Prof. Girardi	Math 142	Fall 2008	09.25.08	Exam 1

MARK BOX		
PROBLEM	POINTS	
1 a–y	25	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
take home	10	
Extra Credit	5	
%	100	

NAME (legibly printed): James Bond

class PIN: 007

(*) Extra Credit: 5 point for knowing yor PIN number.

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  (b) if a line/box is provided, then:
    — show you work BELOW the line/box
    — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.7, 8.8. .

Problem Inspiration: If I told you here, you would know what method to use. So see the solution key, which will be available from the course homepage shortly after the exam.

Hints:

(1) **You can check your answers to the indefinite integrals by differentiating.**

(2) **+ C**

(3) **For more partial credit, box your $u - du$ substitutions.**

Similar to problem #1 from last several semester I taught this class.

1. Fill in the blanks (each worth 1 point).

1a. $\int \frac{du}{u} =$ <u>$\ln$</u> $|u| + C$

1b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u \, du =$ <u>$\frac{a^u}{\ln a}$</u> $+C$

1c. $\int \cos u \, du =$ <u>$\sin u$</u> $+C$

1d. $\int \sec^2 u \, du =$ <u>$\tan u$</u> $+C$

1e. $\int \sec u \tan u \, du =$ <u>$\sec u$</u> $+C$

1f. $\int \sin u \, du =$ <u>$-\cos u$</u> $+C$

1g. $\int \csc^2 u \, du =$ <u>$-\cot u$</u> $+C$

1h. $\int \csc u \cot u \, du =$ <u>$-\csc u$</u> $+C$

1i. $\int \tan u \, du =$ <u>$-\ln|\cos u| + C$ or $\ln|\sec u| + C$</u> $\cancel{+C}$

1j. $\int \cot u \, du =$ <u>$\ln|\sin u| + C$ or $-\ln|\csc u| + C$</u> $\cancel{+C}$

1k. $\int \sec u \, du =$ <u>$\ln|\sec u + \tan u| + C$ or $-\ln|\sec u - \tan u| + C$</u> $\cancel{+C}$

1l. $\int \csc u \, du =$ <u>$-\ln|\csc u + \cot u| + C$ or $\ln|\csc u - \cot u| + C$</u> $\cancel{+C}$

1m. If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} \, du =$ <u>$\arcsin \frac{u}{a}$</u> $+C$

1n. If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} \, du =$ <u>$\frac{1}{a} \arctan \frac{u}{a}$</u> $+C$

1o. If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} \, du =$ <u>$\frac{1}{a} \operatorname{arcsec} \frac{|u|}{a}$</u> $+C$

1p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do <u>long division</u>

1q. Integration by parts formula: $\int u \, dv =$ <u>$uv - \int v \, du$</u>

1r. Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves $a^2 - u^2$, then one makes the substitution $u =$ <u>$a \sin\theta$</u>

1s. Trig substitution:
if the integrand involves $a^2 + u^2$, then one makes the substitution $u =$ <u>$a \tan\theta$</u>

1t. Trig substitution:
if the integrand involves $u^2 - a^2$, then one makes the substitution $u =$ <u>$a \sec\theta$</u>

1u. trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) =$ <u>$2 \sin\theta \cos\theta$</u>.

1v. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) =$ <u>$\frac{1 + \cos(2\theta)}{2}$</u>.

1w. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) =$ <u>$\frac{1 - \cos(2\theta)}{2}$</u>.

1x. trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is <u>$1 + \tan^2\theta = \sec^2\theta$</u>.

1y. $\arctan(-\sqrt{3}) =$ <u>$-\pi/3$ radians</u>. (your answer should be an angle)

$\frac{c \, \phi \, t}{\, \, n}$ $\Rightarrow -\frac{\pi}{2} < \arctan \# < \frac{\pi}{2}$

$\pi/3 \; (\frac{1}{2}, \frac{\sqrt{3}}{2}) \Rightarrow \tan\frac{\pi}{3} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

$\pi/6 \; (\frac{\sqrt{3}}{2}, \frac{1}{2}) \Rightarrow \tan\frac{\pi}{6} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

from 100 Integrals - # 35

2. $\boxed{\displaystyle\int (\sec^3 x)\,(\tan^3 x)\,dx = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x \qquad + C}$

$$\boxed{\begin{aligned} s &= \sec x \\ ds &= \sec x \tan x \, dx \end{aligned}} \qquad \cancel{\boxed{\begin{aligned} t &= \tan x \\ dt &= \sec^2 x \, dx \end{aligned}}}$$

$$\int \sec^3 x \tan^3 x \, dx$$

$$= \int \sec^2 x \tan^2 x \; [\sec x \tan x \, dx]$$

$$= \int \sec^2 x \, (\sec^2 x - 1) \; [\sec x \tan x \, dx]$$

$$= \int s^2 (s^2 - 1) \, ds$$

$$= \int (s^4 - s^2) \, ds$$

$$= \frac{s^5}{5} - \frac{s^3}{3} + C$$

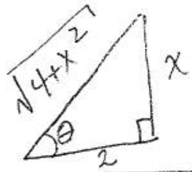
Homework problem §8.4 #5

3. $$\boxed{\int \frac{1}{(4+x^2)^2}\,dx = \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(4+x^2)} + C}$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$4+x^2 = 4+(2\tan\theta)^2 = 4+4\tan^2\theta = 4(1+\tan^2\theta) = 4\sec^2\theta$



$\tan\theta = \frac{x}{2}$

$$\int \frac{dx}{(4+x^2)^2} = \int \frac{2\sec^2\theta\, d\theta}{(4\sec^2\theta)^2} = \frac{2}{4^2}\int \frac{\sec^2\theta}{\sec^4\theta}\,d\theta = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta}$$

$$= \frac{1}{8}\int \cos^2\theta\, d\theta = \frac{1}{8}\cdot\frac{1}{2}\int [1+\cos 2\theta]\, d\theta$$

$$= \frac{1}{16}\int d\theta + \frac{1}{16}\,\frac{1}{2}\int \cos 2\theta\,(2\,d\theta)$$

$$= \frac{1}{16}\theta + \frac{1}{16}\cdot\frac{1}{2}\cdot\sin 2\theta + C$$

$$= \frac{1}{16}\theta + \frac{1}{16}\cdot\frac{1}{\cancel{2}}\cdot\cancel{2}\cdot\cos\theta\cdot\sin\theta + C$$

$$= \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{16}\,\frac{2}{\sqrt{4+x^2}}\cdot\frac{x}{\sqrt{4+x^2}} + C$$

from 100 Integrals # 71

4. $$\boxed{\int \frac{x^3+x^2+2x+1}{x^4+2x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + \frac{1}{2}\frac{(x^2+1)^{-1}}{-1} + C}$$

Hint: $x^4+2x^2+1 = (x^2+1)^2$.

bigger bottoms? yes ✓

$$\frac{x^3+x^2+2x+1}{x^4+2x^2+1} = \frac{x^3+x^2+2x+1}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)^1} + \frac{Cx+D}{(x^2+1)^2} = \frac{(Ax+B)(x^2+1)+Cx+D}{(x^2+1)^2}$$

$$\Rightarrow x^3+x^2+2x+1 = (Ax+B)(x^2+1) + Cx+D$$

x^3 : $1 = A$

x^2 : $1 = B$

x : $2 = A + C \longrightarrow C = 1$

constant : $1 = B + D \longrightarrow D = 0$

$$\int \frac{x^3+x^2+2x+1}{x^4+2x^2+1}\,dx = \int \left(\frac{1x+1}{x^2+1} + \frac{1x+0}{(x^2+1)^2}\right)dx$$

$$= \frac{1}{2}\int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1} + \frac{1}{2}\int (x^2+1)^{-2}(2x\,dx)$$

Homework Problem § 8.2 # 27.

5. $\boxed{\int x^3 e^{x^2}\,dx = \dfrac{x^2 e^{x^2} - e^{x^2}}{2} \qquad + C}$

$$\boxed{\begin{array}{ll} u = \frac{1}{2}x^2 & dv = e^{x^2}(2x)\,dx \\ du = x\,dx & v = e^{x^2} \end{array}}$$

$$\int x^3 e^{x^2}\,dx = \frac{x^2 e^{x^2}}{2} - \int e^{x^2} x\,dx$$

$$= \frac{x^2 e^{x^2}}{2} - \frac{1}{2}\int e^{x^2}(2x\,dx)$$

$$= \frac{x^2 e^{x^2}}{2} - \frac{1}{2} e^{x^2} + C$$

Homework Problem § 8.2 # 23

6. $\boxed{\int \sin(\ln x)\,dx = \frac{1}{2}\left(x\sin(\ln x) - x\cos(\ln x) \right) + C}$

Hint: bring to the other side idea.

$$\boxed{\begin{array}{ll} u = \sin(\ln x) & dv = dx \\ du = \dfrac{\cos(\ln x)}{x}\,dx & v = x \end{array}}$$

$$\boxed{\begin{array}{ll} \tilde{u} = \cos(\ln x) & d\tilde{v} = dx \\ d\tilde{u} = \dfrac{-\sin(\ln x)}{x}\,dx & \tilde{v} = x \end{array}}$$

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - \int \cos(\ln x)\,dx$$

$$= x\sin(\ln x) - \left[x\cos(\ln x) - -\int \sin(\ln x)\,dx \right]$$

$$= x\sin(\ln x) - x\cos(\ln x) - \int \sin(\ln x)\,dx$$

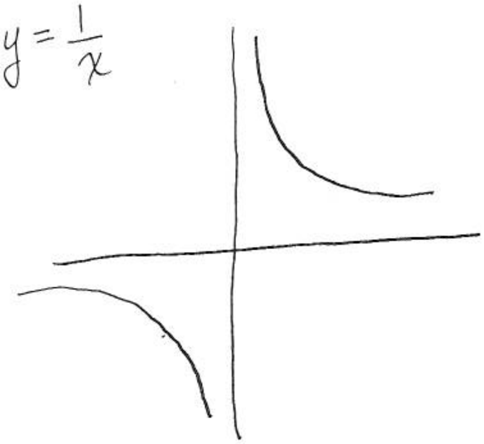
Hint above

$$2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + C$$

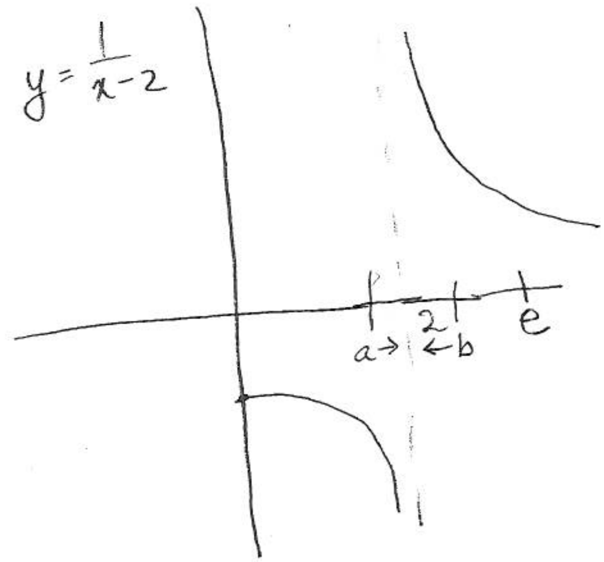
7. $\boxed{\displaystyle\int_0^e \frac{dx}{x-2} = \text{DNE}}$

HINT: $e \approx 2.7 > 2$

$y = \frac{1}{x}$



$2 \to$



$$\int_0^e \frac{dx}{x-2} = \lim_{a\to 2^+} \int_0^a \frac{dx}{x-2} + \lim_{b\to 2^-} \int_b^e \frac{dx}{x-2}$$

$$= \lim_{a\to 2^+} \ln|x-2| \Big|_{x=0}^{x=a} + \lim_{b\to 2^-} \ln|x-2| \Big|_{x=b}^{x=e}$$

$$= \lim_{a\to 2^+} \left[\ln|a-2| - \ln 2 \right] + \lim_{b\to 2^-} \left[\ln|e-2| - \ln|b-2| \right]$$

$$= \underbrace{\left[\lim_{a\to 2^+} \ln|a-2| \right]}_{= -\infty} - \ln 2 + \ln|e-2| - \underbrace{\lim_{b\to 2^-} \ln|b-2|}_{= -\infty}$$

MARK BOX		
PROBLEM	POINTS	
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TOTAL	10	

NAME (legibly printed): Key

class PIN: 007

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(3) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Section 8.7 .

Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.
I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : James Bond

7. Use inequality (11), page 563, to find an upper bound on the error in part 3. Do not use a calculator.

$$|T_6 - I| \le \frac{(4-1)^3}{12(6)^2} \cdot \frac{6}{2^4} \overset{or}{=} \frac{3}{2^7} \overset{or}{=} \frac{3}{128}$$

$$K_2 = \max_{1 \le x \le 4} |f^{(2)}(x)| = \max_{1 \le x \le 4} \frac{6}{(x+1)^4} = \frac{6}{2^4}$$

$$\Rightarrow 1 \le x \le 4 \Rightarrow 2 \le x+1 \le 5 \Rightarrow \frac{1}{5} \le \frac{1}{x+1} \le \frac{1}{2}$$

8. Use your calculator to approximate the error estimate in part 7 to as many decimal places as your calculator will give you.

$$|T_6 - I| \lessapprox 0.0234375$$

9. Use inequality (12), page 563 to find an upper bound on the error in part 5. Do not use a calculator.

$$|S_6 - I| \le \frac{(4-1)^5}{180(6)^4} \cdot \frac{120}{2^6} \overset{or}{=} \frac{1}{2^9}$$

$$K_4 = \max_{1 \le x \le 4} |f^{(4)}(x)| = \max_{1 \le x \le 4} \frac{120}{(x+1)^6} = \frac{120}{2^6}$$

10. Use your calculator to approximate the error iestimate in part 9, to as many decimal places as your calculator will give you.

$$|S_6 - I| \lessapprox 0.001953125$$

5. Approximate I using Simpon's Rule S_{2n} with $2n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

$$S_6 = \left(\frac{1}{3}\right)\left(\frac{4-1}{6}\right)\left[\frac{1}{2^2} + \frac{4}{(2.5)^2} + \frac{2}{3^2} + \frac{4}{(3.5)^2} + \frac{2}{4^2} + \frac{4}{(4.5)^2} + \frac{1}{5^2}\right]$$

6. Find an approximation for S_6, using part 5 and your calculator, to as many decimal places as your calculator will give you.

$$S_6 \approx 0.30021398498$$

***.** Find the first 4 derivatives of the given $y = f(x)$ that we are working with.

$$f(x) = (x+1)^{-2}$$

$$f^{(1)}(x) = -2(x+1)^{-3}$$

$$f^{(2)}(x) = 6(x+1)^{-4}$$

$$f^{(3)}(x) = -24(x+1)^{-5}$$

$$f^{(4)}(x) = 120(x+1)^{-6}$$

Numerical Integration . Let

$$f(x) = \frac{1}{(x+1)^2} \qquad \text{and} \qquad I = \int_{x=1}^{x=4} f(x)\, dx \ .$$

The 10 steps of this problem are similar to the homework but the number of subintervals is **6** and **not 10. On the parts that say "Do not use a calculator", you need not do alot of arthritic.**

1. Find the exact value of I, without using a calculator. Your answer should be a fraction, without decimal places.

$I = \frac{3}{10}$

$$\int_{x=1}^{x=4} (x+1)^{-2}\, dx = \frac{(x+1)^{-1}}{-1} \Big|_{x=1}^{x=4} = \frac{1}{x+1} \Big|_{x=4}^{x=1} = \frac{1}{2} - \frac{1}{5}$$

$$= \frac{5-2}{10} = \frac{3}{10}$$

2. Find an approximation for I, using part 1 and your calculator, to as many decimal places as your calculator will give you.

$I \approx .3$

3. Approximate I using the Trapezoid Rule T_n with $n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

$$T_6 = \frac{4-1}{2(6)} \left[\frac{1}{2^2} + \frac{2}{(2.5)^2} + \frac{2}{3^2} + \frac{2}{(3.5)^2} + \frac{2}{4^2} + \frac{2}{(4.5)^2} + \frac{1}{5^2} \right]$$

Divide [1, 4] into 6 subintervals $\Rightarrow \Delta x = \frac{b-a}{n} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2} = .5$

x_0	x_1	x_2	x_3	x_4	x_5	x_6
$a = 1$	$1\frac{1}{2} = 1.5 = \frac{3}{2}$	2	$2\frac{1}{2} = 2.5 = \frac{5}{2}$	3	$3\frac{1}{2} = 3.5 = \frac{7}{2}$	$4 = b$

4. Find an approximation for T_6, using part 3 and your calculator, to as many decimal places as your calculator will give you.

$T_6 \approx 0.3048132401$