satisfies that $\lim_{x\to\infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n, then $\lim_{n\to\infty} a_n = L$.

T F If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n\to\infty} a_n = L$ and $f\colon [0,\infty) \to \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n, then $\lim_{x\to\infty} f(x) = L$.

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum (a_n + b_n)$ converges.

T F If $\sum (a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $r \neq 1$ and $S_N = \sum_{n=17}^{N} r^n$, then $S_N = \dfrac{r^{17} - r^{N+1}}{1-r}$ for each $N > 17$.

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

3. Find the limit of the following **sequences**. Indicate your reasoning.
Put ANSWER IN BOX and show WORK BELOW BOX.
There is: 3a, 3b, 3c (on this page), and then 3d on the next page.

3a. $\lim_{n\to\infty} (.999917)^n =$

3b. $\lim_{n\to\infty} (1.0000017)^n =$

3c. $\lim_{n\to\infty} \dfrac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} =$

3d IS ON NEXT THE PAGE $\Longrightarrow$

3d. $$\lim_{n\to\infty}\left(\frac{n+6}{n+1}\right)^n \;=$$

Hint: if $\ln(a_n) \to 17$, then $a_n \equiv e^{\ln(a_n)} \to e^{17}$

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{17n^2+n-1}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4+1}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

6. Let

$$a_n = \frac{3^n \; n!}{(2n)!}$$

6a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a fractorial sign (that is a ! sign) in it.
Hint: $(2 \;\; (n+1) \;\;)! = (2n+2)!$

$\frac{a_{n+1}}{a_n} \quad =$

6b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \; \frac{3^n \; n!}{(2n)!}$$

- ☐ absolutely convergent
- ☐ conditionally convergent
- ☐ divergent

7. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n}$$

Hint: $(2x+6)^n \;=\; [2\,(x+3)]^n \;=\; 2^n\,(x+3)^n \;=\; 2^n\,(x-(-3))^n$

The center is $x_0 =$ ____________________ and the radius of convergence is $R =$ ____________________ .
As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.